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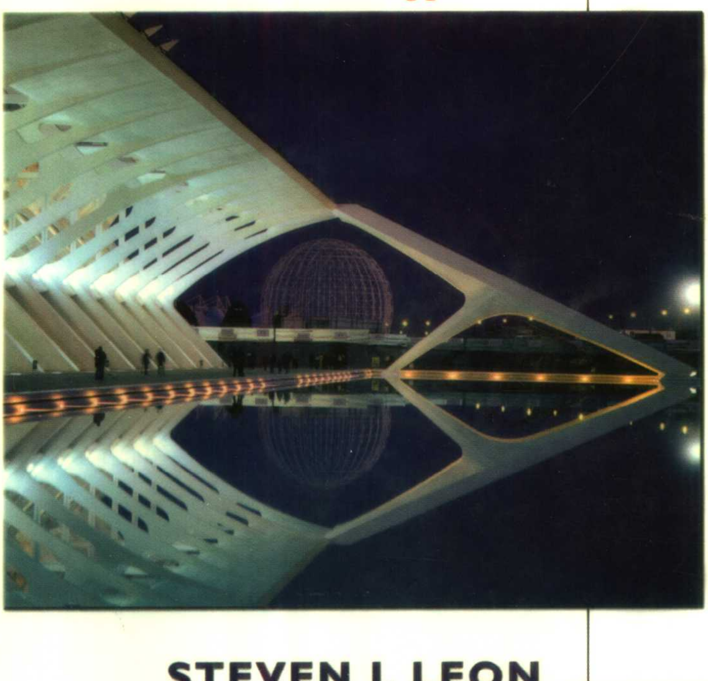
LINEAR ALGEBRA

With Applications



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机械工业出版社
China Machine Press

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Linear Algebra with Applications

(Seventh Edition)

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PREFACE

The author is pleased to see the text reach its seventh edition. The continued support and enthusiasm of the many users has been most gratifying. Linear algebra is more exciting now than at almost any time in the past. Its applications continue to spread to more and more fields. Largely due to the computer revolution of the last half century, linear algebra has risen to a role of prominence in the mathematical curriculum rivaling that of calculus. Modern software has also made it possible to dramatically improve the way the course is taught. The author teaches linear algebra every semester and continues to seek new ways to optimize student understanding. For this edition every chapter has been carefully scrutinized and enhanced. Additionally, many of the revisions in this edition are due to the helpful suggestions received from users and reviewers. Consequently, this new edition, while retaining the essence of previous editions, incorporates a variety of substantive improvements.

WHAT'S NEW IN THE SEVENTH EDITION?

1. New and Expanded Applications

The following new applications have been added for this edition.

(a) The yaw, pitch, and roll of an airplane are presented in terms of linear transformations.
(b) The orientation of a space shuttle is determined as an application involving eigenvectors.
(c) Google™ is currently the leading search engine used by Web surfers. In Chapter 6 we see how the search engine makes use of linear algebra to rank and order Web pages.
(d) The treatment of Markov chains has been expanded and a new theorem is presented.
(e) Markov chains are revisited at the end of Chapter 6.

2. Chapter Tests and New Exercises

The previous edition had a true-false chapter test at the end of each chapter. A second chapter test with 10 to 12 workout problems has been added to each chapter. Additionally new exercises have been added to many of the Exercise Sections in the book.

3. New Figures

Seven new figures have been added for this edition.

4. MATLAB Updates

The MATLAB® Appendix has been updated and expanded. All MATLAB exercises in the book have been checked for compatibility with MATLAB 7.0 (the latest version of MATLAB). Two MATLAB exercises have been replaced with new exercises.

5. Special Web Site and Supplemental Web Materials

Prentice Hall has developed a special Web site to accompany this book. This site includes a host of materials for both students and instructors. The Web pages are being revised for the seventh edition as we go to press. You can also download two supplemental chapters for this book using a link from the Prentice Hall site. The additional chapters are:

- Chapter 8. Iterative Methods
- Chapter 9. Canonical Forms

The URL for the site is listed in the Supplementary Materials section of this Preface.

6. *The ATLAST Companion Computer Manual*, Second edition

ATLAST (Augmenting the Teaching of Linear Algebra using Software Tools) is an NSF sponsored project to encourage and facilitate the use of software in the teaching of linear algebra. During a five year period, 1992–1997, the ATLAST Project conducted 18 faculty workshops using the MATLAB software package. Participants in these workshops designed computer exercises, projects, and lesson plans for software-based teaching of linear algebra. A selection of these materials was published as a manual in 1997. That manual has now been greatly expanded in a new edition, *ATLAST Computer Exercises for Linear Algebra, Second edition* (Prentice Hall, 2003). The ATLAST book is available as a *free* companion volume to this textbook when the two books are wrapped together for class orders. The ISBN for ordering the two-book bundle is given in the Supplementary Materials section of this Preface. The collection of software tools (M-files) developed to accompany the ATLAST book may be downloaded from the ATLAST Web site. Mathematica users can download the collection of *ATLAST Mathematica Notebooks* that has been developed by Richard Neidinger.

7. Companion Manuals

A number of other MATLAB and Maple computer manuals are available from Prentice-Hall. These are offered at no extra cost when ordered with the textbook as a two-book bundle. The list of bundles and the corresponding ISBN's for placing orders are given in the Supplementary Materials section of this Preface.

8. Student Guide to Linear Algebra with Applications

A new student study guide has been developed to accompany this edition. The guide is described in the Supplementary Materials section of this preface.

9. Other Changes

In preparing the seventh edition, the author has carefully reviewed every section of the book. In addition to the major changes that have been listed, many minor improvements have been made throughout the text.

COMPUTER EXERCISES

This edition contains a section of computing exercises at the end of each chapter. These exercises are based on the software package MATLAB. The MATLAB Appendix in the book explains the basics of using the software. MATLAB has the advantage that it is a powerful tool for matrix computations and yet it is easy to learn. After reading the Appendix, students should be able to do the computing exercises without having to refer to any other software books or manuals. To help students get started we recommend one 50 minute classroom demonstration of the software. The assignments can be done either as ordinary homework assignments or as part of a formally scheduled computer laboratory course.

As mentioned previously, the ATLAST book is available as a companion volume to supplement the computer exercises in this book. Each of the eight chapters of the ATLAST book contains a section of short exercises and a section of longer projects.

While the course can be taught without any reference to the computer, we believe that computer exercises can greatly enhance student learning and provide a new dimension to linear algebra education. The Linear Algebra Curriculum Study Group has recommended that technology be used for a first course in linear algebra. and this view is generally accepted throughout the greater mathematics community.

OVERVIEW OF TEXT

This book is suitable for either a sophomore-level course or for a junior/senior-level course. The student should have some familiarity with the basics of differential and integral calculus. This prerequisite can be met by either one semester or two quarters of elementary calculus.

If the text is used for a sophomore-level course, the instructor should probably spend more time on the early chapters and omit many of the sections in the later chapters. For more advanced courses a quick review of many of the topics in the first two chapters and then a more complete coverage of the later chapters would be appropriate. The explanations in the text are given in sufficient detail so that beginning students should have little trouble reading and understanding the material. To further aid the student, a large number of examples have been worked out completely. Additionally, computer exercises at the end of each chapter give students the opportunity to perform numerical experiments and try to generalize the results. Applications are presented throughout the book. These applications can be used to motivate new material or to illustrate the relevance of material that has already been covered.

The text contains all the topics recommended by the National Science Foundation (NSF) sponsored Linear Algebra Curriculum Study Group (LACSG) and much more. Although there is more material than can be covered in a one-quarter or one-semester course, it is the author's feeling that it is easier for an instructor to leave out or skip material than it is to supplement a book with outside material. Even if many topics are omitted, the book should still provide students with a feeling for the overall scope of the subject matter. Furthermore, many students may use the book later as a reference and consequently may end up learning many of the omitted topics on their own.

In the next section of this preface a number of outlines are provided for one-semester courses at either the sophomore level or the junior/senior level and with either a matrix-oriented emphasis or a slightly more theoretical emphasis. To further aid the instructor in the choice of topics, three sections have been designated as optional and are marked with an dagger in the table of contents. These sections are not prerequisites for any of the following sections in the book. They may be skipped without any loss of continuity.

Ideally the entire book could be covered in a two-quarter or two-semester sequence. Although two semesters of linear algebra has been recommended by the LACSG, it is still not practical at many universities and colleges. At present there is no universal agreement on a core syllabus for a second course. Indeed, if all of the topics that instructors would like to see in a second course were included in a single volume, it would be a weighty (and expensive) book. An effort has been made in this text to cover all of the basic linear algebra topics that are necessary for modern applications. Furthermore, two additional chapters for a second course are available for downloading from the Internet. See the special Prentice Hall Web page discussed earlier.

SUGGESTED COURSE OUTLINES

I. Two-Semester Sequence:

In a two semester sequence it is possible to cover all 39 sections of the book. Additional flexibility is possible by omitting any of the three optional sections in Chapters 2, 5, and 6. One could also include an extra lecture demonstrating how to use the MATLAB software.

II. One-Semester Sophomore-Level Course

A. A Basic Sophomore-Level Course

Chapter 1	Sections 1–5		7 lectures
Chapter 2	Sections 1–2		2 lectures
Chapter 3	Sections 1–6		9 lectures
Chapter 4	Sections 1–3		4 lectures
Chapter 5	Sections 1–6		9 lectures
Chapter 6	Sections 1–3		4 lectures
		Total	35 lectures

B. The LACSG Matrix Oriented Course:

The core course recommended by the Linear Algebra Curriculum Study involves only the Euclidean vector spaces. Consequently, for this course you should omit Section 1 of Chapter 3 (on general vector spaces) and all references and exercises involving function spaces in Chapters 3 to 6. All of the topics in the LACSG core syllabus are included in the text. It is not necessary to introduce any supplementary materials. The LACSG recommended 28 lectures to cover the core material. This is possible if the class is taught in lecture format with an additional recitation section meeting once a week. If the course is taught without recitations, the author feels that the following schedule of 35 lectures is perhaps more reasonable.

Chapter 1	Sections 1–5	7 lectures
Chapter 2	Sections 1–2	2 lectures
Chapter 3	Sections 2–6	7 lectures
Chapter 4	Sections 1–3	2 lecture
Chapter 5	Sections 1–6	9 lectures
Chapter 6	Sections 1,3–5	8 lectures
	Total	35 lectures

III. One-Semester Junior/Senior-Level Courses:

The coverage in an upper division course is dependent on the background of the students. Below are two possible courses with 35 lectures each.

A. Course 1

Chapter 1	Sections 1–5	6 lectures
Chapter 2	Sections 1–2	2 lectures
Chapter 3	Sections 1–6	7 lectures
Chapter 5	Sections 1–6	9 lectures
Chapter 6	Sections 1–7 Section 8 if time allows	10 lectures
Chapter 7	Section 4	1 lecture

B. Course 2

Review of Topics in Chapters 1–3		5 lectures
Chapter 4	Sections 1–3	2 lectures
Chapter 5	Sections 1–6	10 lectures
Chapter 6	Sections 1–7 Section 8 if time allows	11 lectures
Chapter 7	Sections 4–7 If time allows, Sections 1–3	7 lectures

SUPPLEMENTARY MATERIALS

The following books are offered at no extra cost when bundled with this textbook. In each case the ISBN is given for ordering the bundle.

- **ATLAST Computer Exercises for Linear Algebra.** ISBN. 0-13-169817-6.
- **Linear Algebra Labs with MATLAB: 3rd ed.** by David Hill and David Zitarelli, ISBN. 0-13-169816-8.
- **Visualizing Linear Algebra using Maple,** by Sandra Keith, ISBN. 0-13-169818-4.
- **Maple Supplement for Linear Algebra,** by John Maloney, ISBN. 0-13-169819-2.
- **Understanding Linear Algebra Using MATLAB,** by Irwin and Margaret Kleinfeld, ISBN. 0-13-169820-6.
- **Student Guide to Linear Algebra with Applications.** ISBN. 0-13-224555-8 The manual is available to students as a study to accompany this textbook. The manual summarizes important theorems, definitions, and concepts presented

in the textbook. It provides solutions to some of the exercises and hints and suggestions on many other exercises.

Additional Supplements

- The **ATLAST Mathematica Notebooks** contain Mathematica versions of many of the exercises and projects in the ATLAST book. The *Mathematica Notebooks* may be downloaded for free from the ATLAST Web site.

  www.umassd.edu/specialprograms/atlast

- **Instructor's Solution Manual.** ISBN: 0-13-193625-5 A solutions manual is available to all instructors teaching from this book. The manual contains complete solutions to all the nonroutine exercises in the book. The manual also contains answers to many of the elementary exercises that were not already listed in the answer key section of the book.
- **Web Supplements.** The Prentice Hall Web site for this book has an impressive collection of supplementary materials. The URL for the Web site is:

  www.prenhall.com/leon

ACKNOWLEDGMENTS

The author would like to express his gratitude to the long list of reviewers that have contributed so much to all seven editions of this book. Thanks also to the many users who have sent in comments and suggestions.

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Many of the revisions and new exercises in this latest edition are a direct result of their comments and suggestions.

Thanks to Diego Casadei for supplying the space shuttle figure for a new application in Chapter 6. Special thanks to Mathematics Editor, George Lobell, for his help in planning the new edition and to Production Manager Bob Walters for his work on coordinating production of the book. Thanks to the entire editorial, production, and sales staff at Prentice Hall for all their efforts.

The author would like to acknowledge the contributions of Gene Golub and Jim Wilkinson. Most of the first edition of the book was written in 1977–1978 while the

author was a Visiting Scholar at Stanford University. During that period the author attended courses and lectures on numerical linear algebra given by Gene Golub and J. H. Wilkinson. Those lectures have greatly influenced this book. Finally, the author would like to express his gratitude to Germund Dahlquist for his helpful suggestions on earlier editions of this book. Although Germund Dahlquist and Jim Wilkinson are no longer with us, they continue to live on in the memories of their friends.

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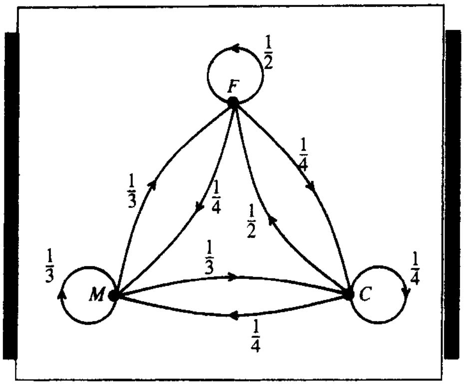
CONTENTS

[†] indicates an optional sections. These sections are not prerequisites for any other sections of the book.
*Web: The Supplemental Chapters 8 and 9 can be downloaded from the web page: (www.prenhall.com/Leon)



C H A P T E R

1

MATRICES AND SYSTEMS OF EQUATIONS

Probably the most important problem in mathematics is that of solving a system of linear equations. Well over 75 percent of all mathematical problems encountered in scientific or industrial applications involve solving a linear system at some stage. By using the methods of modern mathematics, it is often possible to take a sophisticated problem and reduce it to a single system of linear equations. Linear systems arise in applications to such areas as business, economics, sociology, ecology, demography, genetics, electronics, engineering, and physics. Therefore, it seems appropriate to begin this book with a section on linear systems.

1 SYSTEMS OF LINEAR EQUATIONS

A *linear equation in n unknowns* is an equation of the form

$$a_1x_1 + a_2x_2 + \cdots + a_nx_n = b$$

where $a_1, a_2, \ldots, a_n$ and b are real numbers and $x_1, x_2, \ldots, x_n$ are variables. A *linear system* of m equations in n unknowns is then a system of the form

$$\begin{array}{rcccccccl} a_{11}x_1 & + & a_{12}x_2 & + & \cdots & + & a_{1n}x_n & = & b_1 \\ a_{21}x_1 & + & a_{22}x_2 & + & \cdots & + & a_{2n}x_n & = & b_2 \\ \vdots & & & & & & & & \\ a_{m1}x_1 & + & a_{m2}x_2 & + & \cdots & + & a_{mn}x_n & = & b_m \end{array} \qquad (1)$$

where the a_{ij}'s and the b_i's are all real numbers. We will refer to systems of the form (1) as $m \times n$ linear systems. The following are examples of linear systems:

$$\textbf{(a)}\quad \begin{aligned} x_1 + 2x_2 &= 5 \\ 2x_1 + 3x_2 &= 8 \end{aligned} \qquad \textbf{(b)}\quad \begin{aligned} x_1 - x_2 + x_3 &= 2 \\ 2x_1 + x_2 - x_3 &= 4 \end{aligned} \qquad \textbf{(c)}\quad \begin{aligned} x_1 + x_2 &= 2 \\ x_1 - x_2 &= 1 \\ x_1 \quad &= 4 \end{aligned}$$

System **(a)** is a 2×2 system, **(b)** is a 2×3 system, and **(c)** is a 3×2 system.

By a solution to an $m \times n$ system, we mean an ordered n-tuple of numbers $(x_1, x_2, \ldots, x_n)$ that satisfies all the equations of the system. For example, the ordered pair $(1, 2)$ is a solution to system **(a)**, since

$$\begin{aligned} 1 \cdot (1) + 2 \cdot (2) &= 5 \\ 2 \cdot (1) + 3 \cdot (2) &= 8 \end{aligned}$$

The ordered triple $(2, 0, 0)$ is a solution to system **(b)**, since

$$\begin{aligned} 1 \cdot (2) - 1 \cdot (0) + 1 \cdot (0) &= 2 \\ 2 \cdot (2) + 1 \cdot (0) - 1 \cdot (0) &= 4 \end{aligned}$$

Actually, system **(b)** has many solutions. If α is any real number, it is easily seen that the ordered triple $(2, \alpha, \alpha)$ is a solution. However, system **(c)** has no solution. It follows from the third equation that the first coordinate of any solution would have to be 4. Using $x_1 = 4$ in the first two equations, we see that the second coordinate must satisfy

$$\begin{aligned} 4 + x_2 &= 2 \\ 4 - x_2 &= 1 \end{aligned}$$

Since there is no real number that satisfies both of these equations, the system has no solution. If a linear system has no solution, we say that the system is *inconsistent*. If the system has at least one solution, we say that it is *consistent*. Thus system **(c)** is inconsistent, while systems **(a)** and **(b)** are both consistent.

The set of all solutions to a linear system is called the *solution set* of the system. If a system is inconsistent, its solution set is empty. A consistent system will have a nonempty solution set. To solve a consistent system, we must find its solution set.

2×2 Systems

Let us examine geometrically a system of the form

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 &= b_1 \\ a_{21}x_1 + a_{22}x_2 &= b_2 \end{aligned}$$

Each equation can be represented graphically as a line in the plane. The ordered pair (x_1, x_2) will be a solution to the system if and only if it lies on both lines. For example, consider the three systems

$$\text{(i)}\quad \begin{aligned} x_1 + x_2 &= 2 \\ x_1 - x_2 &= 2 \end{aligned} \qquad \text{(ii)}\quad \begin{aligned} x_1 + x_2 &= 2 \\ x_1 + x_2 &= 1 \end{aligned} \qquad \text{(iii)}\quad \begin{aligned} x_1 + x_2 &= 2 \\ -x_1 - x_2 &= -2 \end{aligned}$$

The two lines in system (i) intersect at the point (2, 0). Thus {(2, 0)} is the solution set to (i). In system (ii) the two lines are parallel. Therefore, system (ii) is inconsistent and hence its solution set is empty. The two equations in system (iii) both represent the same line. Any point on this line will be a solution to the system (see Figure 1.1.1).

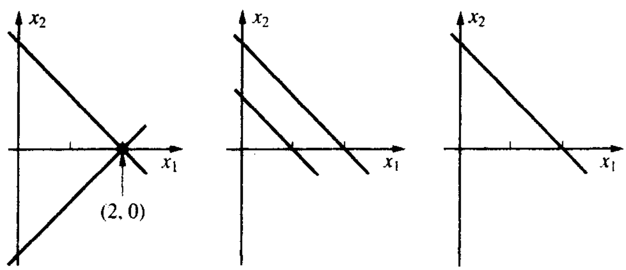


Figure 1.1.1

In general, there are three possibilities: the lines intersect at a point, they are parallel, or both equations represent the same line. The solution set then contains either one, zero, or infinitely many points.

The situation is the same for $m \times n$ systems. An $m \times n$ system may or may not be consistent. If it is consistent, it must either have exactly one solution or infinitely many solutions. These are the only possibilities. We will see why this is so in Section 2 when we study the row echelon form. Of more immediate concern is the problem of finding all solutions to a given system. To tackle this problem, we introduce the notion of *equivalent systems*.

Equivalent Systems

Consider the two systems

$$\textbf{(a)}\quad \begin{aligned} 3x_1 + 2x_2 - x_3 &= -2 \\ x_2 &= 3 \\ 2x_3 &= 4 \end{aligned} \qquad \textbf{(b)}\quad \begin{aligned} 3x_1 + 2x_2 - x_3 &= -2 \\ -3x_1 - x_2 + x_3 &= 5 \\ 3x_1 + 2x_2 + x_3 &= 2 \end{aligned}$$

System **(a)** is easy to solve because it is clear from the last two equations that $x_2 = 3$ and $x_3 = 2$. Using these values in the first equation, we get

$$\begin{aligned} 3x_1 + 2 \cdot 3 - 2 &= -2 \\ x_1 &= -2 \end{aligned}$$

Thus the solution to the system is $(-2, 3, 2)$. System **(b)** seems to be more difficult to solve. Actually, system **(b)** has the same solution as system **(a)**. To see this, add the first two equations of the system:

$$\begin{array}{rcrcrcr} 3x_1 & + & 2x_2 & - & x_3 & = & -2 \\ -3x_1 & - & x_2 & + & x_3 & = & 5 \\ \hline & & x_2 & & & = & 3 \end{array}$$

If (x_1, x_2, x_3) is any solution to **(b)**, it must satisfy all the equations of the system. Thus it must satisfy any new equation formed by adding two of its equations. Therefore, x_2 must equal 3. Similarly, (x_1, x_2, x_3) must satisfy the new equation formed by subtracting the first equation from the third:

$$\begin{array}{rcrcrcr} 3x_1 & + & 2x_2 & + & x_3 & = & 2 \\ 3x_1 & + & 2x_2 & - & x_3 & = & -2 \\ \hline & & & & 2x_3 & = & 4 \end{array}$$

Therefore, any solution to system **(b)** must also be a solution to system **(a)**. By a similar argument, it can be shown that any solution to **(a)** is also a solution to **(b)**. This can be done by subtracting the first equation from the second:

$$\begin{array}{rcrcrcr} & & x_2 & & & = & 3 \\ 3x_1 & + & 2x_2 & - & x_3 & = & -2 \\ \hline -3x_1 & - & x_2 & + & x_3 & = & 5 \end{array}$$

Then add the first and third equations:

$$\begin{array}{rcrcrcr} 3x_1 & + & 2x_2 & - & x_3 & = & -2 \\ & & & & 2x_3 & = & 4 \\ \hline 3x_1 & + & 2x_2 & + & x_3 & = & 2 \end{array}$$

Thus (x_1, x_2, x_3) is a solution to system **(b)** if and only if it is a solution to system **(a)**. Therefore, both systems have the same solution set, $\{(-2, 3, 2)\}$.

▶ DEFINITION

Two systems of equations involving the same variables are said to be **equivalent** if they have the same solution set.

Clearly, if we interchange the order in which two equations of a system are written, this will have no effect on the solution set. The reordered system will be