RECENT ADVANCES IN OPTICS

BY

E. H. LINFOOT

JOHN COUCH ADAMS ASTRONOMER IN THE
UNIVERSITY OF CAMBRIDGE

OXFORD
AT THE CLARENDON PRESS
1955

THE
INTERNATIONAL SERIES
OF
MONOGRAPHS ON PHYSICS

THE INTERNATIONAL SERIES OF MONOGRAPHS ON PHYSICS

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Oxford University Press, Amen House, London E.C.4

GLASGOW NEW YORK TORONTO MELBOURNE WELLINGTON

BOMBAY CALCUTTA MADRAS KARACHI CAPE TOWN IBADAN

Geoffrey Cumberlege, Publisher to the University

PRINTED IN GREAT BRITAIN

PREFACE

THE main purpose of this book is to give an account of some of the more interesting developments in instrumental optics during the last twenty years. A complete survey would have taken up more than the available space, and the book therefore deals with selected topics. Many of these reflect the author's special interests; in particular the discussion of image assessment and error balancing at the beginning of Chapter I is based on previously unpublished work by him and by P. A. Wayman. It is hoped, however, that the choice is wide enough to convey a good general idea of the renaissance in instrumental optics which has taken place since F. Zernike's pioneer work on the phase-contrast microscope and on the analysis of partial coherence. The remainder of Chapter I deals with what is commonly called the diffraction theory of aberrations and with the imaging of coherent and partially coherent object-surfaces.

It has not seemed appropriate to refer specifically to the techniques of practical optics, but the importance of new types of optical system of very high theoretical performance is closely bound up with the existence of a high level of practical figuring technique. This in turn depends on the availability and adequate understanding of test procedures, among which the Foucault knife-edge test still occupies a prominent place. An account of the diffraction theory of this test is given in Chapter II.

Among the newer types of optical system introduced in recent years, the Schmidt camera stands out as one of the most successful, especially in astronomy, where Schmidt telescopes of large aperture and small fast Schmidt spectrograph cameras have set up an entirely new standard of photographic performance. A systematic account of the fifth-order optics of the Schmidt camera and of the field-flattened Schmidt camera is given in Chapter III.

In Chapter IV, C. R. Burch's method of plate-diagram analysis is developed and applied to discuss the Seidel properties of Schmidt–Cassegrain systems, of the Schmidt camera with aspherized mirror, and of coma-free two-mirror systems.

My grateful thanks are due to Dr. R. Kingslake, Professor F. Zernike, Professor A. Maréchal, and Dr. C. R. Burch for their kindness in supplying original photographs and drawings reproduced in Chapters I and IV. Thanks are also due to the Societies concerned for permission

to reproduce long extracts from papers published in the Proceedings of the Royal Society, the Proceedings of the Physical Society, and the Monthly Notices of the Royal Astronomical Society, and to the Clarendon Press for their unfailing care and courtesy.

E. H. L.

THE OBSERVATORIES
UNIVERSITY OF CAMBRIDGE
October 1954

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I

THE OPTICAL IMAGE

1. Geometrical theory

1.1. *Introduction*

In spite of its mathematical elegance, general ikonal theory does not provide a very satisfactory starting-point for a discussion of the images formed by optical instruments. In its general form, the theory does not show any natural tendency to centre round the special preoccupations of instrumental optics; when it is made to do this by appropriate mathematical restatement, the analysis loses most of its elegance and still does not give much insight into the actual working of optical systems. A more physical approach is therefore adopted in the present section, and ikonal theory only makes brief appearances as a convenient analytical tool.

In instrumental optics, we usually have to deal with what is effectively a single infinity of pencils of rays issuing from the separate points of a symmetrical object surface, passing through a centred optical system, and emerging as bundles of rays each of which is approximately concurrent. The points of approximate concurrence (in some agreed sense) form a thin shell or image-layer in the image-space and the receiving surface or 'image surface' may be supposed to be anywhere in this layer, though of course some positions will be preferable to others.

The first problem is evidently the determination of the image-layer corresponding to a prescribed object surface for a given optical system, and the choice within this layer of the best receiving surface according to a prescribed method of assessing image quality.

It is easy to show that the Petzval surface lies in the image-layer and that, as Conrady first pointed out, it forms a convenient and natural field reference surface. But a comparison of the relative merits of the different receiving surfaces within this layer requires the setting up of analytical formulae which describe the behaviour in the image-layer of the rays issuing from an arbitrary point of the object surface. Exact formulae are almost useless for this purpose, even in the investigation of two-component systems like the Schmidt camera, because of their formidable complexity. With more elaborate systems such as a Taylor triplet lens the situation would be much worse. A general analytical discussion of these systems, if it is to be made at all, should from the nature of the problem be based on an appropriate use of approximations.

It is obvious that mere inequalities giving upper limits to the size of the geometrical images do not give enough information; they will not even prescribe the best choice of the receiving surface within the image-layer, still less indicate optimum values for the design-constants of the system.

We need approximate formulae, valid in the image-layer, consisting of a leading term plus an error-term. Such formulae may allow the best receiving surface to be determined, with an error which is too small to affect seriously the performance of the system, and the structure of the image in this surface to be analysed.

For the present, we suppose the light monochromatic; the effects of chromatism will be considered later.

1.2. *Notation*

Fig. 1 represents an axially symmetric system **S** which images the points of a spherical or flat object surface on to a receiving surface in the image-layer, both object surface and receiving surface being symmetrical about the optic axis of **S**. We suppose that **S** works over a field of angular diameter $O(\mu)$ radians, where $O(\mu)$ means 'not exceeding a moderate multiple of μ' and μ is the numerical aperture of **S**, suitably normalized. Then all the rays which pass through **S** make angles $O(\mu)$ with the optic axis.

Slightly different choices of μ-normalization are preferable in different applications. In a Schmidt camera, μ may be conveniently defined as H/R, where H is the semi-aperture and R the radius of curvature of the spherical mirror; this makes μ nearly equal to half the numerical aperture, whence $\mu^2 \simeq \frac{1}{64}$ in an f/2 Schmidt and our approximate formulae are accurate to a few per cent. In an f/1 Schmidt, $\mu^2 \simeq \frac{1}{16}$ and the accuracy of these formulae is correspondingly reduced. In a refracting system, a definition of μ which puts its value near to the numerical aperture improves the verisimilitude of the picture given by the error-term assessments. In a general discussion, it seems better to leave the μ-normalization arbitrary to the extent of a factor comparable with unity, and this will be done here.

By Gauss imaging of its aperture stop, the entry and exit pupils of **S** are obtained. We set up Cartesian coordinates (x, y, z) in the image-space of **S**, taking the origin O' at the centre of the exit pupil and the axis $O'z$ along the optic axis (see Fig. 1). The exit pupil then fills a circle $x^2+y^2 \leqslant H'^2$ in the plane $xO'z$, where $H' = O(f\mu)$ and f is the focal length.

In this plane, and in the space near it, we introduce scale-normalized lateral coordinates u, v by means of the equations

$$x = H'u, \qquad y = H'v. \tag{1.1}$$

The scale-normalized polar coordinates r, ϕ in the space surrounding the exit pupil are connected with u, v by the equations

$$u = r\cos\phi, \qquad v = r\sin\phi, \qquad r = +\sqrt{(u^2+v^2)}. \tag{1.2}$$

The exit pupil occupies the region $u^2+v^2 \leqslant 1$ of the plane $xO'y$.

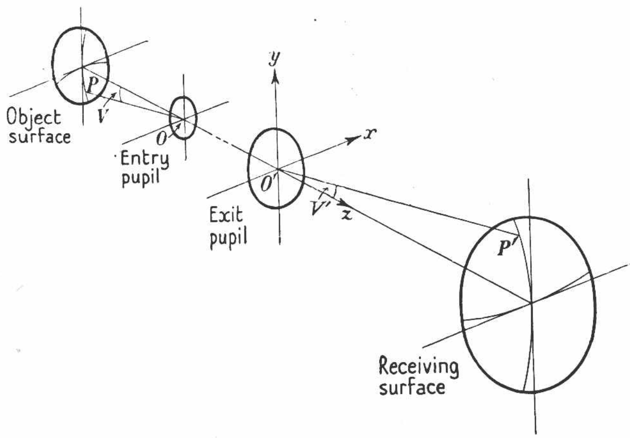

FIG. 1.

From the off-axis object point P a pencil of rays passes through the system **S** to form an image-patch on the receiving surface. We call the ray through O' the *principal ray* of the emerging pencil and its intersection P' with the receiving surface the *principal point* of the image. Because of the axial symmetry of **S**, there is no loss of generality in supposing the off-axis displacement of P' to be in the positive y-direction.

By the *angular off-axis distance of the image* we mean the angle $V' = P'O'z$. As P runs over the object field, V' ranges over a certain interval $(0, V'_0)$. We define $\Theta = V'/V'_0$, so that the field is defined by the inequalities $0 \leqslant \Theta \leqslant 1$.

Because of the aberrations of the system, the rays of the emerging pencil trace out in the plane $xO'y$ a region which does not exactly coincide with the exit pupil; their boundaries may differ by $O(f\mu^3)$.

Some or all of the optical surfaces of **S** may be figured; we suppose that no figuring depths exceed $O(f\mu^4)$. This condition is satisfied by all 'useful' figurings in systems for which angular aperture and angular field radius are both $O(\mu)$.

We restrict the discussion to 'useful' receiving surfaces, namely those sufficiently close to the best focal surface not to increase the size of the geometrical image-spreads by more than a factor $O(1)$. This means that we consider only receiving surfaces lying within the *image layer* of § 1.1.

Such receiving surfaces can be specified by means of a displacement function $f\mu^2\epsilon(\Theta)$† which, for each value of the normalized off-axis distance Θ, measures to a sufficient approximation the focus shift of the receiving surface relative to a selected *field-reference surface.*

As field-reference surface we choose the Petzval surface of **S** imaging from the given object surface, namely the spherical surface, of curvature $1/\rho_P$ given by the equation

$$\frac{1}{\rho_P}-\frac{1}{\rho_0}=\sum_i\left[\frac{n_i-1}{n_i}\left(\frac{1}{r_2^{(i)}}-\frac{1}{r_1^{(i)}}\right)\quad\text{or}\quad\frac{-2}{r_i}\right], \tag{1.3}$$

which cuts the optic axis orthogonally at the paraxial focus. The quantities r on the right of (1.3) stand for the paraxial radii of curvature of the optical surfaces (including figurings); these radii, like ρ_P, are taken positively when the surface is concave towards the incident light;‡ $1/\rho_0$ is the curvature of the object surface.

When the Seidel aberrations dominate the image, or, in particular, when **S** is an aplanat,§ the order of magnitude of the image-spreads at best focus is $f\mu^3$ and the thickness of the image-layer is $O(f\mu^2)$. In this case we could use the Gauss plane as field-reference surface, with some gain in simplicity in those cases where a flat field is aimed at in the design. But when **S** is an anastigmat, with image-spreads of order $f\mu^5$ at best focus, the image-layer lies everywhere within a distance $O(f\mu^4)$ of the Petzval surface, and the Gauss plane could only be used as field-reference surface in systems for which the Petzval curvature $1/\rho_P = O(\mu^2/f)$. We call such systems *flat-fielded anastigmats.*

When the Petzval surface and not the Gauss plane is chosen as field-reference surface, the approximation technique which is used for the aplanats and for general centred systems can be adapted without

† Or $f\mu^4\epsilon(\Theta)$ in an anastigmat; it is more convenient to have $\epsilon(\Theta) = O(1)$ in both cases.

‡ We do not define the Petzval surface by applying (1.3) to the optical surfaces divested of their figurings, because it is desirable (e.g. in discussing the Schmidt camera and plate-mirror systems generally) to permit the presence of an r^2-term, of coefficient not exceeding $O(f\mu^4)$, in the function which defines the figuring depths. The position of the Petzval surface would not then be determined uniquely by the system and the object surface. Conrady's definition of the Petzval surface by means of 'thin radial pencils' has the practical inconvenience that the surface so defined is not strictly spherical.

§ By an aplanat we mean a system in which the first two Seidel errors (spherical aberration and coma) are reduced to very small values and the biggest remaining aberration is the off-axis astigmatism used to flatten the field; the image-spreads are $O(f\mu^3)$. By an anastigmat we mean a system in which the image-spreads are $O(f\mu^5)$ in the selected receiving surface. This approximates to the present commercial use of the terms.

essential change to the anastigmats. In the aplanats the displacement function $f\mu^2\epsilon(\Theta) = O(f\mu^2)$; in the anastigmats the displacement function $f\mu^4\epsilon(\Theta) = O(f\mu^4)$.

1.3. *The aberration function*

With each pencil of rays is associated an orthogonal family of surfaces called the *geometrical wave surfaces* or *wave-fronts*. If the rays of a pencil all pass through a single point, the wave-fronts are evidently spheres. This is the case of an aberration-free geometrical image. In the more usual case where the rays are not strictly concurrent, but pass within a distance $O(f\mu^3)$ of each other, the wave-fronts are no longer strictly spherical, but are distorted by amounts $O(f\mu^4)$ from the spherical form. Near a focus, the wave-fronts may now develop singularities; to avoid this complication we exclude from the discussion wave-fronts whose distance from a focus is small compared with f.

Each wave-front W is cut in a unique point Q by the ray through the point (u, v) in the exit pupil; we call (u, v) the *coordinate numbers* of the ray, and of the point Q on W. In its passage through the system, the ray also defines coordinate numbers (u, v) on the optical surfaces and in the entry pupil. In the entry pupil u, v agree, to within $O(\mu^2)$, with the values of the corresponding scale-normalized Cartesian coordinates there.

The *principal point* of W is the point of coordinate numbers $(0, 0)$ on W; when W is in the image-space, this is the intersection with W of the principal ray $O'P'$.

The *reference sphere of* W is defined in the image-space as the sphere, centred on P', which passes through the principal point of W. It can be defined in the intermediate image-spaces of the system as the sphere centred on the intersection of the ray $(0, 0)$ with the corresponding intermediate Petzval surface and passing through the principal point of W.

In a general centred system, W lies everywhere within a distance $O(f\mu^4)$ of its reference sphere; in the (final) image-space of an anastigmat, within a distance $O(f\mu^6)$.†

The *aberration function* $\phi(u, v; \Theta)$ is defined for each u, v as the distance, measured along the optical ray of coordinate numbers (u, v), by which W lags behind its reference sphere. Evidently $\phi(0, 0; \Theta) = 0$. As the wave surface progresses, $\phi(u, v; \Theta)$ remains unchanged to within $O(f\mu^6)$ in a general centred system; in the image-space of an anastigmat it remains unchanged to within $O(f\mu^{10})$.

† The truth of this statement depends on the fact that the receiving surface lies in the image-layer.

In an aplanat or in a general centred system we can write

$$\phi(u,v;\Theta) = f\mu^4\Phi(u,v;\Theta);$$

in an anastigmat we can write

$$\phi(u,v;\Theta) = f\mu^6\Phi(u,v;\Theta),$$

the function Φ being $O(1)$ in both cases.

We call ϕ a 'smooth' function when the order of magnitude of $\partial\phi/\partial u$ and $\partial\phi/\partial v$ is the same as that of ϕ itself. If the optical surfaces of **S** are spherical, then ϕ will be smooth. ϕ will still be smooth if the surfaces of **S** carry figurings of the type represented by an equation of the form

$$\zeta = f\mu^4\left[c_2\frac{x^2+y^2}{h^2}+c_4\left(\frac{x^2+y^2}{h^2}\right)^2\right]+f\mu^6c_6\left(\frac{x^2+y^2}{h^2}\right)^3+f\mu^8\chi\left(\frac{x^2+y^2}{h^2}\right), \quad (1.4)$$

where the coefficients c_2, c_4, c_6 are $O(1)$, h denotes the semi-diameter of the axial pencil at the surface under consideration, χ is $O(1)$ and a smooth function of its argument $(x^2+y^2)/h^2$, and ζ denotes the figuring depth, measured parallel to the z direction, at the surface-point of Cartesian coordinates x, y. When only the Seidel errors of the system are under discussion, the value of c_6 is irrelevant and (1.4) may be used in the simplified form

$$\zeta = f\mu^4\left[c_2\frac{x^2+y^2}{h^2}+c_4\left(\frac{x^2+y^2}{h^2}\right)^2\right]+O(f\mu^6). \quad (1.5)$$

Useful surface deformations of more general type than (1.5) are possible, for example those of the form

$$\zeta = f\mu^4\left[c_2\frac{x^2+y^2}{h^2}+c_4\left(\frac{x^2+y^2}{h^2}\right)^2+c_6'\left(\frac{x^2+y^2}{h^2}\right)^3\right]+O(f\mu^6),$$

where the error-term is a smooth function of $(x^2+y^2)/h^2$. Their effect is to introduce third-order aberrations of a different type from those occurring 'naturally' (that is to say, in centred systems with spherical surfaces); the latter are of the third order and the third degree.† When the object of introducing figurings is the better control of the classical aberration coefficients, it is appropriate to use figurings of the special form (1.4). We call these *normal figurings*.

The quantities x/h, y/h are evidently scale-normalized Cartesian coordinates on the figured surface, which agree to within $O(\mu^2)$ with the coordinate numbers u, v imprinted on this surface by the pencil from the axial object point. If the object point moves off axis in the negative

† That is to say, of the form $f\mu^3P(u,v;\Theta)+O(f\mu^5)$, where $P = O(1)$ is a polynomial of maximum total degree 3 in u, v, Θ and the error-term is a smooth function of u, v. Compare p. 16, below.

y-direction, the new coordinate numbers u, v on the surface are connected with x/h, y/h by equations

$$u = \frac{x}{h} + O(\mu^2), \qquad v = \frac{y}{h} + l\Theta + O(\mu^2),$$

in which the value of the constant l is given to the necessary accuracy by Gauss theory.

In a general centred system with normal figurings, imaging on the Petzval surface, the monochromatic aberration function

$$\phi(u, v; \Theta) = f\mu^4 \Phi_0^*(u, v; \Theta) + O(f\mu^6), \tag{1.6}$$

where

$$\Phi_0^*(u, v; \Theta) = \frac{H}{f\mu}[\tfrac{1}{4}a_1(u^2+v^2)^2 + a_2\,\Theta v(u^2+v^2) + \tfrac{1}{2}a_3\,\Theta^2(3u^2+v^2)] \tag{1.7}$$

and the coefficients a_1, a_2, a_3 are $O(1)$.† H stands for the semi-diameter of the entry pupil,‡ so that the factor $H/f\mu$ is comparable with unity.§

We call $f\mu^4\Phi_0^*(u, v; \Theta)$ the *Seidel aberration function* or the *fourth-order aberration function* of the system working on the Petzval surface. The corresponding aberration-deviations ξ_0, η_0 are given by the equation

$$(\xi_0, \eta_0) = \frac{f\mu}{H} f\mu^3 \left(\frac{\partial}{\partial u}, \frac{\partial}{\partial v}\right)\Phi_0^* + O(f\mu^5) \tag{1.8}$$

or

$$\xi_0 = \xi_0^* + O(f\mu^5), \qquad \eta_0 = \eta_0^* + O(f\mu^5),$$

where

$$\left.\begin{aligned} \xi_0^* &= f\mu^3[a_1 u(u^2+v^2) + 2a_2\,\Theta uv + 3a_3\,\Theta^2 u] \\ \eta_0^* &= f\mu^3[a_1 v(u^2+v^2) + a_2\,\Theta(u^2+3v^2) + a_3\,\Theta^2 v] \end{aligned}\right\}. \tag{1.9}$$

If, introducing scale-normalized polar coordinates (r, ϕ) in the exit pupil, we write

$$u = r\cos\phi, \qquad v = r\sin\phi, \tag{1.10}$$

equations (1.9) can be given the more compact form

$$\xi_0^* + i\eta_0^* = f\mu^3[a_1 r^3 e^{i\phi} + ia_2 r^2\Theta(2-e^{2i\phi}) + a_3 r\Theta^2(2e^{i\phi}-e^{-i\phi})]. \tag{1.11}$$

When the receiving surface is displaced forward from the Petzval surface by an amount $f\mu^2\epsilon(\Theta) + O(f\mu^4)$, the main term $\Phi^*(u, v; \Theta)$ is replaced by

$$\Phi^*(u, v; \Theta) = \Phi_0^*(u, v; \Theta) + \tfrac{1}{2}(u^2+v^2)\left(\frac{H}{f\mu}\right)^2 \epsilon(\Theta) \tag{1.12}$$

† See for example A. E. Conrady, *M.N.* **80** (1919), 320–8. The presence of the error-term in (1.6) allows a certain freedom of choice in the values assigned to a_1, a_2, a_3; they may be changed by amounts $O(\mu^2)$ without invalidating the representation.

‡ Not the exit pupil.

§ H/f is equal to the numerical aperture of the emerging pencil.

and approximate aberration displacements ξ_0^*, η_0^* by ξ^*, η^*, where

$$\xi^*+i\eta^* = f\mu^3\left[a_1 r^3 e^{i\phi}+ia_2 r^2\Theta(2-e^{2i\phi})+a_3 r\Theta^2(2e^{i\phi}-e^{-i\phi})+\frac{H}{f\mu}\epsilon(\Theta)re^{i\phi}\right], \tag{1.13}$$

the approximation error being $O(f\mu^5)$.

The equations

$$\xi = \xi^*+O(f\mu^5), \qquad \eta = \eta^*+O(f\mu^5) \tag{1.14}$$

give an evaluation in the desired form (viz. a leading term of manageable complexity plus an error-term) of the monochromatic aberrations in any axially symmetric receiving surface lying in the image-layer. Inside the square bracket of (1.13) the term $a_1 r^3 e^{i\phi}$ represents the primary spherical aberration, $ia_2 r^2\Theta(2-e^{2i\phi})$ the Seidel coma, $[2a_3\Theta^2+(H/f\mu)\epsilon(\Theta)]e^{i\phi}$ the focus shift relative to the reference surface, and $-a_3 r\Theta^2 e^{-i\phi}$ the off-axis astigmatism, the deviations being measured from the principal point of the image-patch. It makes no difference to the form of (1.13) and (1.14) if ξ, η be measured perpendicular to the normal drawn from the receiving surface at some point P'' of the image-patch (the η-direction being taken, as before, in the meridional plane through P''). For this alters the values of ξ, η only by $O(f\mu^5)$.

(1.13), (1.14) allow the determination, to a sufficient accuracy, of the best field surface corresponding to a given definition of image quality. A procedure which is both analytically convenient and physically acceptable in many practical applications is to define the effective radius of a single monochromatic image-patch as the square root of the expression

$$\rho^2 = \frac{1}{\pi}\iint\limits_{u^2+v^2\leqslant 1}(\xi^2+\eta^2)\,dudv \tag{1.15}$$

and the effective monochromatic image radius over the working field $V'\leqslant V_0'$ on a given receiving surface as the square root of

$$E = \frac{2}{\pi}\int\limits_0^1 \Theta\,d\Theta \iint\limits_{u^2+v^2\leqslant 1}(\xi^2+\eta^2)\,dudv. \tag{1.16}$$

This amounts to defining the effective radius of each image-patch as the radius of gyration of its ray-density distribution about its principal point† and the effective monochromatic image radius over the working field as the root mean square average of these effective radii over the field area $\Theta\leqslant 1$.

† The alternative (and in some ways more natural) definition by means of the radius of gyration about the centre of gravity of the image-patch will be considered later.