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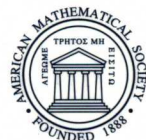
688

Groups, Rings, Group Rings, and Hopf Algebras

International Conference in Honor of
Donald S. Passman's 75th Birthday
Groups, Rings, Group Rings, and Hopf Algebras
October 2–4, 2015

Loyola University, Chicago, IL
AMS Special Session in Honor of
Donald S. Passman's 75th Birthday
Groups, Rings, Group Rings, and Hopf Algebras
October 3–4, 2015
Loyola University, Chicago, IL

Jeffrey Bergen
Stefan Catoiu
William Chin
Editors



This volume contains the proceedings of the International Conference on Groups, Rings, Group Rings, and Hopf Algebras, held October 2–4, 2015 at Loyola University, Chicago, IL, and the AMS Special Session on Groups, Rings, Group Rings, and Hopf Algebras, held October 3–4, 2015, at Loyola University, Chicago, IL. Both conferences were held in honor of Donald S. Passman's 75th Birthday.

Centered in the area of group rings and algebras, this volume contains a mixture of cutting edge research topics in group theory, ring theory, algebras and their representations, Hopf algebras and quantum groups.

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Groups, Rings, Group Rings, and Hopf Algebras

Dedicated to Professor Donald S. Passman, University of Wisconsin-Madison
on the occasion of his 75th birthday

Preface

This proceedings volume is based on the talks presented at the International Conference on Groups, Rings, Group Rings and Hopf Algebras, dedicated to Professor Donald S. Passman of the University of Wisconsin-Madison on the occasion of his 75th birthday. The conference was held at Loyola University, Chicago, on October 2-4, 2015, and was organized as a joint special session with the same name at the AMS Central Sectional Meeting # 1112, Chicago, October 3-4, 2015.

Don Passman has been a renowned expert on group algebras for more than five decades. His work has touched many other adjacent areas, especially group theory, ring theory, enveloping algebras of Lie algebras and Hopf algebras. The conference has brought together leading research mathematicians from around the world to discuss the recent developments in these fields and the status of known open problems. The present volume focuses on the general framework of groups, rings, algebras and their representations, and has a particular emphasis on group rings, enveloping algebras of Lie algebras, and Hopf algebras. All articles are refereed.

The volume is useful to research mathematicians working in the general areas of group theory, ring theory, associative and non-associative algebras, representation theory, Hopf algebras, quantum groups and other related fields. The great mixture of related topics and a number of articles with a significant expository component makes the whole material accessible to graduate students and interested research mathematicians of all sorts.

Our greatest thanks go to Georgia Benkart of the AMS, for her assistance in merging the conference with the AMS Sectional Meeting. Without this support, both the conference and the proceedings book would have never been possible.

We are grateful to our Loyola colleagues, Steve Doty, Tony Giaquinto and Aaron Lauve, for their help during the conference, and especially to Ann Rogers of DePaul University, for her time and effort with the room setting and refreshments.

We thank the speakers for their talks, and all participants for making it all happen. Many of them have traveled long distances to be a part of this three-day event in the middle of the academic year.

We are very thankful to the referees, Jason Bell (Waterloo), Andrew Carroll (DePaul), Sorin Dăscălescu (Bucharest), Angel del Rio (Murcia), Antonio Giambruno (Palermo), Robert Guralnick (Southern California), Miodrag Iovanov (Iowa), S. K. Jain (Ohio), Apoorva Khare (Stanford), Mikhail Kochetov (Newfoundland), Alexander Lichtman (Wisconsin-Parkside), Chia-Hsin Liu (Taiwan Normal), Leo Margolis (Stuttgart/Murcia), Ian M. Musson (Wisconsin-Milwaukee), Siu-Hung Ng (Louisiana State), Donald S. Passman (Wisconsin-Madison), César Polcino Milies (Sao Paulo) and Sarah Witherspoon (Texas A&M), for their time, careful reading and suggestions for improvement.

Our special thanks go to Christine Thivierge of the AMS Contemporary Mathematics series, for her invaluable support during the planning and execution of this volume. Her continuous assistance goes back to almost one year before the conference started.

Jeffrey Bergen, Stefan Catoiu and William Chin

List of Participants

Marcelo Aguiar
Cornell University

Andreas Bächle
University of Brussels

John Beachy
Northern Illinois University

Allen Bell
University of Wisconsin-Milwaukee

Jason P. Bell
University of Waterloo

Esther Beneish
University of Wisconsin-Parkside

Georgia Benkart
University of Wisconsin-Madison

Allan Berele
DePaul University, Chicago

Jeffrey Bergen
DePaul University, Chicago

William Blair
Northern Illinois University

Daniel Britten
University of Windsor

Andrew Carroll
DePaul University, Chicago

Stefan Catoiu
DePaul University, Chicago

Xueqing Chen
University of Wisconsin-Whitewater

William Chin
DePaul University, Chicago

Steve Cohen
Roosevelt University, Chicago

Angel del Rio
University of Murcia

Stephen Doty
Loyola University, Chicago

Anthony Giaquinto
Loyola University, Chicago

George Glauberman
University of Chicago

Jairo Z. Goncalves
University of Sao Paulo

Alfred W. Hales
CCR La Jolla

Tom Halverson
Macalester College

Allen Herman
University of Regina

Joshua Herman
University of Illinois at Chicago

Miodrag C. Iovanov
University of Iowa

Adam M. Jacoby
Temple University

S. K. Jain
Ohio University

Geoffrey Janssens
University of Brussels

Eric Jespers
University of Brussels

Yevgenia Kashina DePaul University, Chicago	Donald S. Passman University of Wisconsin-Madison
Vladislav K. Kharchenko UNAM, Mexico	Peter Plaumann UAEB0, Mexico
Apoorva Khare Stanford University	César Polcino Milies University of Sao Paulo
Ellen E. Kirkman Wake Forest University	Jean-Baptiste Priez University of Paris-Sud
Mikhail Kochetov Memorial University of Newfoundland	Declan Quinn Syracuse University
Robert Krone Queen's University, Canada	David E. Radford University of Illinois at Chicago
Leonid Krop DePaul University, Chicago	Ann E. Rogers DePaul University, Chicago
Aaron Lauve Loyola University, Chicago	David Riley University of Western Ontario
Alexander Lichtman University of Wisconsin-Parkside	Paul Schrader Bowling Green State University
Chia-Hsin Liu National Taiwan Normal University	Lance W. Small University of California, San Diego
Martin Lorenz Temple University	Yorck Sommerhäuser Memorial University of Newfoundland
Leo Margolis University of Stuttgart	Bhama Srinivasan University of Illinois at Chicago
Joseph Mayne Loyola University, Chicago	Ashish K. Srivastava St. Louis University
Susan Montgomery University of Southern California	Earl J. Taft Rutgers University
Ian M. Musson University of Wisconsin-Milwaukee	Nathaniel Thiem University of Colorado
Siu-Hung Ng Louisiana State University	Peter Tingley Loyola University, Chicago
Van Nguyen Northeastern University	Luis Valero-Elizondo University of Michoacana, Mexico
Venrata Ranya P. Ogirala Loyola University, Chicago	
James Osterburg University of Cincinnati	

Jeanne Wald
Michigan State University

Sarah Witherspoon
Texas A&M University

Daniel Yee
University of Wisconsin-Milwaukee

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The Dixmier-Moeglin equivalence for extensions of scalars and Ore extensions

Jason P. Bell, Kaiyu Wu, and Shelley Wu

Dedicated to Don Passman on the occasion of his 75th birthday.

ABSTRACT. An algebra A satisfies the Dixmier-Moeglin equivalence if we have the equivalences:

$$P \text{ primitive} \iff P \text{ rational} \iff P \text{ locally closed for } P \in \text{Spec}(A).$$

We study the robustness of the Dixmier-Moeglin equivalence under extension of scalars and under the formation of Ore extensions. In particular, we show that the Dixmier-Moeglin equivalence is preserved under base change for finitely generated complex noetherian algebras. We also study Ore extensions of finitely generated complex noetherian algebras A . If $T : A \rightarrow A$ is either a $\mathbb{C}$ -algebra automorphism or a $\mathbb{C}$ -linear derivation of A , we say that T is *frame-preserving* if there exists a finite-dimensional subspace $V \subseteq A$ that generates A as an algebra such that $T(V) \subseteq V$. We show that if A is of finite Gelfand-Kirillov dimension and has the property that all prime ideals of A are completely prime and A satisfies the Dixmier-Moeglin equivalence then the Ore extension $A[x; T]$ satisfies the Dixmier-Moeglin equivalence whenever T is a frame-preserving derivation or automorphism.

1. Introduction

The Dixmier-Moeglin equivalence is an important result in the representation theory of enveloping algebras of finite-dimensional Lie algebras that has since been extended to many other settings. At its core, the motivation for this equivalence lies in trying to understand the irreducible representations of an algebra—this is often a very difficult problem. As a means to deal with such thorny representation-theoretic issues, Dixmier proposed that one should instead find a “coarser” understanding by finding the annihilators of simple modules—these are the so-called primitive ideals and they form a subset of the prime spectrum. This approach has been used successfully in the study of enveloping algebras and their representations. A concrete characterization of the primitive ideals of enveloping algebras of finite-dimensional complex Lie algebras was given by Dixmier [5] and Moeglin [13]. In this case, we have the following equivalence.

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Key words and phrases. Primitive ideals, Nullstellensatz, base change, Ore extensions, Dixmier-Moeglin equivalence.

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THEOREM 1.1. *Let $\mathcal{L}$ be a finite-dimensional complex Lie algebra and let P be a prime in $\text{Spec}(U(\mathcal{L}))$. Then the following are equivalent:*

- (1) P is primitive;
- (2) $\{P\}$ is locally closed in $\text{Spec}(R)$;
- (3) P is rational.

We recall, for the reader's benefit, that a subset V of a topological space Z is *locally closed* if we have $V = C_1 \setminus C_2$ for some closed subsets C_1 and C_2 of Z ; as stated earlier, a prime ideal P in a ring R is (left) *primitive* if P is the annihilator of some simple left R -module. Finally, a prime ideal P of a noetherian k -algebra R is *rational* if the centre of the Artinian ring of quotients of A/P , which we denote $Q(A/P)$, is an algebraic extension of the base field. We let $Z(Q(A/P))$ denote the centre of this ring of quotients and we call it the *extended centre* of A/P . We point out that the Dixmier-Moeglin equivalence gives both a topological and a purely algebraic characterization of the primitive ideals of an algebra.

A noetherian algebra for which properties (1)–(3) are equivalent for all prime ideals of the algebra is said to satisfy the *Dixmier-Moeglin equivalence*. More precisely, we have the following definition.

DEFINITION 1.2. Let R be a noetherian algebra. We say that R satisfies the *Dixmier-Moeglin equivalence* if for every $P \in \text{Spec}(R)$, the following properties are equivalent:

- (A) P is primitive;
- (B) P is rational;
- (C) $\{P\}$ is locally closed in $\text{Spec}(R)$.

The Dixmier-Moeglin equivalence has been shown to hold for many classes of algebras, including many quantized coordinate rings and quantized enveloping algebras, algebras that satisfy a polynomial identity, and many algebras that come from noncommutative projective geometry [3, 4, 6, 14]. In general, there are many examples of algebras for which the Dixmier-Moeglin equivalence does not hold. For example, Lorenz [11] constructed a polycyclic-by-finite group whose complex group algebra is primitive but with the property that (0) is not locally closed in the prime spectrum. While the Dixmier-Moeglin equivalence need not hold in general, one always has the implications $(C) \implies (A) \implies (B)$ for algebras satisfying the Nullstellensatz (we recall that a k -algebra A satisfies the *Nullstellensatz* if every prime ideal is the intersection of the primitive ideals above it and the endomorphism ring of every simple module is algebraic over the base field); in particular, this holds for countably generated noetherian algebras over an uncountable base field (cf. [4, II.7.16] and [12, Prop. 6]).

The focus of this paper is to study the robustness of the Dixmier-Moeglin equivalence under extension of scalars and under Ore extensions. We recall for the reader's benefit that if k is a field and A is a k -algebra then an *Ore extension* $A[x; \sigma, \delta]$ is formed by taking a k -linear automorphism σ of A and a k -linear map $\delta : A \rightarrow A$, called a σ -derivation, which satisfies $\delta(ab) = \sigma(a)\delta(b) + \delta(a)b$, and then forming an algebra by taking the polynomial ring $A[x]$ and giving it a new multiplication in which multiplication of elements of A is just the ordinary product in A and multiplication by x is given by

$$xa = \sigma(a)x + \delta(a) \quad \text{for } a \in A.$$

In the case when σ is the identity, then δ is just a derivation of the algebra and we suppress σ and write $A[x; \delta]$; in the case when δ is zero, we suppress δ and write $A[x; \sigma]$.

In general, it is known that the Dixmier-Moeglin equivalence is not necessarily preserved under typical extensions used in ring theory. For Ore extensions, an example of Lorenz gives a Laurent polynomial ring $\mathbb{C}[x^{\pm 1}, y^{\pm 1}]$ with an automorphism σ such that the algebra $\mathbb{C}[x^{\pm 1}, y^{\pm 1}][t, t^{-1}; \sigma]$ does not satisfy the Dixmier-Moeglin equivalence (Lorenz' example is stated in the language of group algebras of polycyclic groups). On the other hand, there is a recent example of a finitely generated commutative complex domain with a derivation δ such that $A[x; \delta]$ does not satisfy the Dixmier-Moeglin equivalence [2]. Our main result for extension of scalars shows that for uncountable algebraically closed base fields of characteristic zero, the Dixmier-Moeglin equivalence is preserved under extension of scalars for finitely generated algebras (see Theorem 2.3 for a slightly more general statement).

THEOREM 1.3. *Let A be a finitely generated complex noetherian algebra and suppose that A satisfies the Dixmier-Moeglin equivalence. Then $A \otimes_{\mathbb{C}} F$ satisfies the Dixmier-Moeglin equivalence for all extensions F of $\mathbb{C}$.*

One of the nice consequences of this result is that it shows that if A is a finitely generated complex noetherian algebra that satisfies the Dixmier-Moeglin equivalence then, up to finitely many exceptions, the height one primes of A are parametrized by the extended centre in some natural sense. This agrees with the case of many quantized coordinate rings, where Goodearl and Letzter showed in this setting that after inverting a normal element (i.e., removing a finite set of height one primes and the primes that contain them), the prime spectrum of the resulting algebra is homeomorphic to the prime spectrum of its centre, with the homeomorphisms coming from expansion and contraction of prime ideals. More precisely, we have the following result.

COROLLARY 1.4. *Let A be a finitely generated complex noetherian algebra and suppose that A satisfies the Dixmier-Moeglin equivalence. If P is a prime ideal of A and $B = A/P$ then ZB has finitely many height one primes, where ZB is the subalgebra of $Q(B)$ generated by B and the centre, Z , of $Q(B)$.*

The second focus of this paper is the study of the Dixmier-Moeglin equivalence under Ore extensions. Here we use Theorem 1.3 and Corollary 1.4 to obtain our results. As mentioned earlier, there are examples of algebras that satisfy the Dixmier-Moeglin equivalence and that have Ore extensions that do not. In practice, however, the Dixmier-Moeglin equivalence is generally preserved under the process of taking Ore extensions that arise naturally. For example, the work of Goodearl and Letzter [6] shows that the class of iterated Ore extensions now known as CGL extensions all satisfy the Dixmier-Moeglin equivalence.

When one compares the class of CGL extensions with the pathological examples appearing in [2, 11], an immediate observation one makes is that in the pathological examples above, the automorphism and derivation do not preserve a frame of the algebra. We recall that in a finitely generated k -algebra A , a *frame* is simply a finite-dimensional subspace V of A that contains 1 and that generates A as a k -algebra. Most Ore extensions that arise in the setting of quantum groups and enveloping algebras are *frame-preserving*; that is there is frame of the algebra which