

Wiley Finance Series

Derivatives Analytics with Python

*Data Analysis, Models, Simulation,
Calibration and Hedging*

YVES HILPISCH

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Calibration and Hedging*

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Preface

This book is an outgrowth of diverse activities of myself and colleagues of mine in the fields of financial engineering, computational finance and Python programming at our company The Python Quants GmbH on the one hand and of teaching mathematical finance at Saarland University on the other hand.

The book is targeted at practitioners, researchers and students interested in the market-based valuation of options from a practical perspective, i.e. the single numerical and technical implementation steps that make up such an effort. It is also for those who want to learn how Python can be used for derivatives analytics and financial engineering. However, apart from being primarily practical and implementation-oriented, the book also provides the necessary theoretical foundations and numerical tools.

My hope is that the book will contribute to the increasing acceptance of Python in the financial community, and in particular in the analytics space. If you are interested in getting the Python scripts and IPython Notebooks accompanying the book, you should visit <http://wiley.quant-platform.com> where you can register for the Quant Platform which allows browser-based, interactive and collaborative financial analytics. Further resources are found on the website <http://derivatives-analytics-with-python.com>. You should also check out the open source library DX Analytics under <http://dx-analytics.com> which implements the concepts and methods presented in the book in standardized, reusable fashion.

I thank my family—and in particular my wife Sandra—for their support and understanding that such a project requires many hours of solitude. I also want to thank my colleague Michael Schwed for his continuous help and support. In addition, I thank Alain Ledon and Riaz Ahmad for their comments and feedback. Discussions with participants of seminars and my lectures at Saarland University also helped the project significantly. Parts of this book have benefited from talks I have given at diverse Python and finance conferences over the years.

I dedicate this book to my lovely son Henry Nikolaus whose direct approach to living and clear view of the world I admire.

YVES HILPISCH
Saarland, February 2015

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