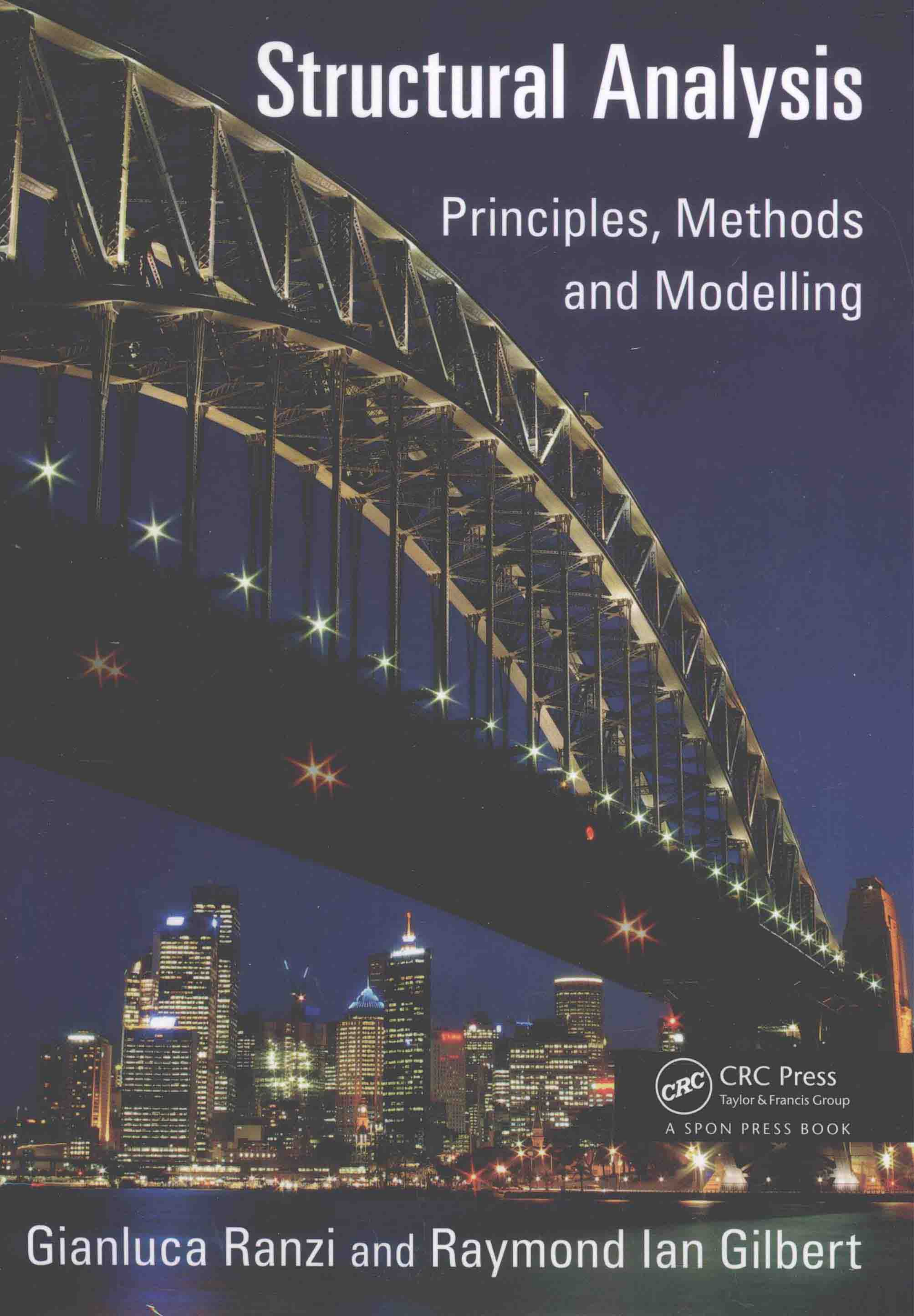
Structural Analysis

Principles, Methods and Modelling



CRC Press
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Gianluca Ranzi and Raymond Ian Gilbert

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CRC Press
Taylor & Francis Group
Boca Raton London New York

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CRC Press
Taylor & Francis Group
6000 Broken Sound Parkway NW, Suite 300
Boca Raton, FL 33487-2742

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Printed on acid-free paper
Version Date: 20140617

International Standard Book Number-13: 978-0-415-52644-9 (Paperback)

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Preface

This book is intended as a text for undergraduate students of Civil or Structural Engineering about to embark on the adventure of learning how to analyse engineering structures. It provides a unique in-depth treatment of structural analysis where fundamental aspects and derivations of the analytical and numerical formulations are outlined and illustrated by numerous simple, yet informative, worked examples.

The book is divided into four parts. The first part comprises Chapters 1 to 4 and covers the analysis of statically determinate structures. Although it is assumed that the student has already completed courses in statics and mechanics of solids, some of the material revises concepts and procedures that have been covered previously. The second part of the book includes Chapters 5 to 9 and deals with the classical methods for the analysis of statically indeterminate structures. These methods are suitable for hand calculation, where the deformation characteristics and the geometry of the structure, as well as considerations of equilibrium, are used to establish the internal actions and structural deformations. Although practising structural engineers usually use computer software packages to analyse structures, these classical methods provide the background knowledge that is essential for the preparation of appropriate input for structural analysis software and the correct interpretation of the output. The third part (Chapters 10 to 12) covers the stiffness method of analysis that underpins most computer applications and commercially available structural analysis software, while the fourth part (Chapters 13 to 15) deals with more advanced topics, including the finite element method, structural stability and problems involving material nonlinearity. Finally, three appendices are included that provide additional background material that is of use throughout the book.

Every topic is illustrated with *numerous worked examples* that lead the student step by step through the solution process. Sections entitled *Reflection Activities* invite students to reflect on the material covered by questioning some of the details of the procedures or extending their applicability to a broader range of problems. The detailed sequence of steps required by different methods of analysis are described in particular sections entitled *Summary of Steps*. At the end of most chapters, a wide range of tutorial problems are set to assist the student to practise the various analysis techniques and to build critical thinking.

The book is complemented by a comprehensive set of educational support material for both instructors and students as described below. We hope that the book will prove useful to both students and instructors.

ADDITIONAL RESOURCES AVAILABLE FOR STUDENTS AND INSTRUCTORS

Resources available for download at http://www.crcpress.com/product/isbn/9780415526449

- *MATLAB scripts* of the worked examples included in Chapters 10 to 13 and 15 are available and will enable students to gain a clear understanding of all steps involved in the structural analysis solution process when implemented in a computer program.

Resources for instructors who adopt the book are available from CRC Press upon request. Please send an email to orders@crcpress.com or contact your sales representative.

- *Solutions Manual* presents detailed solutions for every tutorial problem included in the book.
- *PowerPoint presentations* files available for face-to-face lectures.
- *A PowerPoint presentation* to introduce MATLAB to students with no prior knowledge of it.
- *Videos* supported by a voice narration and are available to enable students to review selected material covered in the book at their own pace. These videos are viewable with computers and smart devices.

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FEEDBACK

As this is the first edition of the book, we would welcome your feedback. Please feel free to send us any comments, criticisms or suggestions regarding any aspect of the book. We will greatly appreciate your input. Please send your feedback to gianluca.ranzi@sydney.edu.au.

Acknowledgments

Thanks are extended to the colleagues and students who provided valuable assistance in the preparation of the textbook and of the solution manual, in particular, Peter Ansourian, Massimiliano Bocciarelli, Graziano Leoni, and Alessandro Zona, who reviewed parts of the manuscript; Lingzhu Chen, Glen Clifton, Anthony Joseph, Charles K. S. Loo Chin Moy, and Osvaldo Vallati, who assisted in the preparation of the material for the solution manual and in the formatting of the figures.

The authors acknowledge the support given by their respective institutions, the University of Sydney and the University of New South Wales.

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Chapter 1

Introduction

1.1 STRUCTURAL ANALYSIS AND DESIGN

Structural engineering involves the *analysis* and *design* of structures and is one of the core sub-disciplines of *civil engineering*. Civil engineering structures take a variety of forms and include buildings, bridges, towers, marine structures, dams, tunnels, retaining walls and other infrastructure. The most common materials used for the construction of these structures are concrete, steel and timber, although a variety of other materials are used including stone, aluminium, polymers, carbon fibre, glass and many more.

Structural engineering underpins and sustains the built environment, where bridges, buildings and other structures must be safe, serviceable, durable, aesthetically pleasing and economical. It is concerned primarily in developing structural solutions to resist loads and other forces, and in devising ways to provide safe load paths for these forces. It is an applied science, founded on mathematical laws and physical concepts applied to engineering materials, both traditional and advanced, for the provision of infrastructure and technological innovation. The demands of new and existing structures imposed by society and by economics and the use of new or advanced materials require solutions that challenge and unite creativity and scientific rigour.

Structural design involves the determination of the type of structure that is suitable for a particular purpose, the materials from which the structure is to be constructed, the loads and other actions that the structure must sustain and the arrangement, layout and dimensions of the various components of the structure. This involves detailed calculations to ensure that the structure is stable and that every structural member, and every connection between members, has adequate strength to resist the design loads. It also involves determination of the deformation of each part of the structure to ensure that the structure remains serviceable throughout its design life and is able to perform its intended function. Structural design involves careful detailing of every part of the structure, including the preparation of detailed structural drawings that effectively communicate the engineering design to the contractors who are engaged to build the structure.

Structural analysis is an integral part of structural design. It involves the calculation of the response of the structure to the design loads and imposed deformations that it will be required to resist during its lifetime. This involves the determination of the internal forces within the various components of the structure and the deformation of these components. Calculation of the internal forces in a structure will allow the structural designer to select materials and member sizes that provide the structure with adequate strength and ensure that the chances of collapse are acceptably small. Calculation of the deformation of the structure will permit the assessment of serviceability. Whether or not a structure is acceptable for a particular purpose depends on its deformation, as well as its strength.

The mathematical algorithms used for structural analysis range from classical methods, suitable for manual calculation (often assuming linear elastic material behaviour), to more complex non-linear numerical analysis, using modern matrix methods and high-performance computers. The choice depends on the type and complexity of the structure and the computational power available to the structural analyst. All methods involve the application of structural mechanics to an idealised structure, where approximations are made concerning the geometry of the structure and its support conditions, the applied loads and deformations, and the material modelling laws. The interpretation of the results of the analysis requires both experience and engineering judgment.

1.2 STRUCTURAL IDEALISATION

It is not possible to undertake an exact analysis of a structure. Real structures have complex geometries that are never known exactly at the time of the analysis. Even structural members that are supposed to be straight are never exactly straight; cross-sectional dimensions that are supposed to be uniform along a member are never exactly uniform; and the dimensions and rotational capacity of connections between structural members and structural supports are never known precisely. Real materials have properties that vary from point to point in a structure and the actual variation and distribution of material properties is never known with a great deal of precision. In addition, the magnitude and distribution of the loads imposed on a structure are rarely known accurately. Structural analysis is therefore undertaken on an *idealised structure*, where simplifying assumptions are made concerning the geometry of the structure and its supports, the material properties and the applied loads so that the conditions approximate those of the real structure. These simplifying approximations introduce errors, some small and some not so small. However, the aim of the idealisation is to simplify the analysis, so that the calculated loads, internal actions, reactions, stresses and deformations are not too different from those in the real structure and adequately describe the behaviour of the structure.

In this process, *loads* applied to any structure can take a variety of forms and may be *idealised* as either concentrated loads or distributed loads, including line loads and surface loads. The structure is *idealised* as a combination of various *components* and *members*, adequately connected to each other and capable of transferring the applied actions through the structure to the supporting foundations. The magnitudes and directions of the forces exerted on a structure by its supports depend on the *types of support*. These structural idealisations (members, loads and supports) are briefly discussed in the following sections.

1.3 STRUCTURAL MEMBERS AND ELEMENTS

Structures are composed of various components and members, that can be categorised according to their dimensions and the way they carry loads.

Many common structural members can be adequately described as *one-dimensional elements*. This is an appropriate classification for elements whose lengths are larger than their cross-sectional dimensions. This is illustrated in Figure 1.1 for a steel member and a reinforced concrete member, whose lengths L are greater than the size of their cross-sections depicted by B and D. For the purpose of structural idealisation, it is possible in these cases to replace the member by a one-dimensional line element.

Line elements are further classified according to the internal actions they resist. For example, a line element that carries only axial forces (either compression or tension) is

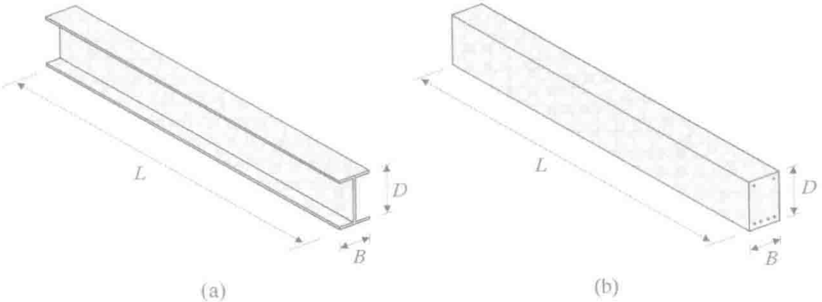

Figure 1.1 One-dimensional (line) elements. (a) Structural steel. (b) Reinforced concrete.

usually referred to as a *bar* or a *truss element*. Figure 1.2 illustrates a truss element before and after deformation. The element elongates when resisting a tensile force and shortens when resisting a compressive force. A bar carrying axial tensile loads applied at each end, with each cross-sections subjected only to axial tension, is called a *tie* (Figure 1.2a), while a bar carrying axial compression applied at each end, with each cross-sections subjected only to axial compression, is called a *strut* (Figure 1.2b).

Whether an axially loaded element in a structure is classified as a tie or a strut often depends on its position in the structure and the way the loads are transferred to it. Let us consider a bookshelf attached to a wall as shown in Figure 1.3a. The weight of the shelf is resisted by compressive forces induced in the diagonal members below the shelf, and the diagonal elements are classified as struts. Let us now consider the arrangement shown in Figure 1.3b. To resist the weight of the shelf, the diagonals above the shelf are in tension and are therefore classified as ties. Sometimes, flexible wires, chains or ropes are used to carry axial tension and are referred to as *cables*. Because of their flexibility, cables are unable to resist compressive loads.

A one-dimensional line element that carries transverse loads in bending and shear (with or without torsion) is called a *beam* or a *girder* and these are very common in structures. Their main features can be illustrated with the help of a simple ruler. If you hold the ruler at its ends and bend it, as depicted in Figure 1.4a, you will apply a couple at each end and a constant curvature will exist along the length of the ruler. The ruler is subjected to a constant bending moment along its length and there are no transverse loads applied to the ruler and

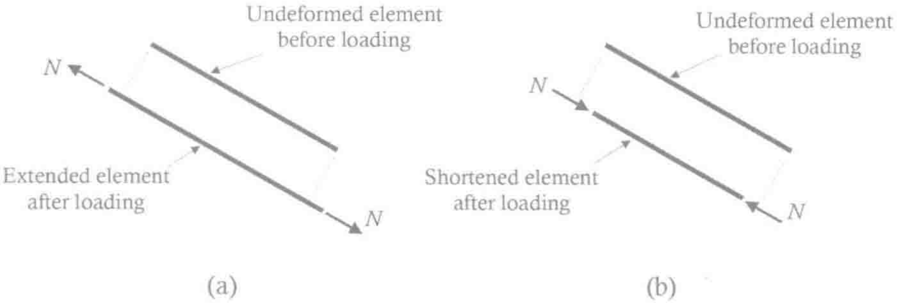

Figure 1.2 One-dimensional (line) elements — ties and struts. (a) Bar in axial tension: a tie. (b) Bar in axial compression: a strut.

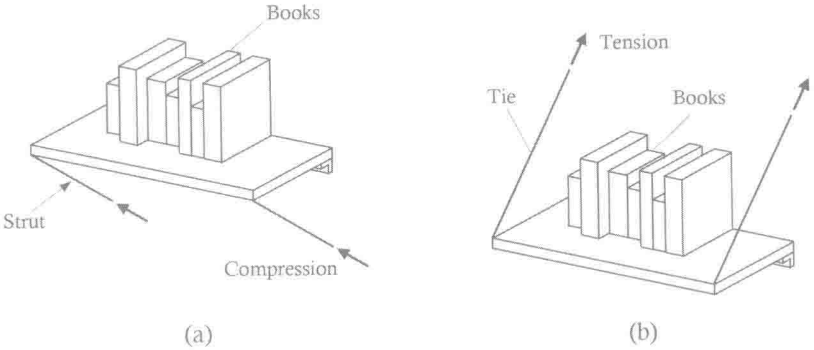

Figure 1.3 Example of struts and ties. (a) Struts supporting shelf from below. (b) Ties supporting shelf from above.

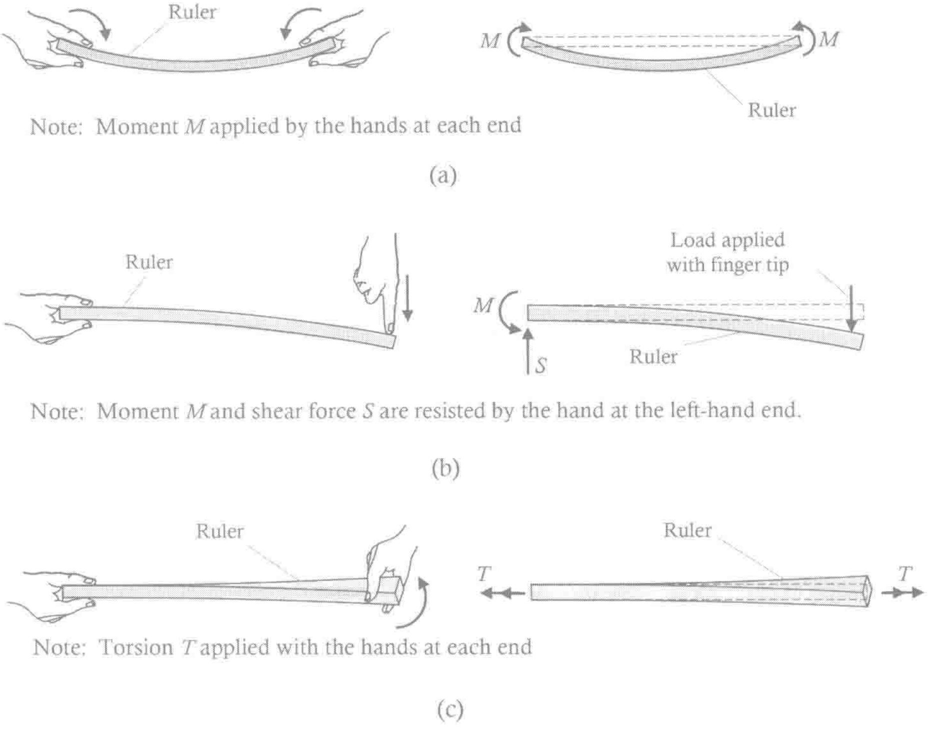

Figure 1.4 Example of line elements acting as beams or girders. (a) Constant moment. (b) Shear force and varying. (c) Constant torsion.

consequently no shear forces. If you are unfamiliar with the concepts of bending moment and shear force, do not worry as they are introduced and explained in Chapters 2 and 3.

If you now hold the same ruler at one end only and apply a downward force with your fingertip at the other end, as shown in Figure 1.4b, each cross-sections of the ruler will be subjected to a constant shear force and the bending moment increases linearly with distance from the applied downward load. A beam can also be subjected to torsion or twisting about its longitudinal axis. Torsion will be induced in the ruler, if you hold it at its ends and twist one side relative to the other, as illustrated in Figure 1.4c.

Columns are one-dimensional elements primarily loaded in axial compression but may also carry bending moments, shear forces and torsion.

Members with one dimension (thickness t) smaller than the other two (length L and width B) can be represented by two-dimensional elements, also referred to as planar elements. It is common to refer to two-dimensional flat elements as *plates* (Figure 1.5a) and curved elements as *shells* (Figure 1.5b). Plates can be subdivided into *slabs* and *walls*, where *slabs* are usually in a horizontal plane and withstand transverse loads by a combination of axial force, bending moments, torsion and shear at their cross-sections. *Walls* consist of planar elements, usually in a vertical plane, and resist both in-plane and transverse forces. Slabs are commonly found in the floor systems of buildings and form part of most bridge decks, while walls also form part of most buildings and are often used to retain soils and water. *Shells* usually have curved surfaces capable of resisting axial forces, bending, torsion and shear.

Membranes are two-dimensional elements that support applied loads by means of a biaxial tensile state. A simple example of this kind of structure is the jumping bed depicted in Figure 1.6, which is capable of supporting the transverse loads induced by the self-weight of a person by stretching its material as shown. The tension induced in the membrane material of the jumping bed by virtue of its deformation is resisted by a compressive force in the ring strut.

Arches are curved structural members that transfer the applied loads by means of compressive forces along the arch. Depending on the actual geometry of the arch and on the nature of the applied loads, the compressive axial force might be combined with different

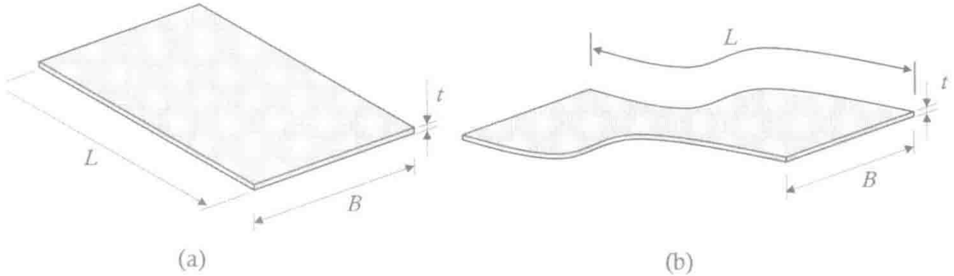

Figure 1.5 Two-dimensional (planar) elements. (a) Plate. (b) Shell.

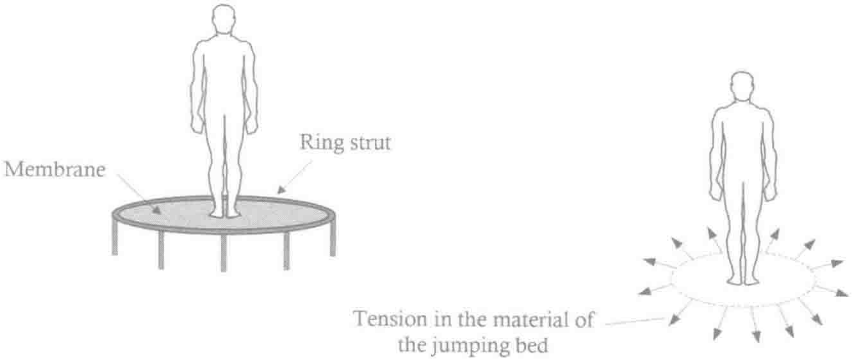

Figure 1.6 A membrane structure.

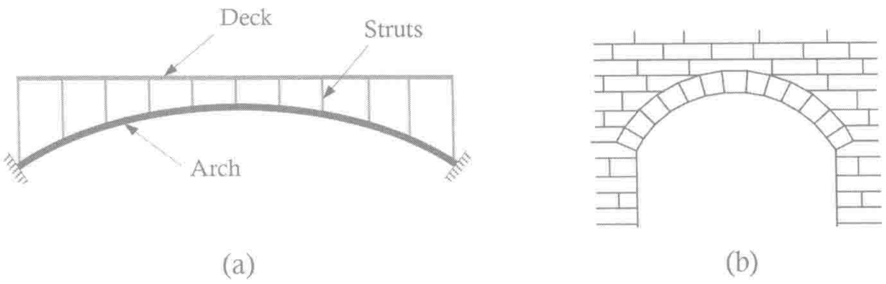

Figure 1.7 Common arches. (a) Arch supporting a bridge deck. (b) Arch in a masonry wall.

levels of moment and shear forces that need to be carefully evaluated. Arches are popular in bridge applications. A typical arch supporting a bridge deck is shown in Figure 1.7a. Arches are also commonly used around openings in masonry construction, as depicted in Figure 1.7b.

1.4 STRUCTURAL SYSTEMS

Structural systems consist of combinations of structural members and are classified according to the type of elements and the way these elements are connected to each other. A real structure is usually represented by a combination of one-, two- and three-dimensional elements. Only some of the most common structural systems are considered in this section and these include *trusses*, *frames*, *arch and cable structures* and *surface structures*.

Trusses are two- or three-dimensional systems of struts and ties connected at their ends by simple pinned connections. This implies that the ends of the members connected together at a node can rotate relative to each other. It is common to assume that loads are mainly applied at the nodes and, under these conditions, the truss elements are subjected to either axial tensile or compressive forces. In reality, truss members may be loaded, between the nodes and must also resist their self-weight. Depending on the members' dimensions, these *member loads* may need careful consideration for an accurate prediction of the structural response. The main components of a simple plane truss are shown in Figure 1.8. The horizontal top and bottom members are usually referred to as *top and bottom chords*, respectively. The distance between the top and bottom chord greatly affects the magnitude of the axial forces in the chords and the structural performance of the truss. Under the same

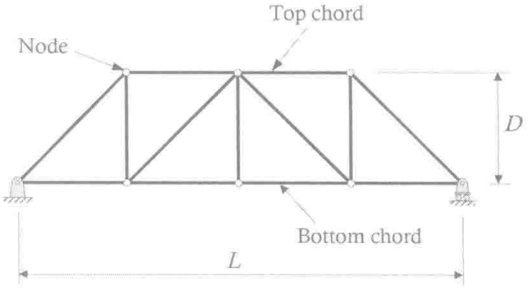

Figure 1.8 Typical components of a plane (two-dimensional) truss.