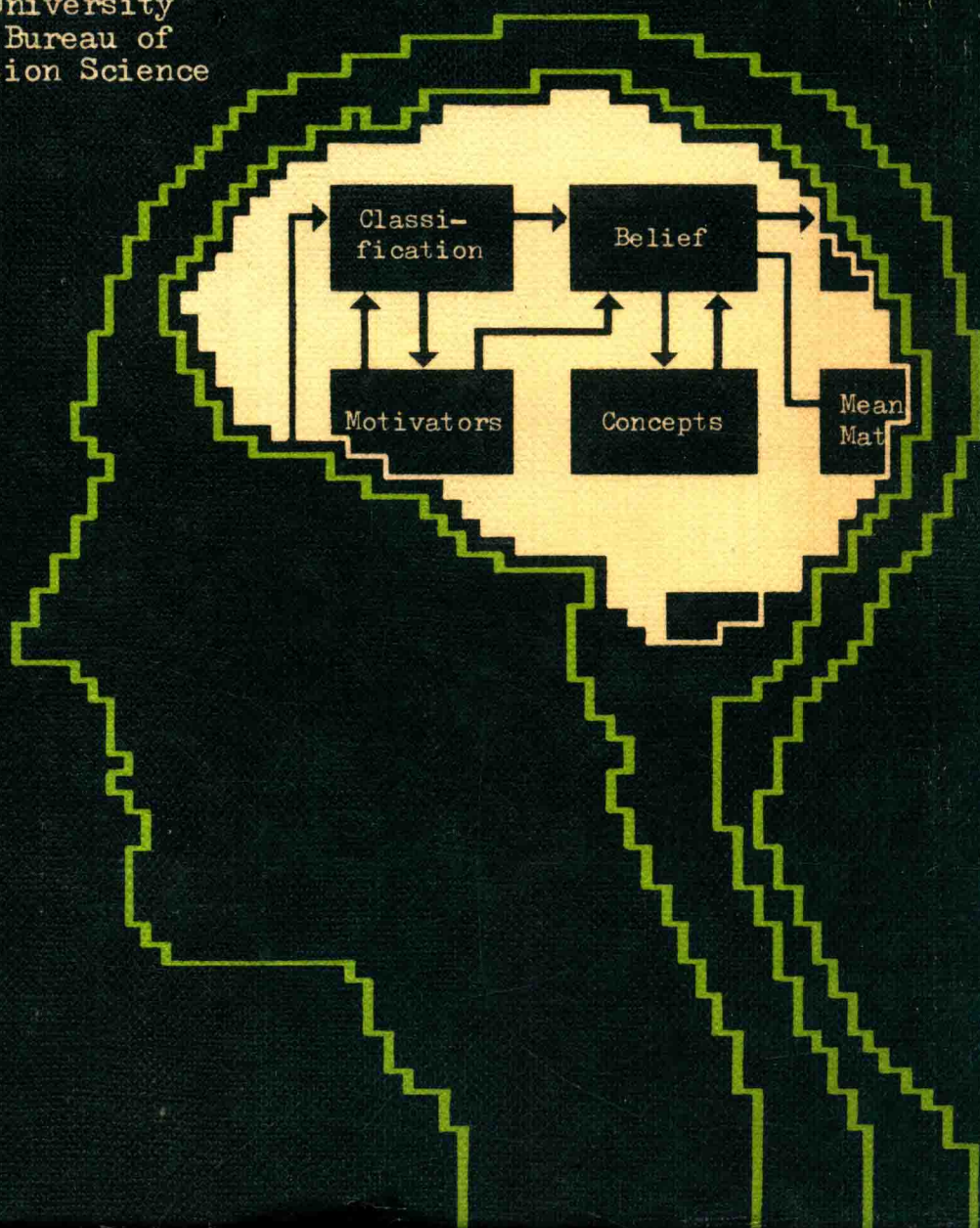


# PRECISION, LANGUAGE & LOGIC

**F. H. George**

Brunel University  
and the Bureau of  
Information Science



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by

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and

*The Bureau of Information Science*



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**PRECISION, LANGUAGE  
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## DEDICATION

I would like to dedicate this book to the Beaconsfield Golf Club, not only because I spent so many pleasant hours writing the notes for this book and the book itself there, but also because a large part of the original impetus for writing the book sprang from discussions with some of the other members which made me realise - and I mean this in the nicest and most humble way - the need for such a book.

F.H. George

Beaconsfield,

1976

## FOREWORD

This book is intended to provide a text to guide people in the main ingredients of clear thinking and logical discussion. It emphasises logic itself, both informal and formal, including a certain amount of symbolic logic. There is enough of the latter (with the references) to guide the reader as to its further development. The book though is intended for the undergraduate and the graduate student as well as the layman. To this end the book can be approached from more than one point of view.

The reader who is interested in everyday debate and discussion only, and not concerned with the formal technical details of logistic systems can "skip over" (preferably glancing through the pages and getting the gist of the contents) Chapters 4, 5 and 6 of Part I, and possibly the first few pages of Chapter 9 of Part II, but the rest is essential reading - even the section on Bayes Rule should be reasonably easy to follow and certainly relevant to his needs. In fact even Chapter 4 (and as a result Chapter 5) should be attempted seriously by almost everyone. Only the first part of Chapter 6 (up to the section on Predicate Calculus) can reasonably be skipped over by the person who is definitely disinterested in the technical development of logic.

If on the other hand the reader is interested in the technicalities of logic and clear thinking, which are kept at a fairly simple level, he should read everything. Those interested primarily in logic concentrating on the first and second parts, while those with philosophical or psychological interests might concentrate on the second and third parts.

In a sense the book has two roles to play. One is to show how a knowledge of logic coupled to a knowledge of semantics, philosophy and pragmatics can clarify one's thinking and the second is to show how such logical and semantics considerations lead to the process of formalisation (or making precise) which takes you from a theory to its underlying (logical) model.

In taking pragmatics (or behaviour) for our theory, we have, as it were, tried to indicate the way we formalise our own use of language and logic. We have in a sense gone full circle, if only in outline, to try to show how scientific method, explanation and the like are closely akin to common sense and ordinary sensible behaviour and also to precise logic.

In fact we should guess that those readers interested in science as such would find Part III of more interest and Part II of slightly less interest; those readers primarily interested in clear thinking would probably find the opposite. We would however like both to read both, as we believe both are relevant.



Some mention should be made of summaries, exercises and references. Summaries have been supplied in those chapters which are, as it were, "broad brush" chapters which lend themselves best to summarisation. In the latter part of the book these have been dropped because the text did not seem to be suitable for summaries. The exercises were maintained throughout (should the reader wish to try his hand at them) but they become rather more abstract as the text changes especially on moving into Book III. Finally, references presented some difficulty. In the early chapters these were clearly intended as "further reading" and no references were made in the text. In the later chapters, a compromise was made between "further reading" references which were not referred to (by date) and others which were. But still the references at the end of the chapter were chosen on a "further reading" basis and the remainder not already referred to at the end of chapters are collected together at the end of the book under the heading "additional references". Thus if a reference is made in the text to some author, and his work is not quoted at the end of the chapter, the reader should look in the "Additional References" list at the end of the book.

Finally we should say that this book deals with its subject at more than one level. In ending the book on the subject of neural nets and automata it is completing a sort of circle, which has been traversed in rather general terms. We have moved from logic and clear thinking to philosophy, which enriches the backcloth to both, and then on, more speculatively and more controversially, to abstract models of the brain and nervous system. This work is one aspect of the suggested link between language and logic, brains and nervous systems and the world around us. It has been suggested that language, brains and the physical world are all in some way structurally similar to each other; and we have taken the implications of this view seriously. But it should again be emphasised that this book could simply be read as a treatise on logic and clear thinking.



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## CHAPTER 1

# A BACKGROUND TO LOGIC

This book is intended to describe logical, or what is sometimes called clear, thinking. The word 'logic' is often used rather broadly to cover a wide field of activities including semantics, scientific method, probabilities and statistics, and even human behaviour. We shall certainly be thinking of logic in this broad sense, although we shall also be thinking of it in the more precise and narrow sense of classical logic, of both an informal and a formal variety. We shall try to emphasize the complex relationship that exists between all these aspects of the problem of thinking and talking in a clear and logical manner.

The basic ideas of logic go back a very long time indeed over a period of two thousand years; such basic ideas are concerned with the ability to carry out a rational discourse when something, such as a statement or set of statements, is proposed; that is when something is set out in terms of propositions using terms or classes. All these central concepts such as rational discourse, terms and classes will be explained.

The purpose of logic is to apply correct reasoning to certain assumptions, and the purpose of rational discourse is to derive certain valid consequences from those assumptions which are themselves true, or at least highly confirmed. It should be made absolutely clear that valid consequences, that is the result of applying logical principles to assumptions, do not necessarily lead to true statements. By 'truth' we mean something like the ability to accurately describe, in the form of a statement or proposition, a state of affairs. A true statement is a statement which represents affairs accurately, and we shall not primarily concern ourselves with various notions of truth in this book, because we are mainly concerned with logical validity and ability to carry out rational discussion and debate. A valid argument is one that follows logically from its assumptions and is independent of the truth of the propositions (or statements) involved.

Although we shall not make concepts like "truth" and "meaning" a main concern of our discussion, we should try to indicate our attitude to such matters as they are important to the broader contexts of logic.

'Truth' will be used to designate true propositions on one hand and reality on the other. Thus we shall say that propositions (or statements) are true if they correctly describe reality. This is, in essence, what is called the Correspondence Theory of truth. It also embodies the Semantic Theory in that it refers to the statements insofar as, for example, the statement "It is raining" is true if and only if it is raining. That we can specify the place and time and other details can, of course, be assumed.

There is also a theory of truth known as the Coherence Theory which asserts that true statements are consistent with - or cohere with - statements we already believe to be true. We shall think of such coherence as applying to the extent to which statements are confirmed by evidence rather than as to whether or not they are true.

The distinction between confirmation and truth is of some importance. Empirical statements are true or false, assuming them to be meaningful or testable, but we never certainly know whether empirical statements are true or false. All we can ever know of empirical statements is the extent to which they are confirmed. The actual process of confirmation (or the opposite process of infirmation) is certainly something which is susceptible to coherence, since the extent to which a statement coheres with statements already accepted is most certainly a part of the measure of the extent to which we believe in a statement, or a measure of the extent to which it is confirmed.

At the same time as we have expressed our attitude to the concept of truth, we should perhaps also express our attitude to the related problem of meaning. We are not expecting this to be universally acceptable, but it at least indicates our way of thinking. A statement or proposition is meaningful we shall say if it is capable of being tested for its truth. We reserve the right to accept 'testable' as meaning directly testable, or "testable in principle" or to imply (as we shall generally expect it to mean) "capable at least in principle of being confirmed or infirmed".

Meaning, like explanation, is a matter of degree of precision, and the meaning of statements can be tackled at various degrees of precise definition as accords with the precision of the discourse in which the statements arise.

We should make clear that we shall think of both meaning and explanation as contextual in the sense that words and propositions (expressed in sentences for example) depend for their effectiveness on the nature of the context in which they occur. Words can change their meaning (by qualification, for example) according to the context, and explanations need only to be as detailed as their use requires. Finally, on this point, we should add that any statement needs a degree of precision that (like explanations) is as precise as the discourse requires.

There is something to be said for the view that meaningfulness refers to complete propositions and not to isolated terms, except insofar as those terms are interpreted as shorthand for a complete proposition. Much of what is meaningful in language can be formalised. This means it can be subject to formation rules which eliminate meaningless combinations of words and hopefully leave only meaningful ones.

"The the the the the"

is clearly meaningless, whereas

"The man sat on the sofa"

is clearly meaningful, whether or not it is true.

We shall sharply distinguish between meaning and truth, since a statement can be meaningful and either true or false, but if it is meaningless it can be neither true nor false. This is what creates the notion of testability, since we argue that if an empirical statement cannot be said, even in principle, to be true or false then it is meaningless. We should note the following meaningful statements:-

"Get off the field immediately"

or matters of opinion such as

"Beethoven is the greatest composer".

We have here a criterion of meaning for statements which may be neither true nor false.

We are very much concerned with formal logic, since this is the background subject which eventually led to symbolic logic, which will represent a part of our study of logic. By formal logic we mean to assert that an argument is formal if it is independent of the factual content of the assertions it makes. In a sense this is much the same as saying that a formal argument is independent of the truth or falsehood of the assertions it makes. In a sense this is much the same as saying that a formal argument is independent of the meaning the assertions involved may contain by virtue of the interpretation placed upon them. In other words, if we make statements such as:-

"Charles is the father of Jack"

or

"Harry is the brother of Camille"

then the relationship of "being a father to" or "being a brother of" is independent of the people actually mentioned, or the names actually used in the propositions. "Being a brother of" or "being a father to" or any other sort of relationship which might be stated in words, may be independent of the people actually involved in that relationship. What is entailed by the relationship is independent of the people so named. This is the study of form, or formal logic, as it is sometimes called.

We have to be fairly careful in making this distinction between formal and what is sometimes called "factual" information, because in many arguments the factual features, sometimes said to be governed by the semantic rules which are involved in the interpretation of the formal words used, are so closely associated with those formal words that they affect the logical inference.

To see one aspect of the difficulty of separating the factual from the formal, we only have to remind ourselves of the implications of the above example. "If X is the father of Y and Y is the father of Z" it does not follow that "X is the father of Z"; he is, of course, the grandfather. If on the other hand we had "If X is the brother of Y and Y is the brother of Z" it does follow that "X is the brother of Z". In other words, we can only use our formal logical structures to describe those occurrences or events which have the same logical structure.

A further example may help to clarify the point. If we say Euclidean Geometry in three-dimensions is a formal system we may or may not be able to use it to describe the physical world around us. Whether we can or not depends upon the nature of the physical world. We know such a Euclidean Geometry fits much of our immediate experience very well, but we also know that for astronomical distances forms of Non-Euclidean Geometry are required. Let us now look at another more obviously logical sort of example.

If I say that there are 366 people in a particular room, then it follows, by logical argument (Leap Year excepted) that at least two people in that room have the same birthday. This is logically true and is entailed by the meanings of the terms actually used. This can easily be seen to be so if we say that the word 'birthday' does not mean the day you were born but the hour you were born. If we accepted this new meaning of the word 'birthday', then the logical entailment fails.

The haziness of the distinction between factual and formal arguments comes about because so much of logic is couched in ordinary English, or some other natural language, and as a result cannot be independent of the semantic rules or meanings we ascribe to the terms used. Therefore we will not wish to press too strongly that formal logic is independent of facts when logic is phrased in ordinary language. This in part is what leads us to symbolic logic, where arbitrary symbols are used to replace ordinary English words. This highlights the nature of the formal relationships between the symbols which represent terms or classes or whatever, and thus places greater emphasis on the formal nature, and the relative semantic independence, of the argument.

We have already used a number of new key words such as 'proposition' and 'statement', and also 'terms' and 'classes', which we promised to explain, so we ought now to explain what these mean in the logical context. By 'proposition' we mean something which is exemplified by a statement, whether written or spoken, or a sentence which is the most usual form of statement because of the syntactical form that Indo-European languages take. The proposition is in some sense what is proposed by the utterer of the proposition, something which is a belief or something which can be exemplified in the statement and is thus susceptible of various tests of logical validity and semantic truth. A proposition, therefore, is something prior to its exemplification as a sentence, but it will not lead us too far astray if we think of a sentence or statement as exemplifying a proposition.

Within the proposition, which is expressed in the Indo-European languages as sentences in subject-predicate form, we have the notion of a 'term'. A 'term' is a word or collection of words that makes up a subject or predicate, and most often refers to a 'class'. The word 'class' usually refers to a collection of objects or events. "All red objects", "all tall objects", "all circular objects" are classes of objects which may be referred to by class names such as 'red', 'tall' or 'circular'. A class is a combination or collection of objects which is relatively well defined. In most of logic, although we shall explain later in our discussion that there are exceptions, we mean by a 'class' something which is complete and well-defined. We should bear in mind that in symbolic logic we expect these classes to be independent of their meaning, though there is an intended meaning in mind in most cases. We can then assume

that by using letters such as a, b, ..., n, we are referring to classes of things which are always precisely defined. Indeed if we have classes of things which are not wholly precise, it may be said that they are not appropriate to the interpretation that will be placed upon them by a precisely defined class logic.

Thus it is that combinations of propositions are put together, such as in our example of "if there are 366 people in this room..." which allow valid or invalid arguments, fallacies and the like, to emerge. Logic is the study of these relationships with a view to trying to show that certain combinations are valid, and if possible, to characterize sets of such valid combinations by some sort of general formula.

Rational discourse is our main aim, and this requires the ability to be precise - by definition and by use of explanation - in one's statements and descriptions. This involves logic, but also involves, as we shall see, a great deal more besides.

It is not easy, because of the shadowy distinction between factual and formal logic, to distinguish logic as a study independent of science, from science and scientific methods. Thus it is important to realize that scientific method involves deductive argument, where by 'deductive argument' we mean the ability to draw inferences, and this means deductive inferences from assumptions, in order to derive their logical entailments or consequences. Scientific method, of course, involves more than this; it also involves inductive arguments which proceed from the particular to the general and is necessarily probabilistic in form. Such inductive arguments too are necessarily descriptive and refer to the empirical world in which they occur.

Insofar as we are concerned with the factual features of the world, then of course we are also involved in the definition of terms and the explanation of certain - often causal - features of the environment. These two concepts of definition and explanation are fairly closely related. We can mean many different things by the word 'definition', but more often than not we mean something like a synonym for a term used. This is very much a lexicographer's type of definition and is exemplified by dictionaries and thesauri which say roughly what a word means by telling us its synonyms. A rather more precise form of definition is a substitutional definition where we substitute one word for a complete phrase, where the phrase is more descriptive of the word than the word alone, which is in a sense symbolic of the class of objects to which the phrase refers; the phrase is therefore the definition of the word. The substitutional type of definition is a sort of shorthand; so rather than say "people who are related to other people by such and such a relationship", we can in shorthand simply say 'brother'.

There are other forms of definition besides the lexicographers' definition and the substitutional form of definition, as exemplified by ostensive definitions where we point to things and say "by X I mean that" and point at a particular feature which we are concerned with defining, whether it be a colour like red or whether it be a shape like circular. Another form of definition is an operational definition whereby we try to describe a feature of our environment or our experience in terms of how it actually affects us. We can describe an electric current in terms of the effect it has up to certain terminal points, for example, and indeed it

has sometimes been argued that words like 'intelligence' could be defined by that which is tested by intelligence tests. One can see the danger of circularity here, sometimes such circularity is wholly unhelpful, but sometimes it supplies a useful operational definition of the concept underlying the term which is being defined.

Explanations are somewhat similar to definitions but they are usually in the form of statements or propositions which characterize a process rather than a single concept. Thus we try to explain a concept by saying what it does under a whole set of circumstances. We try to explain the behaviour of people in terms of their experience and their relationships to other people in a particular social context. Explanatory statements are propositions of a factual kind which are related to each other causally in order to provide an historical picture of how such activities come about.

In general, in our discussion of logic in this book, definitions play an important part, particularly formal definitions, relating classes of objects to each other or defining new operators in terms of their effect on classes of objects, whereas factual definitions or explanations tend to play a slightly less central role. It is, however, important to realize that logic and scientific method certainly have a considerable degree of overlap.

Two other terms which we should mention here are 'symbols' and 'signs'. By the word 'symbol' we mean any mark on paper or any sound which is used to stand for, in any sense, something other than itself, although one can also have self-referring symbols. Words are most often used in ordinary language to stand for objects, and therefore there is a sense in which the word 'Chicago' refers to Chicago, the city in America, and the word 'pencil' refers to a class of objects called pencils which are made of wood, lead, etc., and there is also a sense in which the words are symbols. They are symbols in the sense that they are arbitrarily chosen, even if they have grown up over a period of time there is a sense in which they are arbitrary, since we could quite easily decree at any time that the word 'pen' would stand for what we have normally called a pencil and the word 'pencil' would stand for pen. They are, however, also signs in that they represent things and they refer to things, and the actual letters which make up the words are manifestly symbols; they have no "meaning" as they stand. Signs tend to be of two kinds in general. They are either linguistic signs in the form of words or they are natural signs in the form of such things as clouds which signify the possibility of rain. The overlap of symbols and signs occurs in linguistic signs where the linguistic sign is in a sense being used as a symbol and is certainly made up of individual letters which are symbols, and yet has some of the properties of being a sign by virtue of referring to, or signifying, a class of objects or even a single particular individual.

We shall now draw attention to a convention which we have already introduced and that is the use of quotation marks. We shall use single quotation marks to indicate the word rather than the thing signified or referred to by the word, we shall use double quotation marks for phrases or sentences or actual quotations, and we shall also use double quotation marks to draw attention to the fact that a term may be vague or ambiguous in some sense. It is important to recognize this distinction between



single and double quotation marks since it is the intention to use this convention consistently throughout the book. It becomes especially important when we discuss semantics and pragmatics as opposed to syntax and formal logic, as we shall see.

One other fairly traditional convention we shall adopt is to underline (i.e. put into italics) words that we want especially emphasized.

One further point we should mention by way of warning. Logical discourse does not depend upon the use of true assumptions. We can start with false assumptions and apply logical inferences and draw the valid conclusions. This is standard practise among scientists who, by such means, discover the likely truth of the assumptions. For example, it is quite usual to consider alternative hypotheses in situations where we know both cannot be true and deduce the (logical) consequences of both in order to examine, in effect, the plausibility of the assumptions.

We are embarking on a fairly complex field of thinking, reasoning and talking, which is intended to produce a degree of precision into our thinking reasoning and talking. It is particularly intended to sharpen our capacity for rational discourse or logical argument. Not that we expect or wish to encourage tedious pedantry or needless precision. What is required is the ability to recognize the degree of detail or precision of definition which is required in any particular debate.

#### SUMMARY

This chapter attempts to set the scene for our whole analysis of what should be the rational man's equipment.

We need to use language - both so-called "natural languages" such as English and French, and we need as a result to define the terms involved in our linguistic discourse. This in turn involves us in problems of meaning, truth and logic. It also involves us in the relationship between formal and factual statements, as well as other sorts of statements involving, for example, opinions.

#### EXERCISES

1. What are the essential ingredients of a valid argument? What in an argument (or even a dispute) depends upon "fact" and what on "reasoning"? Provide a check list.
2. To what extent can we separate "factual" from "formal" features in a carefully argued discourse? Give some examples of both, and if possible, a "borderline" case.
3. Definitions are obviously important to precise discussion. Explain what the concept of a "definition" entails. Write down some examples.
4. The notion of context is going to be important to us in rational discourse. What do we mean by a "contextual definition" or a

"contextual explanation"?

5. What thoughts do you have on "meaning" and "truth": you ought to be prepared to say what they mean for you since they are basic to our ways of describing "things".
6. The two basic questions which arise in most debates are:-
  - i) What do you mean?
  - ii) How do you know?

Do you regularly use these two questions as a guide to sensible discussion (as well as to destroy an opponent)? If not, why not?

#### REFERENCES

- |                       |   |
|-----------------------|---|
| BARKER, S.F.          | <u>The Elements of Logic.</u> McGraw-Hill, 1965               |
| COHEN, M. & NAGEL, E. | <u>An Introduction to Logic.</u> Routledge, 1963              |
| KLEEN, G.B.           | <u>Language and Reasoning.</u> Van Nostrand, 1961             |
| LUCE, A.A.            | <u>Teach Yourself Logic.</u> English Universities Press, 1958 |
| STRAWSON, P.F.        | <u>Introduction to Logical Theory.</u> Methuen, 1953          |