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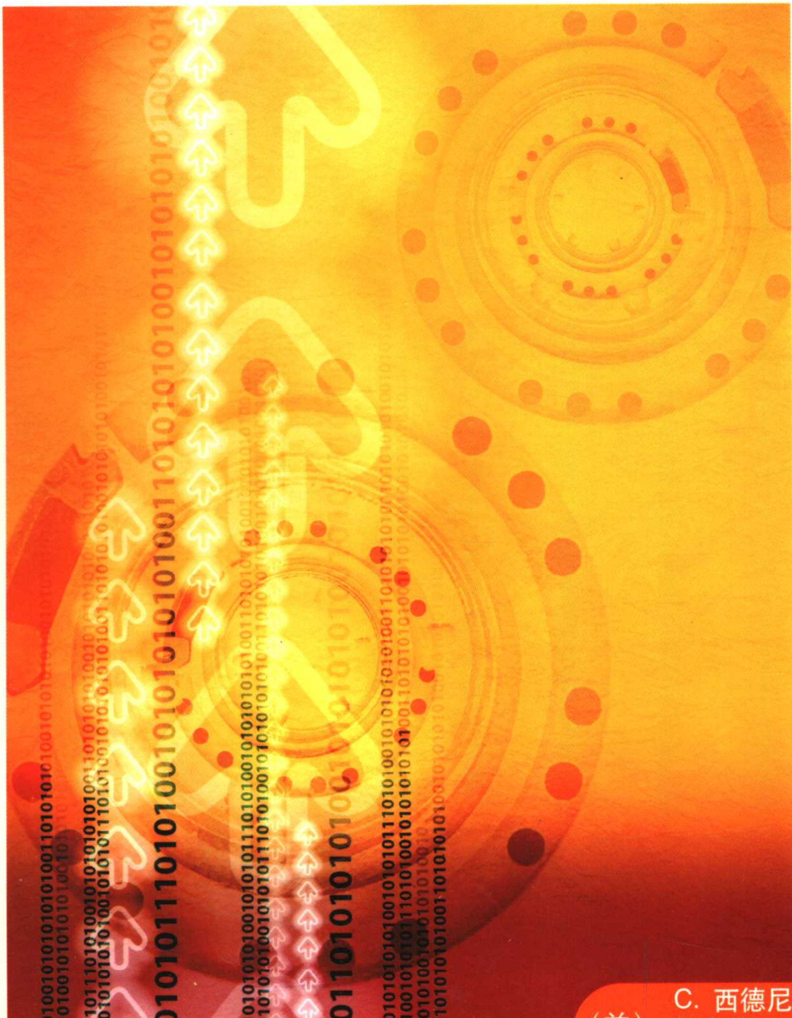
小波与小波变换导论

（英文版）



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（美） C. 西德尼·伯罗斯 拉米什 A. 戈皮那思 郭海涛 著

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前　言

本书详尽地讨论小波隐含的思想与小波的性质，并说明如何将小波作为信号处理、数值分析和数学建模的分析工具。我们力求采用工程师、科学工作者和应用数学研究人员都容易接受的一种方式来阐述，既作为一种理论步骤，也作为一种可能的解决问题的实用方法。虽然这门学科的起源可以上溯到较早的年代，但重新引起人们的兴趣并且有所进展仅有几年的光景。

小波方面早期的研究工作是由Morlet、Grossmann、Meyer、Mallat以及其他一些人在20世纪80年代完成的，但是1988年Ingrid Daubechies [Dau88a]的论文才在信号处理、统计学和数值分析等多个应用数学界引起更多人的注意。许多早期工作是在法国[CGT89，Mey92a]和美国[Dau88a，RBC*92，Dau92，RV91]开展的。与很多新兴学科一样，开创工作同特定的应用或传统的理论框架有着紧密的联系。本书中，我们将考察从应用中抽象和从研究中发展出来的理论，并且讨论这种理论同其他相关思想的关系。我们自己在信号处理方面的兴趣和背景无疑会影响本书的阐述方式。

小波研究的最新目标是创建一组基本函数（或通称为展开函数）和变换，用以对某个函数或信号给出丰富的、有效的和有用的描述。如果信号被表示成时间的函数，小波则在时间和频率（或尺度）两方面提供很有用的局部化。另外一个中心思想是多分辨，其中信号是通过细节的分辨而分解的。

对于傅里叶级数，选择正弦函数作为基本函数，然后考察所得展开式的性质。对于小波分析，人们先提出欲求的性质，然后推导出基本函数。小波的基本函数所具备的一个重要性质就是能够提供多分辨分析。出于多种原因，通常要求基本函数是正交的。在这些目标之下，你将会见到若干相关的技术，包括傅里叶变换、快速傅里叶变换、离散傅里叶变换、Wigner分布、滤波器组、子带编码以及由此产生的其他信号展开式和信号处理方法。

对于数学研究人员、科学工作者和工程师而言，基于小波的分析是新出现的一种令人兴奋的问题求解工具。它天生就能与数字计算机配合，因为其基本函数是由求和而不是求积和求导定义的。与大多数传统的展开式系统不同，小波分析的基本函数不是微分方程的解。就很多方面而言，它是我们多年来第一个真正意义上的新工具。事实上，使用小波和小波变换时，需要采用一种新观点和新方法来解释我们仍然在学习如何利用的表达形式。

新近由Donoho、Johnstone、Coifman及其他人进行的研究，为小波分析为何有如此广泛的应用和如此强大的功能增添了理论依据，并且对仍在进行的工作进行了概括。他们证明了小波系统有着某些固有的普适优势，而且对于一类广泛的问题而言是接近最优的[Don93b]。他们同时证明自适应工具可用于创建特殊信号和信号类的特定小波系统。

多分辨分解看起来是一种分离信号成分的方法，它比分析、处理和压缩信号的其他方法更为优越。由于离散小波变换能够在不同的独立尺度分解信号，而且非常灵活，所以Burke把小波称为“数学显微镜”[Bur94，Hub96]。正是因为这种强有力的和灵活的分解，在小波变换领域

内，对信号的线性和非线性处理为信号的检测、滤波和压缩提供了各种新方法[Don93b，Don95，Don93a，Sai94b，WTWB97，Guo97]。同时这可以用来作为健壮数值算法的基础。

读者也会看到这与数字信号处理的滤波器组理论之间有一种有趣的联系和等价性[Vai92，AH92]。其实，用滤波器组获得的某些结果与用离散时间小波所得结果相同，而且这在信号处理界已由Vetterli、Vaidyanathan、Smith、Barnwell和其他人研究得到。滤波器组以及计算小波变换的大多数算法，是更为一般的多频率系统和时变系统的组成部分。

对于那些具有一定技术背景但对小波知之甚少或者全然不知的人而言，本书可作为一本入门书或自学辅导教材。假定读者具备傅里叶级数和傅里叶变换、线性代数和矩阵论的知识，同时假定读者达到工学、理学或应用数学学士的同等技术水平。掌握一定的信号处理知识对阅读本书很有帮助，但并非是必需的。我们提出用实时变函数或复时变函数模拟一维信号的思想，但这样的思想以及方法在二维、三维甚至四维的图像表示和图像处理中被证明也是有效的[SA92，Mal89a]。向量空间已证明是研究小波理论与应用的天然工具。具有这个领域的某些背景知识是有益的，不过也可以在需要的时候补习。用某种小波软件系统运行实例和进行实验是大有裨益的。书后附有MATLAB库函数的程序，这些程序在我们的网站（在本前言后面提及）上也能找到。其他几种软件系统在第10章介绍。

介绍小波理论有几种不同的方式。我们选用连续时间信号或函数为级数展开式，同傅里叶分析中所用的傅里叶级数非常相似。我们从这种表达形式可以转到一种离散变量（如某种信号的样本）函数的展开式和滤波器组理论，以此有效地计算与解释展开式系数。这种情况类似于离散傅里叶变换（DFT）及其有效实现快速傅里叶变换（FFT）。我们也可以从级数展开式获得称为连续小波变换的积分变换，这同傅里叶变换或傅里叶积分相似。我们感到，从级数展开式出发可以充分领悟小波理论，而且很容易看清小波分析与傅里叶分析之间的异同。

本书分为若干相对自成一体的章节。前面几章对离散小波变换（DWT，这种变换把信号展开为小波和尺度函数的级数）进行了非常全面的讨论。后面几章简述离散小波变换的推广及其应用。各章均引用了许多其他著作，可以作为一种有注解的参考文献。由于本书旨在作为小波变换的导论，而在这个领域已经积累了大量的文献，所以我们在书后附上一个很长的文献目录。但是这个目录很快就会变成不完备的，因为有大量不断发表的论文。无论如何，对于作为导论这一目标而言，提供一个文献指南是非常重要的。

新近由美国科学院出版的一本书，书中由Barbara Burke撰写的一章[Bur94]对小波分析原理及其发展的历史作了很好的概述。Burke还写了此章的精彩扩充版本[Hub96]，这是任何对小波理论感兴趣的人应该阅读的。Daubechies在[Dau96]中对早期研究工作的历史有简要的介绍。

本书提出的很多结果和关系，是以定理和证明或以推导的形式阐述的。我们把重点放在定理陈述的正确性方面，而对定理的证明往往只给出推导的轮廓，其目的在于了解实质而非形式证明。事实上，为了不致使阐述凌乱，我们把很多证明放在附录中。但愿这种方式有助于读者深入理解信号处理中这个非常有趣但有时又显得含糊不清的新教学工具。

我们在书中采用的记号兼有信号处理文献和数学文献所用的记号，这样做是希望阐述思想与结果更易于理解，但是这会丧失某些一致性和清晰性。

作者对AFOSR、ARPA、NSF、Nortel公司、德州仪器公司和Aware公司所提供的支持表示

感谢。我们特别要感谢H. L. Resnikoff，是他首次把我们领入小波领域，而他又准确地预见到我们的能力并会取得成功。我们也感谢W. M. Lawton、小R. O. Wells、R. G. Baraniuk、J. E. Odegard、I. W. Selesnick、M. Lang、J. Tian和莱斯大学计算数学实验室的各位成员，本书中介绍的很多思想与结果是他们提出的。第一署名作者感谢家人Maxfield和Oshman的无私支持。莱斯大学EE-531和EE-696班的学生们提供了极有价值的反馈，他们是：Brucc Francis、Strela Vasily、Hans Sch ü ssler、Petter Steffen、Gary Sitton、Jim Lewis、Yves Angel、Curt Michel、J. H. Husoy、Kjersti Engan、Ken Castleman、Jeff Trinkle、Katherine Jones，与此有关的还有莱斯大学和其他地方的同事。

我们还要特别感谢Tom Robbins及其Prentice Hall的同事们的支持与帮助，他们的评审意见使本书的内容大为充实。

我们将乐于知道任何读者发现的本书中的任何错误或使人误解的论述。我们真诚欢迎对本书提出的任何改进意见。各种建议和评论可用电子邮件发至csb@rice.edu。涉及本书的软件、文章、勘误以及有关莱斯大学小波研究工作的其他信息，可以从网站http://www-dsp.rice.edu/和链接到正在展开小波研究的其他网站上找到。

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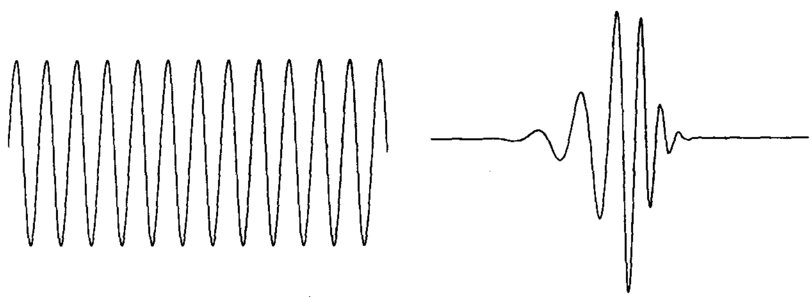
Contents

Chapter 1

Introduction to Wavelets

This chapter will provide an overview of the topics to be developed in the book. Its purpose is to present the ideas, goals, and outline of properties for an understanding of and ability to use wavelets and wavelet transforms. The details and more careful definitions are given later in the book.

A *wave* is usually defined as an oscillating function of time or space, such as a sinusoid. Fourier analysis is wave analysis. It expands signals or functions in terms of sinusoids (or, equivalently, complex exponentials) which has proven to be extremely valuable in mathematics, science, and engineering, especially for periodic, time-invariant, or stationary phenomena. A *wavelet* is a "small wave", which has its energy concentrated in time to give a tool for the analysis of transient, nonstationary, or time-varying phenomena. It still has the oscillating wave-like characteristic but also has the ability to allow simultaneous time and frequency analysis with a flexible mathematical foundation. This is illustrated in Figure 1.1 with the wave (sinusoid) oscillating with equal amplitude over $-\infty \leq t \leq \infty$ and, therefore, having infinite energy and with the wavelet having its finite energy concentrated around a point.



(a) A Sine Wave (b) Daubechies' Wavelet ψ_{D20}

Figure 1.1. A Wave and a Wavelet

We will take wavelets and use them in a series expansion of signals or functions much the same way a Fourier series uses the wave or sinusoid to represent a signal or function. The signals are functions of a continuous variable, which often represents time or distance. From this series expansion, we will develop a discrete-time version similar to the discrete Fourier transform where the signal is represented by a string of numbers where the numbers may be samples of a signal, samples of another string of numbers, or inner products of a signal with some expansion set. Finally, we will briefly describe the continuous wavelet transform where both the signal and the transform are functions of continuous variables. This is analogous to the Fourier transform.

1.1 Wavelets and Wavelet Expansion Systems

Before delving into the details of wavelets and their properties, we need to get some idea of their general characteristics and what we are going to do with them [Swe96b].

What is a Wavelet Expansion or a Wavelet Transform?

A signal or function $f(t)$ can often be better analyzed, described, or processed if expressed as a linear decomposition by

$$f(t) = \sum_{\ell} a_{\ell} \, \psi_{\ell}(t) \tag{1.1}$$

where ℓ is an integer index for the finite or infinite sum, a_{ℓ} are the real-valued expansion coefficients, and $\psi_{\ell}(t)$ are a set of real-valued functions of t called the expansion set. If the expansion (1.1) is unique, the set is called a *basis* for the class of functions that can be so expressed. If the basis is orthogonal, meaning

$$\langle \psi_k(t), \psi_{\ell}(t) \rangle = \int \psi_k(t) \, \psi_{\ell}(t) \, dt = 0 \qquad k \neq \ell, \tag{1.2}$$

then the coefficients can be calculated by the *inner product*

$$a_k = \langle f(t), \psi_k(t) \rangle = \int f(t) \, \psi_k(t) \, dt. \tag{1.3}$$

One can see that substituting (1.1) into (1.3) and using (1.2) gives the single a_k coefficient. If the basis set is not orthogonal, then a dual basis set $\tilde{\psi}_k(t)$ exists such that using (1.3) with the dual basis gives the desired coefficients. This will be developed in Chapter 2.

For a Fourier series, the orthogonal basis functions $\psi_k(t)$ are $\sin(k\omega_0 t)$ and $\cos(k\omega_0 t)$ with frequencies of $k\omega_0$. For a Taylor's series, the nonorthogonal basis functions are simple monomials t^k, and for many other expansions they are various polynomials. There are expansions that use splines and even fractals.

For the *wavelet expansion*, a two-parameter system is constructed such that (1.1) becomes

$$f(t) = \sum_k \sum_j a_{j,k} \, \psi_{j,k}(t) \tag{1.4}$$

where both j and k are integer indices and the $\psi_{j,k}(t)$ are the wavelet expansion functions that usually form an orthogonal basis.

The set of expansion coefficients $a_{j,k}$ are called the *discrete wavelet transform* (DWT) of $f(t)$ and (1.4) is the inverse transform.

What is a Wavelet System?

The wavelet expansion set is not unique. There are many different wavelets systems that can be used effectively, but all seem to have the following three general characteristics [Swe96b].

1. A wavelet system is a set of *building blocks* to construct or represent a signal or function. It is a two-dimensional expansion set (usually a basis) for some class of one- (or higher) dimensional signals. In other words, if the wavelet set is given by $\psi_{j,k}(t)$ for indices of $j,k = 1,2,\cdots$, a linear expansion would be $f(t) = \sum_k \sum_j a_{j,k}\,\psi_{j,k}(t)$ for some set of coefficients $a_{j,k}$.

2. The wavelet expansion gives a time-frequency *localization* of the signal. This means most of the energy of the signal is well represented by a few expansion coefficients, $a_{j,k}$.

3. The calculation of the coefficients from the signal can be done *efficiently*. It turns out that many wavelet transforms (the set of expansion coefficients) can calculated with $O(N)$ operations. This means the number of floating-point multiplications and additions increase linearly with the length of the signal. More general wavelet transforms require $O(N\log(N))$ operations, the same as for the fast Fourier transform (FFT) [BP85].

Virtually all wavelet systems have these very general characteristics. Where the Fourier series maps a one-dimensional function of a continuous variable into a one-dimensional sequence of coefficients, the wavelet expansion maps it into a two-dimensional array of coefficients. We will see that it is this two-dimensional representation that allows localizing the signal in both time and frequency. A Fourier series expansion localizes in frequency in that if a Fourier series expansion of a signal has only one large coefficient, then the signal is essentially a single sinusoid at the frequency determined by the index of the coefficient. The simple time-domain representation of the signal itself gives the localization in time. If the signal is a simple pulse, the location of that pulse is the localization in time. A wavelet representation will give the location in both time and frequency simultaneously. Indeed, a wavelet representation is much like a musical score where the location of the notes tells when the tones occur and what their frequencies are.

More Specific Characteristics of Wavelet Systems

There are three additional characteristics [Swe96b, Dau92] that are more specific to wavelet expansions.

1. All so-called first-generation wavelet systems are generated from a single scaling function or wavelet by simple *scaling* and *translation*. The two-dimensional parameterization is achieved from the function (sometimes called the generating wavelet or mother wavelet) $\psi(t)$ by
$$\psi_{j,k}(t) = 2^{j/2}\,\psi(2^j t - k) \qquad j,k \in \mathbf{Z} \tag{1.5}$$
where $\mathbf{Z}$ is the set of all integers and the factor $2^{j/2}$ maintains a constant norm independent of scale j. This parameterization of the time or space location by k and the frequency or scale (actually the logarithm of scale) by j turns out to be extraordinarily effective.

2. Almost all useful wavelet systems also satisfy the *multiresolution* conditions. This means that if a set of signals can be represented by a weighted sum of $\varphi(t-k)$, then a larger set (including the original) can be represented by a weighted sum of $\varphi(2t-k)$. In other words, if the basic expansion signals are made half as wide and translated in steps half as wide, they will represent a larger class of signals exactly or give a better approximation of any signal.

3. The lower resolution coefficients can be calculated from the higher resolution coefficients by a tree-structured algorithm called a *filter bank.* This allows a very efficient calculation of the expansion coefficients (also known as the discrete wavelet transform) and relates wavelet transforms to an older area in digital signal processing.

The operations of translation and scaling seem to be basic to many practical signals and signal-generating processes, and their use is one of the reasons that wavelets are efficient expansion functions. Figure 1.2 is a pictorial representation of the translation and scaling of a single mother wavelet described in (1.5). As the index k changes, the location of the wavelet moves along the horizontal axis. This allows the expansion to explicitly represent the location of events in time or space. As the index j changes, the shape of the wavelet changes in scale. This allows a representation of detail or resolution. Note that as the scale becomes finer (j larger), the steps in time become smaller. It is both the narrower wavelet and the smaller steps that allow representation of greater detail or higher resolution. For clarity, only every fourth term in the translation ($k = 1, 5, 9, 13, \cdots$) is shown, otherwise, the figure is a clutter. What is not illustrated here but is important is that the shape of the basic mother wavelet can also be changed. That is done during the design of the wavelet system and allows one set to well-represent a class of signals.

For the Fourier series and transform and for most signal expansion systems, the expansion functions (bases) are chosen, then the properties of the resulting transform are derived and

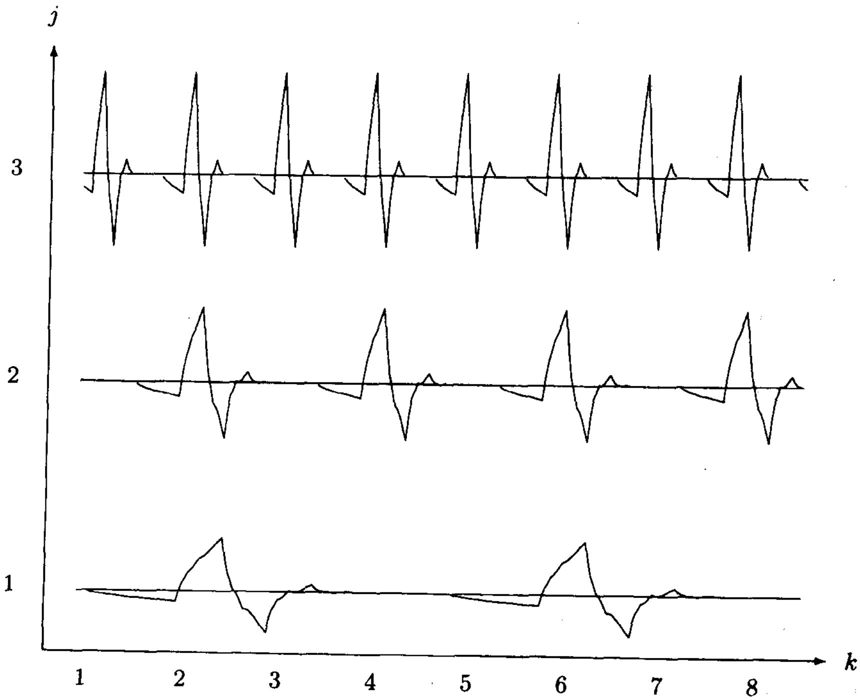


Figure 1.2. Translation (every fourth k) and Scaling of a Wavelet ψ_{D4}

analyzed. For the wavelet system, these properties or characteristics are mathematically required, then the resulting basis functions are derived. Because these constraints do not use all the degrees of freedom, other properties can be required to customize the wavelet system for a particular application. Once you decide on a Fourier series, the sinusoidal basis functions are completely set. That is not true for the wavelet. There are an infinity of very different wavelets that all satisfy the above properties. Indeed, the understanding and design of the wavelets is an important topic of this book.

Wavelet analysis is well-suited to transient signals. Fourier analysis is appropriate for periodic signals or for signals whose statistical characteristics do not change with time. It is the localizing property of wavelets that allow a wavelet expansion of a transient event to be modeled with a small number of coefficients. This turns out to be very useful in applications.

Haar Scaling Functions and Wavelets

The multiresolution formulation needs two closely related basic functions. In addition to the wavelet $\psi(t)$ that has been discussed (but not actually defined yet), we will need another basic function called the *scaling function* $\varphi(t)$. The reasons for needing this function and the details of the relations will be developed in the next chapter, but here we will simply use it in the wavelet expansion.

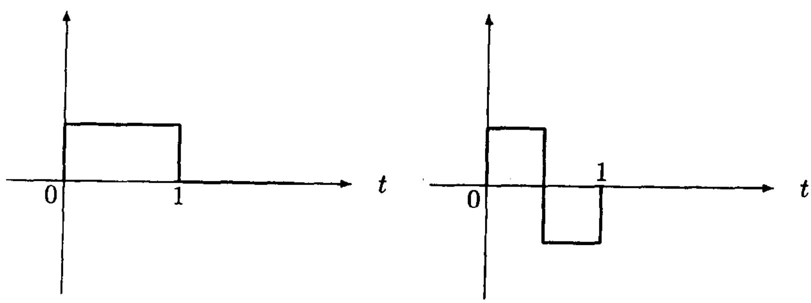
The simplest possible orthogonal wavelet system is generated from the Haar scaling function and wavelet. These are shown in Figure 1.3. Using a combination of these scaling functions and wavelets allows a large class of signals to be represented by

$$f(t) = \sum_{k=-\infty}^{\infty} c_k\, \varphi(t-k) + \sum_{k=-\infty}^{\infty} \sum_{j=0}^{\infty} d_{j,k}\, \psi(2^j t - k). \tag{1.6}$$

Haar [Haa10] showed this result in 1910, and we now know that wavelets are a generalization of his work. An example of a Haar system and expansion is given at the end of Chapter 2.

What do Wavelets Look Like?

All Fourier basis functions look alike. A high-frequency sine wave looks like a compressed low-frequency sine wave. A cosine wave is a sine wave translated by 90^o or $\pi/2$ radians. It takes a



(a) $\phi(t)$ (b) $\psi(t)$

Figure 1.3. Haar Scaling Function and Wavelet