Interaction of Nonlinear Oscillatory Systems with Energy Sources

A. A. Alifov
K. V. Frolov

E. Rivin, English Edition Editor

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A. A. Alifov
K. V. Frolov
USSR Academy of Sciences
Moscow

English Edition Editor
E. Rivin
Wayne State University

Translator
O. Efimov

HEMISPHERE PUBLISHING CORPORATION
A member of the Taylor & Francis Group
New York Washington Philadelphia London

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Originally published as Vzaimodeystviye nelineynykh kolebatel'nykh sistem s istochnikami energii by Nauka, Moscow, 1985.

1 2 3 4 5 6 7 8 9 0 B R B R 9 8 7 6 5 4 3 2 1 0

This book was set in Times Roman by Hemisphere Publishing Corporation. The production supervisor was Peggy Rote and the typesetter was Wayne Hutchins.
Cover design by Sharon Martin DePass.
Printing and binding by Braun-Brumfield, Inc.

A CIP catalog record for this book is available from the British Library.

Library of Congress Cataloging-in-Publication Data

Alifov, A. A. (Alishir Ali ogly)
[Vzaimodeĭstvie nelineĭnykh kolebatel'nykh sistem s istochnikami energii. English]
Interaction of nonlinear oscillatory systems with energy sources / A. A. Alifov, K. V. Frolov; English edition editor E. Rivin; translator O. Efimov.
p. cm.
Translation of: Vzaimodeĭstvie nelineĭnykh kolebatel'nykh sistem s istochnikami energii.

1. Vibration, Self-induced. 2. Nonlinear oscillations.
I. Frolov, K. V. II. Rivin, Eugene I. III. Title.
TA355.A48313 1990
620.3—dc20 89-26999
ISBN 0-89116-695-5 CIP

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PREFACE

This book is devoted to a systematic study of the theory of mixed oscillations of mechanical systems, whether or not the properties of the energy source supporting the oscillations are taken into account. The principal goal is to investigate the rich variety of physical phenomena made possible with mixed oscillations and the dynamic interaction of an oscillatory system with an energy source. An attempt is made to combine the theory of mixed oscillations with that of the dynamic interaction of the oscillatory system with the energy source.

Mixed oscillations result from the dynamic interaction of oscillations of various types in the same oscillatory system. Here, new phenomena arise which are essentially different from the case of generating isolated oscillations. With oscillation types, it is not always possible to predict mixed oscillatory process properties since the superposition principle, which is valid for linear systems, is inapplicable for nonlinear systems. In other words, it is impossible to determine the characteristics of mixed oscillatory processes by superposing their modes.

Mixed oscillatory processes were studied systematically in [8–10, 14–28]. The main part of this book which has arisen from these works is the analysis of mixed oscillations on the basis of a self-oscillatory system with a nonideal energy source, as studied by one of the authors [1–13]. It thus becomes possible to study various mixed oscillation classes with a uniform approach, and to demonstrate the general and individual properties of independent and dependent mechanisms of mixed oscillation excitation, ideal and nonideal energy sources, and linear and nonlinear elastic forces with and without a delay. Analysis is made by approximate analytical methods of nonlinear mechanics complemented by solutions of the corresponding nonlinear differential equations on an analog computer (AC). This enables us to compare approximate analytical results with AC simulation results, to discover new phenomena which are either difficult or impossible to reveal analytically, and to establish limits for the applicability of approximate solutions. It is not the analytical methods that are emphasized, but rather the phenomena which arise under the influence of various factors.

The book consists of an introduction, six chapters, a conclusion, four appendixes, and references. Classification of mixed oscillations and a brief survey of works on their analysis are given in the Introduction.

Chapter One is devoted to the mechanical frictional self-oscillatory system with one or many degrees of freedom and nonlinear elastic forces, and viscous and structural friction dampers. Equations correlating the amplitude of self-excited oscillations to the energy source velocity are derived. The influence of a structural friction damper on steady states of self-excited oscillations is studied, and the conditions for the elimination of undesirable self-excited oscillations are found. The stability of steady states of motion and the system transitions from one dynamic state to another, due to the specifics of the energy source, are analyzed.

Chapter Two is devoted to the study of interactions between forced and self-excited oscillations. The cases of dependent and independent oscillations are discussed. The properties of harmonic entrained and almost periodic oscillations generated by the dynamic interaction of the motor with an oscillatory system are demonstrated. A stability analysis of steady-state motion is made, and transitions from one steady state to another are considered.

Chapter Three describes interactions between parametric and self-excited oscillations in systems with one or many degrees of freedom and linear or nonlinear restoring forces. Systems with dependent and nondependent excitations are discussed. The following are also studied: harmonic oscillations; second order subharmonic oscillations; entrainment regions; steady-state oscillations and their stability; transitions between steady-state motions; and almost periodic oscillations. It is shown that, depending on the system parameters, resonant curves acquire a multitude of forms. Simple computational formulas are derived to select dampers for abating spurious oscillations.

Oscillatory processes arising from interactions of forced and parametric oscillations with self-excited oscillations are studied in Chapter Four, including the cases of independent and dependent excitations with linear and nonlinear elastic force characteristics. It turns out that, in such a system, two stable resonant solutions close in amplitude are possible, each arising from specific initial conditions. It is remarkable that their existence depends on the energy source velocity values involved in the expression for the friction force which causes self-excited oscillations. Approximate solutions are constructed in which the second order terms are calculated with respect to a small parameter. Analysis of quasi-stationary motions is made, enabling the discovery of some interesting phenomena.

The same systems as in Chapters 1–4, but taking the elastic force delay into account, are studied in Chapter Five. It is shown that, with continuous changes in delay, amplitude-frequency curves also undergo periodic changes. The influence of a delayed force leads to new phenomena impossible without a delay.

Chapter Six discusses nonlinear parametric and forced oscillations of systems interacting with two energy sources. Steady-state motions and their stability are considered. It is established that branches of well-known resonant curves, which are unstable for one energy source, may turn out to be stable for two sources. It is also shown that if the energy source power is insufficient, then the amplitude-frequency curve peak doubles.

The averaging method for investigating problems arising from the theory of dynamic interaction of oscillatory systems with energy sources is given in the Appendixes, as well as Routh-Hurwicz criteria in Kononenko form generalization for n-order systems, methods for AC modeling of oscillatory systems with limited excitation and for analysis of mixed oscillations, and the entrainment phenomenon for a nonideal energy source.

References include works devoted to oscillations in systems with limited excitation (they are marked by an asterisk). The authors apologize to researchers whose works are not included due to a lack of information. To facilitate reading, understanding the material, and comparing the results, unified notation is introduced whenever possible.

The authors would like to express their gratitude to N. V. Butenin and K. Sh. Khodzhayev for discussing the manuscript and giving their valuable criticisms. They are also thankful to the researchers, I. T. Chernyavsky and V. P. Gusev, of the A. A. Blagonravov Theory of Machines Institute of the USSR Academy of Sciences, for their assistance in modeling the problems on an AC.

A. A. Alifov
K. V. Frolov

INTRODUCTION

A sufficiently general classification of oscillatory processes is one with respect to an excitation mechanism. Accordingly, oscillatory processes can be divided into the following types: a) free oscillations; b) forced oscillations; c) parametric oscillations; and d) self-excited oscillations.

Mixed-type processes are also possible in physical systems. We will call them mixed. In other words, two or more oscillation types can interact in the same oscillatory system. The following classification of mixed oscillations [30] can then be made:

1. Class FP: the interaction of forced with parametric oscillations.
2. Class FS: the interaction of forced with self-excited oscillations.
3. Class PS: the interaction of parametric with self-excited oscillations.
4. Class FPS: the interaction of forced, parametric, and self-excited oscillations.

This designation of mixed oscillation classes consists of the initial letters of the terms denoting oscillation types. This notation also makes it easier to remember the classes of mixed oscillations and to compare the results of their analysis.

In many branches of modern technology, objects with mixed oscillatory processes are frequently encountered. Typical schemes of mechanical systems whose operation can be, under certain conditions, accompanied by mixed oscillations are given in Fig. 0.1.

The study of systems with mixed oscillatory processes is of important scientific and practical interest and has generated a number of papers. To see the problem clearly, we start with a brief survey of the situation that has evolved in the theory of oscillatory systems with ideal and nonideal energy sources.

Systems with ideal energy sources. Class FP. The work of V. V. Bolotin should be mentioned first. The interaction of forced with parametric oscillations

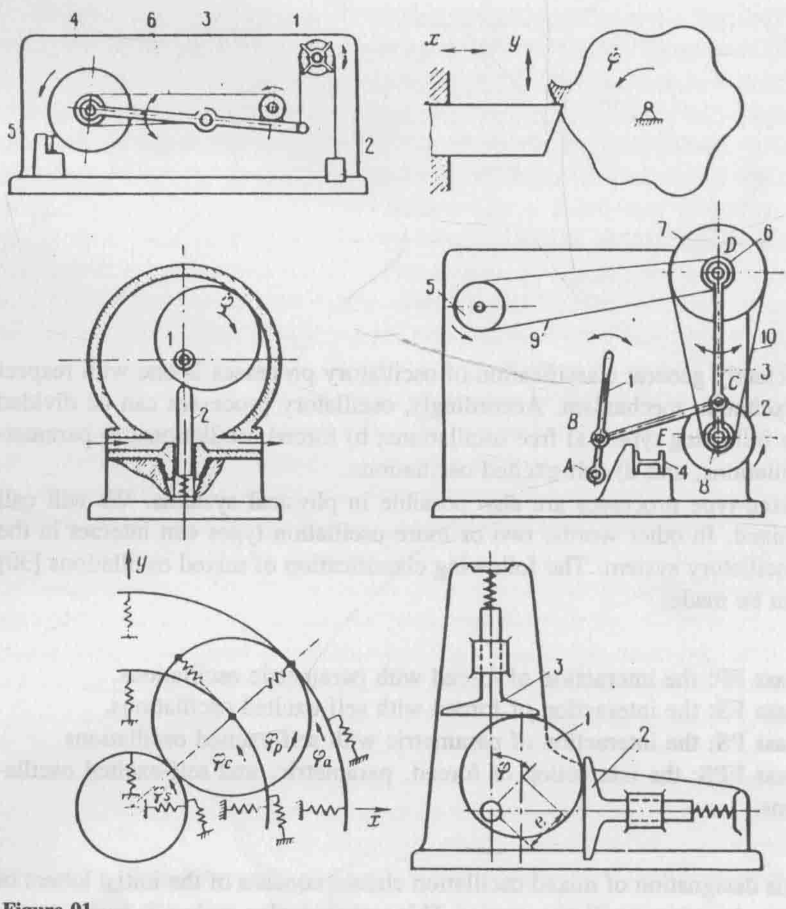

Figure 01

has been considered by taking, as an example, oscillations of an arch and a rod in [45], and a simple mechanical system in [46].

In the case of a rod, the amplitude of steady-state transverse oscillations in domains of dynamic stability is determined by solutions of the equations

$$u'' + \omega_0^2 u + \frac{\pi^2}{8l}(f^2)'' = \frac{P(t)}{M}$$
$$f'' + \omega^2\left(1 - \frac{\omega_0^2 M}{P_*}u\right)f + \gamma f^3 = 0 \tag{1}$$

where $P(t) = P_0 + P_t \cos\theta t$.

For the case of an arch, the system

$$u'' + \frac{(k^2-1)F_1(k)}{2R}(f^2)'' + \omega_0^2 u = \frac{P(t)}{M}$$
$$f'' + \omega^2\left(1 - \frac{\omega_0^2 M}{P_*}u\right)f = 0 \tag{2}$$

is considered, coinciding with (1) but for notation when $\gamma = 0$.

In [131, 132], E. L. Poznyak studied the dynamics of a steel core in a magnetic field, whose motion, if the field is pulsating, can be described by the system

$$\ddot{x} + \omega_x^2(1 + \beta_x\cos 2\Omega t)x = \frac{P_{x_0}a}{m} - \frac{P_{x_0}a}{m}\cos 2\Omega t + \varepsilon\left(\omega^2 + \frac{P_{x_0}}{m}\right)\cos\omega t - \frac{P_{x_0}\varepsilon}{m}\cos\omega t\cos 2\Omega t$$
$$\ddot{y} + \omega_y^2(1 + \beta_y\cos 2\Omega t)y = \frac{P_{y_0}b}{m} - \frac{P_{y_0}b}{m}\cos 2\Omega t + \varepsilon\left(\omega^2 + \frac{P_{y_0}}{m}\right)\sin\omega t - \frac{P_{y_0}\varepsilon}{m}\sin\omega t\cos 2\Omega t$$

making the interaction of forced with parametric oscillations possible.

In the case of oscillations of a core with a shaft in a rotating magnetic field, we again arrive at equations describing the interaction of forced with parametric oscillations.

A number of other works, e.g., [50, 51, 181, 200], are also devoted to the study of this combination but in different systems. The monograph of G. Schmidt [181], generalizing the results of parametric oscillation analysis since the well-known work of V. V. Bolotin [47] should be noted. Schmidt also considers the interaction of forced with parametric oscillations for nonlinear damping, nonlinear parametric excitation, and nonlinear restoring force.

Class FS. Historically, this was apparently the first class of mixed oscillations to be studied in greater detail. The problem was first posed by Van der Pol in

1927 [53]. The combination is the subject of a large number of papers on the analysis of mixed oscillatory processes, e.g., [1, 49, 115, 122, 153, 165]. An important contribution was also made by Ch. Hayashi and A. Tondl.

In his monograph [165], Hayashi analyzed the interaction of forced with self-excited oscillations on the basis of equations

$$\text{a)}\quad \frac{d^2\vartheta}{d\tau^2} - \mu(1 - \beta\vartheta - \gamma\vartheta^2)\frac{d\vartheta}{d\tau} + \vartheta = B\cos\nu\tau$$

$$\text{b)}\quad \frac{d^2\vartheta}{d\tau^2} - \mu(1 - \gamma\vartheta^2)\frac{d\vartheta}{d\tau} + \vartheta^3 = B\cos\nu\tau$$

Unlike the prior researchers, he studied higher order resonances, and suggested approximate methods for investigating quasi-periodic oscillations arising from entrained oscillations, as well as a method for constructing separatrices of domains of attraction.

Tondl also studied the interactions of forced with self-excited oscillations in great detail [153]. However, in contrast with Hayashi, he considered not only a system with one degree of freedom, but also a system with two degrees of freedom, studying the following single degree of freedom systems:

$$\text{a)}\quad \ddot{y} - (\beta - \delta y^2)\,\dot{y} - \alpha y + \mu y^3 = Q_0 + \cos(\eta t + \varphi)$$

$$\text{b)}\quad \ddot{y} - (\beta - \delta y^2)\,\dot{y} - \alpha y + \mu y^3 = Q_0 + \eta^2\cos(\eta t + \varphi)$$

Hayashi's and Tondl's works are distinguished by an extensive use of computers, which makes it possible to complement the obtained approximate analytical results, and consider the system dynamics in cases where analytical studies become either very complicated or impossible if treated by approximate methods of nonlinear mechanics.

Class PS. Problems related to this class are of the highest scientific and practical interest among all problems leading to mixed oscillations, since the parametric and self-oscillatory excitation mechanisms are typical for mechanical systems, and widespread. However, the study of this combination was begun only recently.

In chronological order, the first paper is by K. V. Frolov [157] for a nonideal energy source. The work of V. O. Kononenko and P. S. Kovalchuk [100] then appeared, in which the system

$$\ddot{x} + \omega^2(1 - h\cos\nu t)\,x + \varepsilon f(x, \dot{x}) = 0 \tag{3}$$

was considered.

Its main difference from the previous works on the analysis of FS, and, subsequently, of PS systems is that the study of (3) was made both for the function

$$f(x, \dot{x}) = (\alpha + \beta x^2 + \gamma x^4)\,\dot{x}$$

representing the Van der Pol function with the addition term γx^4 (complex model), and for the function

$$f(x, \dot{x}) = k_0 \operatorname{sgn} (\dot{x} - u_0) - k_1 (\dot{x} - u_0)$$

describing a discontinuous friction force. Such an approach more precisely corresponds to a real situation in mechanical systems.

The combination is considered on the basis of the systems

$$\begin{gathered} m\ddot{x} + c(1 - \varepsilon h \cos \nu t)\, x + \varepsilon f(x_\Delta) = 0 \\ x_\Delta = x(t - \Delta), \quad \Delta = \text{const} \end{gathered} \tag{4a}$$

$$\ddot{x}_j + \omega_j^2 x_j [1 - \Phi_j(\nu t)] + g_i x_j + \sum_{k=1}^{n} b_{jk}(\alpha)\, x_k$$
$$= \varepsilon \Psi_j(x_1, \ldots, x_n, \dot{x}_1, \ldots, \dot{x}_n), \qquad j = 1, 2, \ldots, n \tag{4b}$$

$$m\ddot{x} + c\left(1 - \varepsilon h \cos \frac{\nu}{\mu} t\right) x + \mu f_1(\dot{x}) = 0 \tag{4c}$$

in the work of V. O. Kononenko and P. S. Kovalchuk [99].

In the case of (4a), the self-oscillatory mechanism is due to a quasi-elastic delayed force, and, in the case of (4b), is due to coordinate coupling. In (4c), the self-oscillatory mechanism is of the relaxation type.

This combination was also studied by N. P. Plachtienko in [137] on the basis of

$$\ddot{x} + \omega^2 x = \varepsilon [\delta (1 - \beta x^2)\, \dot{x} + \gamma x^3 + e x \sin \nu t]$$

which takes the nonlinearity γx^3 of the restoring force into account.

This combination was not sufficiently studied in the above works. A more complete study both by approximate analytical methods and by analog modeling was performed by A. Tondl. The research results were included in a monograph [196]. Tondl studied the systems

$$\ddot{y} - (\beta - \delta y^2)\, \dot{y} + \vartheta \operatorname{sgn} \dot{y} + (1 + \gamma y^2 + \mu \cos 2\eta\tau)\, y = 0 \tag{5a}$$

$$\ddot{y} - (\beta - \delta y^2)\, \dot{y} + (1 + \mu \cos 2\eta\tau)(1 + \gamma y^2)\, y = 0 \tag{5b}$$

$$\begin{gathered} m_1\ddot{x}_1 - (\beta_0 - \delta_0 x_1^2)\, \dot{x}_1 + c_1 (1 + \mu \cos 2\omega t)(x_1 - x_2) = 0 \\ m_2\ddot{x}_2 - c_1 (1 + \mu \cos 2\omega t)(x_1 - x_2) + k_2\dot{x}_2 + c_2 x_2 = 0 \end{gathered} \tag{5c}$$

Class FPS. This class of mixed oscillations is the most mathematically complex. It has been studied by V. O. Kononenko and P. S. Kovalchuk [101] on the basis of the system

$$m\ddot{x} + c(1 - \varepsilon q \cos \nu t)\, x + \varepsilon f(x, \dot{x}) = E \cos pt$$

where $f(x, \dot{x}) = -m(\alpha - \beta x^2)\dot{x}$.

This paper is the only one (at least, others are unknown to the authors) devoted to an analysis of this combination.*

In conclusion, we note that the results of [99–101] are also discussed in [65].

Systems with a nonideal energy source. The well-known physical effect discovered experimentally earlier in this century by Sommerfeldt led, almost half a century later, to a new branch of vibration theory, which studies dynamic interactions of an oscillatory system with an energy source. In spite of many works devoted to the Sommerfeldt effect, it was only V. O. Kononenko who began a systematic study of the problem. The results were summarized in his well-known monograph.**

Not much time has elapsed since it was published. Now, the number of studies devoted to various problems posed by modern physics and technology, in which the dynamic interaction of an oscillatory system with an energy source is considered, is increasing. The investigation area has widened, and new spheres of applications have appeared. The theory finds applications in solving numerous problems of technology, and in designing and providing a theoretical basis for a number of devices, mechanisms, and machines such as power plants with internal combustion engines, systems with electromagnetic vibrators, oscillating conveyers, hydromechanical systems, rolling mills, vibroimpact systems, tuning fork clocks, etc. [52].

The published studies are devoted to special, rather small, practical problems. At the same time, the necessity of studies of a more general nature, which would complete the theory, is felt. Such studies, although related to concrete application problems, lead to results partially or completely independent of applications, and account for common effects in different physical systems.

Oscillations of systems with random perturbations, two energy sources, and synchronizing systems have been studied. The studies include, in particular, the influence of dynamic processes in the energy source on the system behavior. A brief survey of works after 1964 is given in [52], and in greater detail in [10]. We can also indicate works not included in [10], and appearing later. However, they often pursue goals not related to the theory in question and do not provide the completeness which is possible at present.

The study of mixed oscillatory processes, taking the energy source characteristics into account, was made in a few papers surveyed below. They do not analyze the FP combination.

The FS combination was considered by V. F. Petrov [133] on the basis of the system described by equations

*This was also indicated by A. Tondl [196].

**Kononenko, V. O. Oscillatory systems with limited excitation. Moscow, Nauka, 1964 (in Russian).

a) $v_0 = \dot{x}$

$$kx - mr\,(\dot{\theta}\sin\theta) = P, \quad -F^0 \leqslant P \leqslant F^*$$
$$J\ddot{\theta} = N\,(\dot{\theta}) \tag{6}$$

b) $v_0 \geqslant \dot{x}$

$$M\ddot{x} + kx = F^0 + \gamma\,(v_0 - \dot{x}) + mr\,(\dot{\theta}\sin\theta)$$
$$J\ddot{\theta} = N\,(\dot{\theta}) + mr\ddot{x}\sin\theta$$

However, (6) was studied in a simplified form with the characteristic of the friction force with a descending portion being replaced by the positive difference between the static friction coefficient and that for an infinitesimal sliding velocity. Certain restrictions were placed on the system forced motion. Actually, the system has little to do with the basic problem of exploring characteristics of mixed oscillations as was done in the above papers for an ideal energy source.

The PS combination was analyzed by K. V. Frolov and G. L. Lifshitz [110] on the basis of the system

$$m\ddot{x} + \tilde{c}x = R\,(v - \dot{x}) - H_1\,(\dot{x})$$
$$J\ddot{\varphi} = L\,(\dot{\varphi}) - H_2\,(\dot{\varphi}) - rR\,(v - \dot{x}), \quad v = r\dot{\varphi}$$

solved on an AC. A variation of natural frequency ($p = \tilde{c}/m$.

$$p = p_0\,(1 \pm \mu t), \quad p = p_0\,(1 + \mu\cos\omega t), \quad p = p_0\,(1 + \mu\varepsilon t^2/2)$$

was considered.

From 1977, mixed oscillatory processes were investigated systematically by A. A. Alifov [8–10, 14, 16, 18, 23, 26–28] and A. A. Alifov and K. V. Frolov [19–21], taking energy source properties into account. The basic analytical results were checked and extended by solving the problems on an AC [15, 17, 22, 24]. The FS combination was studied in [14, 15, 17, 26], the PS combination in [18–22], and the FPS combination in [23, 24]. Thanks to this systematic analysis, general and individual properties of independent and dependent mechanisms of mixed oscillatory process excitation were discovered. Before these studies, attention had not been given to the important fact of the existence of these two excitation classes.

The concepts of separate and dependent excitation mechanisms for mixed oscillations are determined as follows [30]: excitation mechanism for mixed oscillation, distinguished by the absence of a correlation between various parameters of the excitation mechanism, will be called *independent excitations*; mixed excitation mechanisms, characterized by the existence of such correlations, will be called *dependent excitations*.

FP, FS, and PS are characterized by two different excitation mechanisms, whereas FPS is characterized by three. Therefore, various independent and dependent excitations, including combined versions, are possible in the latter case (there are, in total, five different versions, of which three are overlapping).

Accordingly, oscillatory systems will be termed systems with independent or dependent excitations. They can be either nonautonomous or autonomous systems.

The results of analyzing systems with independent or dependent excitation mechanisms demonstrate that, in studying mixed oscillations, the problem of the existence of a direct relation between different excitation mechanisms should be discussed thoroughly. The correctness of a solution of the formulated problem therefore is essentially dependent on the correct determination as to which class the system belongs.

The results of a systematic analysis of mixed oscillations, given in the book, enable us to obtain a sufficiently clear idea of the phenomena occurring in various classes of mixed oscillatory processes with and without an account of the energy source properties in the case of both independent and dependent excitation mechanisms. The monograph is a first attempt of a systematic treatment of these processes. This branch of vibration theory needs further development.

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