Solutions Manual to Accompany

LINEAR ALGEBRA

Ideas and Applications

Fourth Edition

RICHARD C. PENNEY

WILEY

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Purdue University



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STUDENT MANUAL

SYSTEMS OF LINEAR EQUATIONS

1.1 THE VECTOR SPACE OF $m \times n$ MATRICES

Problems begin on page 17

EXERCISES

1.1 (a)
$$\begin{bmatrix} -1 & -4 & -7 & -10 \\ 1 & -2 & -5 & -8 \\ 3 & 0 & -3 & -6 \end{bmatrix}, [3, 0, -3, -6], \begin{bmatrix} -4 \\ -2 \\ 0 \end{bmatrix}$$
(b)
$$\begin{bmatrix} 1 & 8 \\ 4 & 32 \\ 9 & 72 \end{bmatrix}, [9, 72], \begin{bmatrix} 8 \\ 32 \\ 72 \end{bmatrix}$$
(c)
$$\begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} \\ -1 & 0 \end{bmatrix}, [-1, 0], \begin{bmatrix} -\frac{1}{2} \\ -\frac{1}{2} \\ 0 \end{bmatrix}$$

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1.3
$$C = A + B$$
.

1.5 (a)
$$[1, 1, 4] = [1, 1, 2] + 2[0, 0, 1]$$
(c)
$$\begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix} = 0 \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + 2 \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$
(d)
$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} - \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix} + 0 \begin{bmatrix} 9 \\ 12 \\ 15 \end{bmatrix}$$
(f)
$$-3 \begin{bmatrix} 3 & -1 & 2 \\ 0 & 1 & 4 \end{bmatrix} = \begin{bmatrix} -9 & 3 & -6 \\ 0 & -3 & -12 \end{bmatrix}$$

- **1.6** $P_2 = P_5 P_1 P_3 P_4$, where P_i is the *i*th row of P.
- **1.9** Each vector has a nonzero entry in the positions where the other two vectors have zeros.
- 1.10 Suppose first that

$$A_3 = xA_1 + yA_2$$

Then

$$[0,0,8] = [x, 2x + 5y, 3x + 6y]$$

Equating entries yields the system

$$0 = x$$
$$0 = 2x + 5y$$
$$8 = 3x + 6y$$

From the first equation, x = 0. It then follows from the second equation that y = 0, which is impossible due to the third equation. Thus the independence is proved.

- **1.12** [1,-1,0],[1,0,0],[2,-2,0], and [4,-1,0] all belong to the span. [1,1,1] does not because its last entry is nonzero.
- **1.13** (a) -2X + Y = [1, 1, 4] (other answers are possible)].
 - (b) Let [x, y, z] = aX + bY = [-a b, a + 3b, -a + 2b] and substitute into 5x + 3y 2z. You should get 0.
 - (c) Any point [x, y, z] that does not solve the equation 5x + 3y 2z = 0 will work—for example, [1, 1, 1].
- 1.16 No. From the second and third entries aX + bY has positive entries only if both a and b are negative; hence the first entry is negative.

- **1.19** (c) No. If $f(x) = a \cos x + b \sin x$, then f(0) = a and $f(\pi) = -a$ would both be positive which is impossible.
- **1.20** (c) Since the intersection of two planes through the origin is a line, the span of $\{X, Y\}$ must be a line. Hence let X = [x, y, x] where $z \neq 0$, any Y = cX where $c \neq 1$.
- 1.23 For the first part, use various values of a, b, and c in aX + bY + cZ. For the second part note that for all scalars a, b, and c the (2, 1) entry of aX + bY + cZ is zero. Hence any matrix W in M(2, 2) such that $W_{2,1} \neq 0$ will not be in the span.
- **1.25** Let V and W be elements of the span. Then V = aX + bY and W = cX + dY. Then for $s, t \in \mathbb{R}$, sV + tW = (as + ct)X + (bs + dt)Y which belongs to the span of X and Y.
- **1.26** Let the columns of A be A_i , i = 1, 2, 3. Then $3A_3 A_2 = A_1$.
- **1.29** Let one row be a linear combination of the other rows. This is easily done keeping all entries non-zero.
- 1.31 (a) Yes: D = 5A 2B.
 - **(b)** Yes: B = A C, so D = 3A + 2C.
 - (c) You can say nothing about the dependence of A and B. Given A and B, dependent or not, let C = A B and D = 2A + B + 3C.
- **1.33** (b) $\sinh x = \frac{1}{2}(e^x e^{-x}) = \frac{1}{4}(2e^x) \frac{1}{6}(3e^{-x})$
 - (d) From the double angle formula for the cosine function $cos(2x) = -sin^2x + cos^2x$.
 - (f) $(x+3)^2 = x^2 + 6x + 9$.
 - (h) From the angle addition formulas for the sine and cosine functions

$$\sin\left(x + \frac{\pi}{4}\right) = \sin(\pi/4)\cos x + \cos(\pi/4)\sin x$$

$$= \frac{\sqrt{2}}{2}(\cos x + \sin x)$$

$$\cos\left(x + \frac{\pi}{4}\right) = \cos(\pi/4)\cos x - \sin(\pi/4)\cos x$$

$$= \frac{\sqrt{2}}{2}(\cos x - \sin x)$$

$$\sin x = \frac{1}{\sqrt{2}}\left(\sin\left(x + \frac{\pi}{4}\right) - \cos\left(x + \frac{\pi}{4}\right)\right)$$

(i)

$$\ln[(x^2+1)^3/(x^4+7)] = 3\ln(x^2+1) - \ln(x^4+7)$$
$$= 3\ln(x^2+1) - 2\ln\sqrt{x^4+7}.$$

- **1.34** The span is the set of polynomials of degree $d \le 2$. Any pair of such polynomials answers the first question.
- **1.36** (a) Let $B = \begin{bmatrix} x & y \\ z & w \end{bmatrix}$. Then

$$A + B = \begin{bmatrix} x + a & y + b \\ z + c & w + d \end{bmatrix} = \begin{bmatrix} x & y \\ z & w \end{bmatrix}$$

Hence, x + a = x, y + b = y, z + c = c, and w + d = d, which imply that x = y = z = w = 0. Hence, B = 0.

- (b) Solved similarly to (a).
- **1.37** See Example 1.4 on page 12 of the text. For example to prove (i) we let $X \in M(n, m)$, $X = [x_{ij}]$. For scalars k and l

$$(k + l)X = [(k + l)x_{ij}]$$

= $[kx_{ij} + lx_{ij}]$
= $[kx_{ij}] + [lx_{ij}] = kX + lX$

1.40 The steps are as shown below. The vector space properties used were:

Step 1 (a) and (e),

Step 2 (c) and (e),

Step 3 (b), (e), and Proposition 1.2 on page 15,

Step 4 (b), (e), and (g),

Step 5 (f),

Step 6 (h), (g),

Step 7 (j).

$$-(aX) + (aX + (bY + cZ)) = -(aX) + 0$$
$$(-(aX) + aX) + (bY + cZ) = -(aX)$$
$$0 + (bY + cZ) = -1(aX)$$

$$bY + cZ = (-a)X$$

$$(-a)^{-1}(bY + cZ) = (-a)^{-1}((-a)X)$$

$$(((-a)^{-1}b)Y + ((-a)^{-1}c)Z) = 1X$$

$$\left(-\frac{b}{a}\right)Y + \left(-\frac{c}{a}\right)Z = X$$

1.1.2 Applications to Graph Theory I

Problems begin on page 27

SELF-STUDY QUESTIONS

1.1 The matrices for parts (a), (b), and (c) are respectively

$$\begin{bmatrix} 0 & 2 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 & 1 & 2 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

- 1.2 Possible routes are as in Figure 1.1
- 1.3 An airline would not have a flight from a given city A to itself.

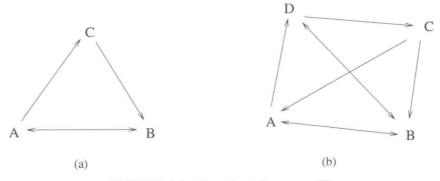


FIGURE 1.1 Exercise 1.2 on page 27.

EXERCISES

1.45 We list the vertices in the order MGM, MGF, PGM, PGF,F, M, S1, S2, D1, D2.

1.46

1.2 SYSTEMS

Problems begin on page 38

In our solutions, Roman numerals refer to equations in a system. Thus, for example "IV" is the fourth equation in a given system.

EXERCISES

- 1.49 *X* is a solution since $4 \cdot 1 2 \cdot 1 1 1 = 0$ and $1 + 3 \cdot 1 2 \cdot 1 2 \cdot 1 = 0$. *Y* is not since $1 + 3 \cdot 2 2 \cdot (-1) 2 \cdot 1 = 7$.
- **1.50** Let Z = aX + bY = [a+b, a+2b, a-b, a+b]. This solves the system if and only if

$$4(a+b) - 2(a+2) - (a-b) - (a+b) = 0$$
$$(a+b) + 3(a+2) - 2(a-b) - 2(a+b) = 0$$

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