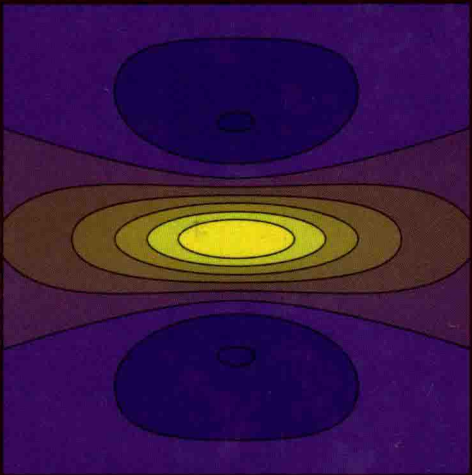
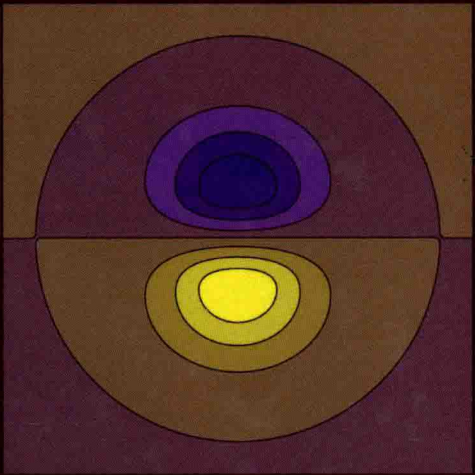
STATIC GREEN'S FUNCTIONS in ANISOTROPIC MEDIA

Ernian Pan and Weiqiu Chen





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STATIC GREEN'S FUNCTIONS IN ANISOTROPIC MEDIA

This book provides the basic theory on static Green's functions in general anisotropic magnetoelectroelastic media and their detailed derivations based on the complex variable method, potential method, and integral transforms. Green's functions corresponding to the reduced cases are also presented, including those in anisotropic and transversely isotropic piezoelectric and piezomagnetic media and those in purely anisotropic elastic, transversely isotropic elastic, and isotropic elastic media. Addressed problem domains are three-dimensional (two-dimensional) infinite, half, and bimaterial spaces (planes). Although the emphasis is on the Green's functions related to the line and point force, those corresponding to the important line and point dislocation are also provided and discussed. This book provides a comprehensive derivation and collection of the Green's functions in the concerned media, and as such, it should be a good reference book for researchers and engineers and a textbook and reference book for both undergraduate and graduate students in engineering and applied mathematics.

Ernian Pan is a professor of civil engineering at the University of Akron and a Fellow of both ASME and ASCE. He received his BS from Lanzhou University, his MS from Peking University, and his PhD from the University of Colorado at Boulder. His research interests are in computational mechanics with applications to anisotropic magnetoelectroelastic solids and nanostructures. As a well-recognized expert in anisotropic and multilayered Green's functions, he has pioneered various benchmark solutions for multiphase and multilayered composites with functionally graded materials and played various active roles in, and contributed to, Green's function research and education. He has published more than 250 journal articles.

Weiqiu Chen is a professor of engineering mechanics at Zhejiang University, China. He received his BS and PhD degrees from Zhejiang University in 1990 and 1996, respectively. He has engaged himself in the mechanics of smart materials/structures and vibrations/waves in structures for more than twenty years. He has coauthored more than three hundred peer-reviewed journal articles, and he has published books on the elasticity of transversely isotropic elastic and piezoelectric solids in 2006 and 2001, respectively. He has received numerous awards, including the National Science Fund for Excellent Young Scholars from the NSFC in 2007 and the Award of Science and Technology (the second grade) from the Ministry of Education, China, in 2012.

In the other class of methods the quantities to be determined are expressed by definite integrals, the elements of the integrals representing the effects of *singularities* distributed over the surface or through the volume. This class of solutions constitutes an extension of the methods introduced by Green in the Theory of the Potential.

—A. E. H. Love, 1944

Preface

As one of the most powerful computational methods, boundary integral equation method (with its discretized version being called boundary element method, or BEM), has been very successfully applied to various practical engineering problems. The BEM has also become a regular senior-level graduate and postgraduate course in various engineering disciplines. Because Green's functions are the key elements in the BEM approach, their derivations and behaviors are important to researchers as well as to students in almost all branches of science and engineering. With advanced materials/composites of general anisotropy being created and fabricated, and novel devices of multiphase coupling being designed, new Green's functions in anisotropic and multiphase materials are in need.

This book is intended to provide the basic theory on static Green's functions in general anisotropic magnetoelectroelastic media and their detailed derivations based on the complex variable method, potential method, and integral transforms. Green's functions corresponding to the reduced (simple) cases are also presented including those in anisotropic and transversely isotropic piezoelectric and piezomagnetic media, and those in purely anisotropic elastic, transversely isotropic elastic and isotropic elastic media. Addressed problem domains are three-dimensional (two-dimensional) infinite, half, and bimaterial spaces (planes). While the emphasis is on the Green's functions related to the line and point forces (the first-order source), those corresponding to the important (line and point) dislocation source are also provided and discussed when convenient. It is the authors' intention that this book provides a relatively comprehensive derivation and collection of the Green's functions in the concerned media, and as such, it should be a good reference book in the hands of researchers and engineers, and a textbook and reference book for both undergraduate and graduate students in engineering and applied mathematics.

The book is divided into nine chapters. Chapter 1 is a brief introduction to the Green's function method and related theorems. Chapter 2 presents the governing equations, including the force and charge balance equations, generalized constitutive relations, and the gradient relations between the extended displacements and strains. While in Chapter 3 we derive the two-dimensional Green's functions in elastic isotropic full and bimaterial planes, the Green's functions in corresponding anisotropic magnetoelectroelastic full and bimaterial planes are presented in Chapter 4. Chapter 5 includes the three-dimensional Green's functions in elastic isotropic full and bimaterial spaces. While Chapter 6 derives the three-dimensional Green's functions in a transversely isotropic magnetoelectroelastic full-space, the three-dimensional Green's functions in a transversely isotropic magnetoelectroelastic bimaterial space are derived in Chapter 7. Chapter 8 presents the three-dimensional Green's functions in the corresponding anisotropic magnetoelectroelastic full-space and Chapter 9 those in the corresponding anisotropic magnetoelectroelastic bimaterial space. Direct and indirect applications of the Green's functions to various science and engineering fields are illustrated.

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Both authors would like to express their sincere thanks to their family members for their consistent and never-ending loves and supports. This is particularly true to the first author as he started to put things together since spring 2011 when he took his sabbatical leave. The first author would also like to thank his former graduate students and colleagues for their contributions as cited in different chapters. Mr. Amir Molavi Tabrizi and Mr. Ali Sangghaleh helped draw, respectively, all the figures on Green's function solutions in Chapter 5 and Chapter 8. Professor Peter Chung of ARL, Professor John Albrecht of AFRL/DARPA, and Mr. Roger Green of ODOT shared their research ideas and collaborated with the first author on various interesting topics. Many colleagues from Green's function and BEM communities provided constant encouragements on writing this Green's function book. In recent summers, the first author visited Zhengzhou University and used his evenings and weekends on the book draft. Constant help and support from colleagues at Zhengzhou University are also key contributions to this book. The second author would like to thank Professor Haojiang Ding, his Ph.D. adviser at Zhejiang University for introducing him into the wonderful field of applied mechanics, especially the mechanics of anisotropic solids with multifield coupling. He is extremely grateful to the National Natural Science Foundation of China for its consistent support over the years.

Finally, both authors would like to thank Senior Editor Peter C. Gordon at Cambridge University Press for his understanding, patience, and encouragement. Our book draft was initially targeted on February 15, 2013, was then extended to October 15, 2013, and finally Peter basically sets "no deadline" for this book. Thanks so much, Peter, for your great patience! Sincere thanks also go to Sara Werden and Jayashree Prabhu for their help with various publication issues.

The first author would like to dedicate this book to his parents. Although they never had the chance to enter school, they have provided the first author with the best possible learning opportunity!

Contents

1 Introduction

1.0 Introduction

Green's function is named after George Green for his fundamental contributions to potential theory, reciprocal relations, singular functions, and representative theorem. Green's functions can be extremely powerful in solving various differential equations and are also the essential components in the boundary integral equation method. As singular solutions to certain differential equations in their most generalized mathematical form, such solutions can find applications in nearly every field of science and engineering. They have been and will be continuously utilized in earth science/geophysics; civil, mechanical, and aerospace engineering; physics and material science; nanoscience/nanotechnology; biotechnology; information technology, and so forth. In this chapter, we define the Green's function and introduce its basic features, along with derivations of some of the common Green's functions in potential problems.

1.1 Definition of Green's Function

Green was a mathematician and physicist of United Kingdom (Cannell 2001; Challis and Sheard 2003). He not only developed this powerful tool for solving linear differential equations, but also contributed to various problems in elasticity. For instance, he offered a derivation of the governing equations of elasticity without using any hypothesis on the behavior of the molecular structure of the solids, and was able to show further that twenty-one elastic constants are required in general to account for the general anisotropy of elastic property (Timoshenko 1953). He further explained how symmetry can reduce the independent number of these constants.

Green's function is also called singular function, which is the fundamental solution of a (partial or ordinary) differential equation (or system of equations) in the problem domain (usually of infinite size) where the inhomogeneous term in the equation is replaced by the Dirac delta function.

As an example, let us consider the following differential equation in a two-dimensional (2D) or three-dimensional (3D) infinite and homogeneous domain,

$$Lu(\boldsymbol{r}) = f(\boldsymbol{r}) \tag{1.1}$$