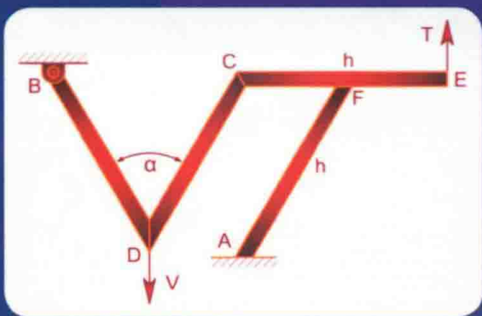
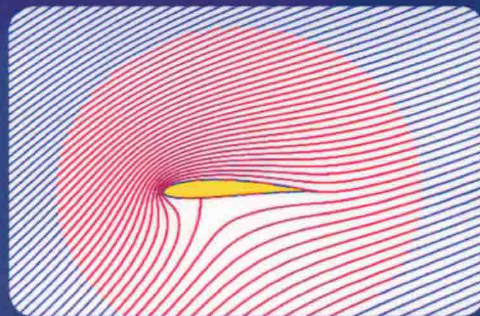
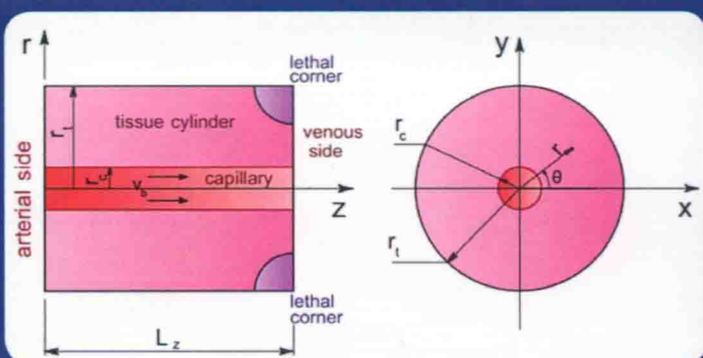
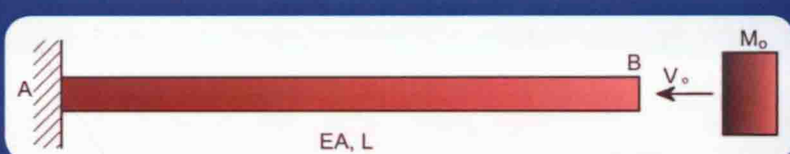
Introduction to Finite Element Analysis for Engineers









Saad A. Ragab and Hassan E. Fayed

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MECHANICAL ENGINEERING

Introduction to
Finite Element Analysis for Engineers

The finite element method is essential for crafting the analysis and design tools that engineers use in computational approaches to problem-solving. Before engineers start using finite-element commercial software, they must know how finite-element models are derived for the basic principles that are usually expressed as differential or integral statements. Thus, with a strong foundation in advanced technological interfaces for such methods, engineering students will be better prepared to tackle complex problems.

Introduction to Finite Element Analysis for Engineers introduces the finite element method as a technique for solving differential equations, which can be broadly applied to problems in civil, mechanical, aerospace and biomedical engineering. Aimed at senior and first-year graduate students in engineering or related disciplines, this inclusive guide shows engineering students how to formulate and solve finite-element models of practical problems and analyze the results. Each chapter focuses on certain types of differential equations including second-order ordinary differential equations (ODE), fourth-order ODE, elliptic partial differential equations (PDE), parabolic PDE, hyperbolic PDE, and differential eigenvalue problems. Throughout the text, the authors emphasize the importance of agreement between computational results and analytic solutions of complex problems, presenting models based on practically-sourced examples that include MATLAB® functionality. This guide

- Presents clear and mathematically simple development of finite-element models
- Is organized by chapter to emphasize mathematical nature of problems
- Verifies finite-element results by comparisons with exact analytic solutions
- Demonstrates new biomedical engineering applications including bio-heat and oxygen uptake in tissues
- Seamlessly transfers among applications in heat transfer, solid and fluid mechanics
- Shows examples in aerodynamics and hydrodynamic stability
- Collects exercises of varying difficulties like hand calculations to computer implementations
- Provides MATLAB codes for solutions and examples

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Suite 300, Boca Raton, FL 33487
711 Third Avenue
New York, NY 10017
2 Park Square, Milton Park
Abingdon, Oxon OX14 4RN, UK

www.crcpress.com

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Fayed

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Saad A. Ragab and Hassan E. Fayed

Virginia Polytechnic Institute and State University

Blacksburg, Virginia

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Introduction to

Finite Element Analysis for Engineers

To our Families
in the United States and Egypt

Preface

Computational approaches to solving engineering problems have become essential analysis and design tools for engineers. The finite-element method is at the center of modern computer analysis techniques. Before embarking on using massive finite-element commercial software, engineers need to know how finite-element models are derived for the basic principles that are usually expressed as differential or integral statements. Having strong mathematical foundation of the finite-element method, engineering students will be better prepared to tackle complex problems. This textbook *Introduction to Finite Element Analysis* has evolved from the first author's lecture notes for finite-element courses that were taught in the department of engineering science and mechanics (now biomedical engineering and mechanics) at Virginia Tech for the past 17 years. The book serves as an introduction to the finite-element method, and presents it as a numerical technique for solving differential equations that describe problems in civil, mechanical, aerospace, and biomedical engineering. It enables engineering students to formulate and solve finite-element models of practical problems and analyze the results. Although commercial finite-element software are not used in this book, it explains the mathematical foundation underpinning such software. Mastering the techniques presented in this book, students will be better prepared to use commercial software as practicing engineers. The book is intended for senior or first-year graduate students in engineering or related disciplines. Thus, mathematical rigor is not compromised but presented at a level consistent with the anticipated mathematics background required in most engineering curricula. The power and versatility of the finite-element method is demonstrated by a large number of examples and exercises of practical engineering problems. Some examples are simple enough so that they can be solved completely in a reasonable time without the use of a computer. Other problems require the use of a PC and programming software such as MATLAB®, *Mathematica* or PYTHON. The MATLAB codes for the examples solved in the book can be downloaded from the publisher's website.

The ability to verify computational results is an especially important facet in using the finite-element method and numerical techniques in general. This book emphasizes the importance of verifying finite-element results by providing comparisons to exact analytic solutions when possible. Excellent agreement between computational results and exact analytic solutions of complex problems strengthens students' confidence in the finite-element method. The book also emphasizes conducting mesh refinement studies so that students become aware of discretization errors, and develop healthy skepticism of physically implausible answers.

The book is organized so that each chapter, after the first introductory chapter, deals with a certain type of differential equation: Second-order ordinary differential equations (ODE), fourth-order ODE, elliptic partial differential equations (PDE), parabolic PDE, hyperbolic PDE, and differential eigenvalue problems. The first chapter is a review of basic mathematical concepts, definitions, and methods in linear algebra and differential equations. For Chapters 2 through 7, each chapter starts with a statement of a standard problem given by a differential equation and associated boundary and/or initial conditions. The weak form of the stated problem (also called the principle of virtual displacements) is derived and used to develop finite-element models.

Chapter 2 deals with second-order ODE. Almost all concepts and definitions of the finite-element method are introduced in that chapter including the method of weighted residuals, weak form, trial solutions, test functions, classification of boundary conditions (essential, natural, mixed), element shape functions, element equations and assembly. Elastic deformation of bars and steady heat conduction in fins and slabs are the main applications. Biomedical engineering applications include oxygen diffusion and consumption in cells and tissues, and pulsatile blood flow in arteries. Chapter 3 deals with fourth-order differential equations; essentially it is the Euler-Bernoulli beam equation. Due to the higher-order equation, the level of complexity is increased because the element shape functions must enable satisfaction of continuity of the trial solution and its first derivative. In addition to beam problems, the finite-element method is extended to plane frames and trusses. The connection between the weak form of the equations of equilibrium and the principle of virtual displacements in solid mechanics is elucidated in that chapter. The principle is derived for a 3D continuum in static equilibrium. Then, it is used to give better understanding of the equivalence between distributed forces and nodal concentrated forces and moments. That chapter also presents the principle of minimum total potential energy, and demonstrates its use for deriving finite-element beam equations.

Chapter 4 deals with elliptic partial differential equations in two space dimensions. Isoparametric elements and numerical integration of element matrices are covered in that chapter. Applications include steady heat conduction, torsion of noncircular sections, laminar viscous flow in ducts, and potential flow around two-dimensional airfoils with lift calculations. Finite-element models are developed for oxygen diffusion and consumption in a Krogh capillary-tissue cylinder in cylindrical coordinates. Linear and nonlinear (Michaelis-Menten) reaction kinetics are covered. Chapter 5 deals with parabolic equations in one space dimension and time. The finite-element method is used for the space operator and the finite-difference method is used for time integration. Applications include transient heat transfer in fins and slabs. Finite-element models for two biomedical engineering problems are presented. First, Pennes heat equation is used to analyze thermal injuries of skin tissues due to flash fire and heat exchange with hot surfaces. Second, transient oxygen diffusion and consumption in cylindrical tissue and spherical cells are analyzed with linear and nonlinear (Michaelis-Menten) reaction kinetics. Hyperbolic equations in one space dimension and time are considered in Chapter 6. The finite-element method is used in space and NewMark method is used for direct time integration. The space operator is either second-order or fourth-order derivative. Waves on strings, and free and forced vibrations of bars and beams are analyzed. Chapter 7 deals with differential eigenvalue problems. Natural frequencies of bars, beams, and frames are presented. Hydrodynamic stability theory is a rich source of differential eigenvalue equations; the Orr–Sommerfeld equation is a well-known example. Finite-element treatment for that equation is presented and applied to Poiseuille channel flow. As previously mentioned, great emphasis is placed on assessment of the accuracy of the finite-element method by comparison with exact analytic solution if available. MATLAB codes used for the examples can be easily modified to solve exercises at the end of each chapter.

In teaching the finite-element method to senior students, we set the learning objectives to be:

1. Develop a clear understanding of the concepts of the weighted residuals and weak form of a differential equation, including defining the physical meaning of primary and secondary variables and essential and natural boundary conditions.
2. Apply the weak form of a differential equation to an element and develop the appropriate element matrices. Assemble element matrices to develop a multi-element model and apply boundary conditions.
3. Be able to determine the consistent equivalent nodal representation of a distributed effect, e.g., equivalent nodal forces due to distributed load.
4. Develop the weak forms of linear partial differential equations (elliptic, parabolic, and hyperbolic) in one dependent variable and two independent variables.
5. Develop element shape functions and element matrices for triangular and rectangular elements.
6. Solve 2D Poisson equation with applications to problems in engineering sciences and mechanics (heat transfer, viscous flow, solid mechanics).
7. Solve transient diffusion problems (e.g., heat transfer in a fin, transient Poiseuille and Couette flows) in one space dimension.
8. Approximate natural frequencies of bars, beams, and frames.
9. Verify finite element results by comparison with exact analytic solutions, and conduct mesh independence studies.

10. Recognize and exploit the versatility of the finite-element method for simultaneously solving problems in different disciplines such as solid and fluid mechanics, heat and mass transfer, etc.

Senior students have been able to easily read a draft manuscript of the book, follow mathematical presentation, and accomplish the above learning objectives. We felt that the notes should be published as a textbook so that other students and practicing engineers could benefit from it.

The authors would like to thank Professor Slimane Adjerid of the Mathematics Department at Virginia Tech for helpful discussions on the finite-element method. We thank Dr. Youssef Bichiou, post-doctoral researcher at Virginia Tech, for reading parts of the book. The first author thanks his sons Ahmad Ragab for reading the entire manuscript and making corrections, and Amr Ragab for discussions on biological and chemical problems. The authors also thank Mr. Jonathan Plant, Ms. Claudia Kisielewicz and Ms. Karen Simon of the editorial staff of Taylor and Francis for their help in the production this book.

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