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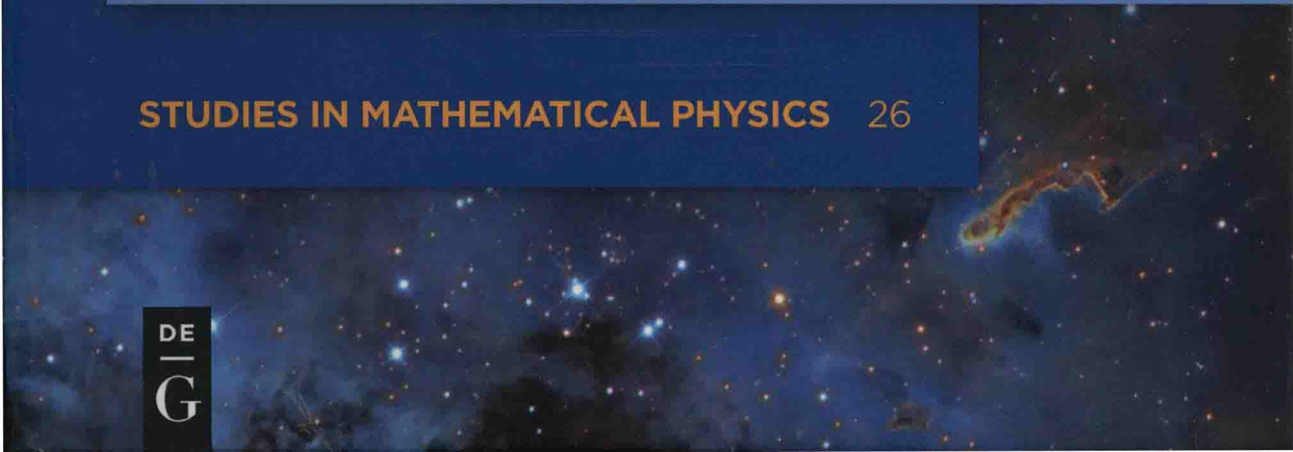
Vincent Laude

PHONONIC CRYSTALS

ARTIFICIAL CRYSTALS FOR SONIC, ACOUSTIC, AND ELASTIC WAVES

STUDIES IN MATHEMATICAL PHYSICS 26



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Preface

Phononic crystals are artificial periodic structures that can efficiently alter the flow of sound, acoustic waves, or elastic waves in solids. They were introduced about twenty years ago and have gained increasing interest since then, both because of their amazing physical properties and because of their potential applications. The topic of phononic crystals stands at the crossroad of physics (condensed matter physics, wave propagation in inhomogeneous and periodic media) and engineering (acoustics, ultrasonics, mechanical engineering, or electrical engineering). Phononic crystals cover a wide range of scales, from meter-size periodic structures for sound in air to nanometer-size structures for information processing or thermal phonon control in integrated circuits. I have tried to reflect this variety in the present book.

Phononic crystals have a definite relation with the topic of photonic crystals in optics. The marriage of phononic and photonic crystals also provides a promising structural basis for enhanced sound and light interaction in the context of optomechanics or of phoxonic crystals. Such interactions are not treated in the present book because the field is still new and rapidly evolving. Depending on how this book on phononic crystals is received, it may be a subsequent book project for the author.

Since the topic of phononic crystal is getting popular, it is nowadays presented and discussed at various international conferences. After the first ten years (1993–2004, approximately) during which the topic remained mainly theoretical with a few proof-of-concept demonstrations appearing in the literature, the latest evolution has been towards applications (including technology development), instrumentation (e.g. "seeing" phonons with optical systems), and novel designs (negative refraction, spatial dispersion control). The physical explanations for various effects are now quite well understood and efficient numerical methods and analysis tools have been developed. These are the reasons why the field has become mature for books. Other books that have appeared recently are general contributions, and although they are collections written by authoritative authors who are also current researchers in the field, they somehow lack the homogeneity of a monograph. The present book was written single handedly, which is both its strength and its weakness. Though it probably contains many mistakes and misses certain developments and contributions, it can but only reflect the sincere knowledge of its author. To all distinguished colleagues, collaborators, and often friends, I wish to present my apologies for any omissions in my text.

As a researcher, I first entered the field of phononic crystals sometime around 2002. Before that, I had been a researcher in optical information processing and wave propagation for a few years. The topic of elastic waves in solids is one I am particularly fond of, because of its formal perfection and simplicity when written in the mathematical language of tensors. The reader will probably identify many traces of this inclination toward applied mathematics in the text. This is especially true of the choice I have made of illustrating most numerical examples by results obtained using the finite element method. Part of the book may be used as an introduction to the Galerkin finite element method as applied to the solution of problems in the physics of wave propagation.

The level at which this monograph aims is rather advanced, at the master level or above. The author is a full-time researcher and has extensive experience in training post-graduate students. It is doubtful that lectures will be given focusing only on the topic of phononic crystals. The subject, however, is of interest to many scientists nowadays and I thought a global reference was missing.

This book is dedicated to those individuals who have had most influence on my scientific life so far. There have been many and I do not wish to enter into the embarrassing process of citing all their names. As an exception to this rule, I wish to express special thanks to Dr Abdelkrim Khelif and to Dr Sarah Benchabane with whom I have been researching phononic crystals for more than ten years. Finally, I am wholeheartedly grateful to my wife and family for their patience and support during all the writing of this book.

Besançon, March 31, 2015 Vincent Laude

To Jean-Pierre Laude, for whom this book represents most after myself

Contents

1 Introduction

Phononic crystals were officially born in 1993 after a famous paper by Kushwaha and co-authors [74] drawing the analogy to acoustic waves of the photonic crystal concept for light, which itself had been inspired by the theory of electrons in crystal lattices. Of course this statement is a simplification, and deciding that phononic crystals were born at a certain time is just putting a name on an idea that was ready to start living its own life as it detached from the tree of the physics of waves in periodic media. Over the last twenty years, the phononic concept has developed gracefully. It has been researched by many authors and has reached maturity in the sense that it became ready to be the subject of a monograph. The purpose of this book is to propose a self-contained presentation of the topic, going from the basics of the physics of waves upto the description of potential applications.

Preliminary comments. The topic of phononic crystals is clearly a branch of physical acoustics, and even more so a branch of the physics of waves. For completeness, in Chapter 2 we have gathered a synthetic presentation of many concepts of wave propagation in periodic media. The material in that chapter is elementary and independent from the concepts and the specific equations of physical acoustics. The presentation is limited to scalar waves, but introduces some important and fruitful tools such as Bloch's theorem, the band structure, evanescent waves, basic mechanisms for band gap formation, the classification of artificial crystals according to their Bravais lattices, and primitive cells and the first Brillouin zone. All these are used abundantly in the subsequent chapters.

The materials inside which waves will be considered to propagate can be broadly classified as either fluids (liquids, gases) or solids (natural crystals, amorphous solids, metals, piezoelectric solids, etc.). A note on terminology is needed here. Although the physical concepts are globally shared, the wave equations in fluids and solids differ mathematically. We will speak of acoustic waves as the pressure waves propagating in fluids; they can be described by a time-dependent scalar field in three-dimensional space, which we denote $p(t, \mathbf{x})$. In Chapter 3 we have gathered a self-consistent presentation of linear acoustic waves in fluids, including water and air. In the case of solids, elastic waves are described most conveniently by a vector field of particle displacements, which we denote $\mathbf{u}(t, \mathbf{x})$. The implication is that elastic waves have a polarization and that even homogeneous solids can be naturally anisotropic, including the important case of natural crystals. We shall try to reserve the adjective "acoustic" for pressure waves, but the practice in the literature is very often to use it for elastic waves in solids as well. Chapter 5 summarizes the topic of elastic waves in homogeneous solids, introducing such important concepts as anisotropy, piezoelectricity, surface elastic waves, and plate waves.

As the material in Chapters 2, 3, and 5 is not specific to phononic crystals, those chapters may be skipped by the knowledgeable reader. Please note, however, that frequent or implicit reference is made to their contents in the other chapters.

Artificial crystals. By artificial crystals we mean periodic material systems that were made by technology or craft. The adjective artificial is used to differentiate them from natural crystals, which are the topic of solid state physics and are the support of phonons – the particles of sound in matter. Artificial crystals usually have a periodicity measured by a length, the lattice constant, that is much larger than the interatomic distances in natural crystals. For this reason they can be treated macroscopically using the equations of continuum mechanics. Still elastic waves in solids can be considered as the asymptotic limit of acoustic phonons, which is a reason for the choice of the name "phononic crystal".

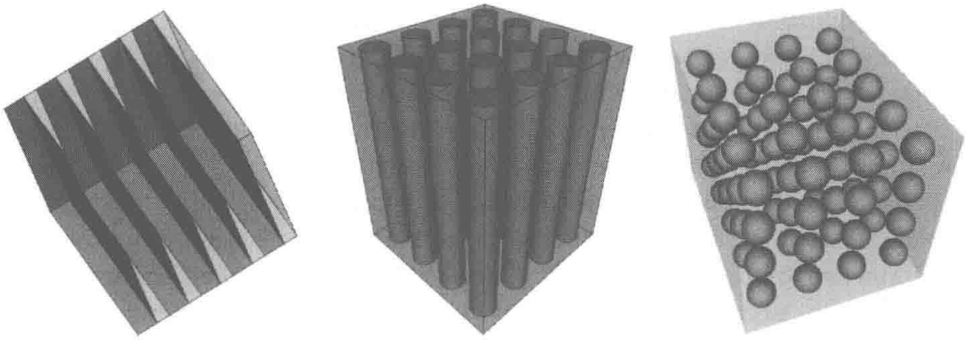

Fig. 1.1: Artificial crystals for waves in 1D, 2D, and 3D. The crystals depicted are composed of a periodic repetition of a certain distribution of solid or fluid matter, or possibly voids. The smallest volume that can be used to pave all space by periodicity is a primitive cell of the crystal. Material distribution is implicitly assumed to be continuous within the different regions but discontinuous at their interfaces.

The phononic crystal appeared around 1993, as we said earlier, and is an artificial crystal for elastic waves in solids. It is typically composed of a periodic alternation of at least two different materials, or a periodic heterostructure, as depicted in Figure 1.1. Depending on the number of periodicities, we shall classify them as one-dimensional (1D), two-dimensional (2D), or three-dimensional (3D) phononic crystals. Of course, it is perfectly possible to consider in theory phononic crystals defined from continuous variations of elastic parameters (mass density and elastic constants), but practical realizations almost always involve discontinuous regions of different materials. It is also perfectly possible to replace one material with holes, as the surface boundary conditions at the surface of the holes will provide the necessary scattering properties. As a note for readers interested in photonic crystals, a vacuum does not support elastic wave propagation, in contrast to optical waves in free space; this is a major difference between phononic and photonic crystals. Another difference is the polarization of elastic waves, which admit three independent modes of propagation in general, whereas there are two independent optical modes of propagation in dielectric materials. Phononic crystals are first introduced in Chapter 6.

Sonic crystals are artificial crystals for pressure waves in fluids. In the literature, the terms phononic crystal and sonic crystal are often used indifferently to encompass all cases of artificial crystals for mechanical waves. For clarity, we shall generally imply more restrictive and separate meanings: sonic crystals for acoustic or pressure waves, and phononic crystals for elastic waves. Sonic crystals are first introduced in Chapter 4 and support only one mode of propagation. They can be composed of a periodic distribution of different fluids, like air bubbles in water, but true voids or holes will not be permitted in practice. By extension, the very common case of solid inclusions in a fluid matrix will also be referred to as a sonic crystal. Indeed, although all three propagation modes of elastic waves exist in the solid inclusions, measurements of the properties of fluid-solid crystals are made via pressure waves. For exactly the inverse reason, fluid inclusions in a solid matrix will be termed a phononic crystal. Coupling of elastic and acoustic waves, and some associated effects, are discussed in Chapter 8.

As artificial crystals, sonic and phononic crystals owe a lot to condensed matter physics and borrow concepts that have been used to describe electronic and photonic band structures, as well as phonons in crystal lattices. The concept of Bloch waves – the natural modes of periodic media – is ubiquitous. Their dispersion properties are presented in diagrams called band structures, plotted with reference to first Brillouin zones, as Chapter 2 explains. Maybe the most salient property of artificial crystals is the existence of band gaps, or frequency ranges inside which propagating Bloch waves are absent whatever the wavenumber and thus propagation is not allowed. A partial band gap is valid for at least one direction in k-space, but not necessarily for others. A complete band gap is valid for all values of the wavevector $\mathbf{k}$, that is for any direction of propagation, but possibly for only one polarization mode. A full band gap is a complete band gap that is valid for all polarization modes and thus for all possible waves in the crystal. The distinction between full and complete band gaps is only meaningful for phononic crystals, both definitions being equivalent for sonic crystals. As a note, the distinction between full and complete band gap is not generally made in the literature, but we shall try to stick to it in this book for clarity. As an example, Figure 1.2 displays the band structure of a 2D square-lattice sonic crystal of steel rods in water, together with the corresponding first Brillouin zone. The diameter of the rods has been chosen such that a complete – or full in this case – band gap appears.

Mechanisms for band gap formation. It is perfectly possible to go a long way with sonic and phononic crystals with just numerical simulations, solving the acoustic or elastodynamic equations to obtain Bloch waves and to plot phononic band structures,

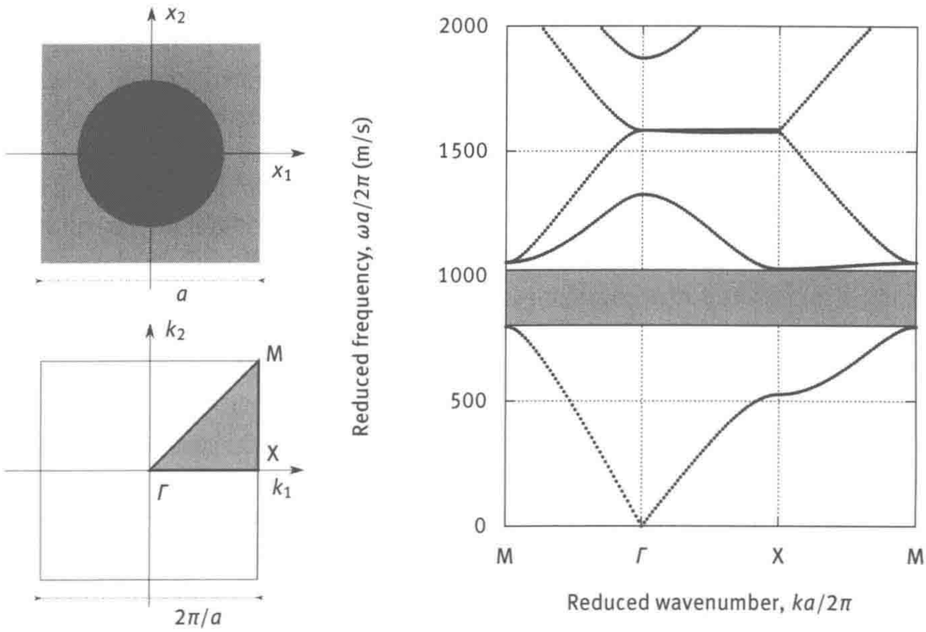

Fig. 1.2: Band structure example for a 2D square-lattice sonic crystal. The sonic crystal is composed of cylindrical steel rods in water. The ratio of the rod diameter to the lattice constant a is 0.83. The first Brillouin zone is sketched at the bottom left. For the square lattice, the first Brillouin zone is also a square whose high symmetry points are noted Γ, X, and M. The irreducible first Brillouin zone is further defined with respect to all symmetries of the unit cell and shaded in gray. The band structure on the right shows the dispersion of propagating Bloch waves as the relation between wavenumber k and angular frequency ω. It is plotted along the boundary of the irreducible first Brillouin zone as path [Γ–X–M–Γ]. The area shaded in gray in the band structure indicates a complete – or full, in the sonic crystal case, – band gap.

or computing the transmission through a finite crystal to locate frequency band gaps. Beyond numerical computation, however, an important question is the physical origin for the opening of a band gap. The most common and the historically first invoked physical mechanism is Bragg interference. Bragg scattering occurs at every periodic plane of scatterers inside a crystal. Bragg band gaps can be created anytime a forward and a backward propagating Bloch wave are phase matched, i.e. when they would have degenerated in the band structure space ($\mathbf{k}$, ω) if they did not couple. In the limit of a vanishingly small material contrast, such band crossings occur in the empty lattice model at every high symmetry point of the first Brillouin zone. This is the phenomenological reason why Bragg band gaps appear at symmetry points in reciprocal space, and hence along the irreducible Brillouin zone. In Figure 1.2, band foldings occur at points Γ, X, and M of the first Brillouin zone. The author however humbly recognizes he does not know of a mathematical demonstration of this property in the general case of medium or strong material contrast.