NUCLEAR ENGINEERING MONOGRAPHS

NUCLEAR REACTOR THEORY

BY

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Preface

With the establishment of programmes of nuclear power both in the United Kingdom and abroad, there is a growing interest in nuclear energy. It is the aim of this Monograph to provide a general introduction to the physics of reactor design. It is based on a series of lectures given to engineers and physicists at a course arranged by the University of Manchester Mechanical Engineering Department.

An attempt has been made to place emphasis on physical concepts rather than mathematics, but some knowledge of mathematics up to elementary calculus is necessary. Some knowledge of elementary nuclear physics is also assumed. After a general account of neutron behaviour in reactors, it is shown how these concepts may be formulated mathematically to provide quantitative data for the physics design of reactors. The control and general kinetic behaviour of nuclear reactors are not discussed since they form the subject of another Monograph in the series. Particular emphasis is given to gas-cooled, natural-uranium-fuelled and graphite-moderated reactors, which form the basis of the United Kingdom nuclear power programme, but the topics discussed are of far more general application; a brief account of possible future development is included in the final chapter.

It is hoped that this Monograph will fulfil its object of providing an elementary introduction to the importance of physics in reactor design, for university and technical college students, and scientists and engineers.

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Harwell
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Notation

A = Atomic weight
a = Radius of sphere or cylinder
a = Radius of uranium rod
a_e = Extrapolated core radius
B^2 = Buckling (or Laplacian)
b = Average cosine of neutron scattering angle
b = Equivalent radius of lattice cell
C_0 = Initial conversion factor
c = Outer radius of coolant gap around fuel
D = Diffusion coefficient
E = Neutron energy
E_0 = Neutron energy corresponding to a velocity of 2200 m/s (i.e. 0·0253 eV)
f_5 = Thermal utilization factor for ^{235}U
H = Height of cylindrical core
J_0 = Zero-order Bessel function of the first kind
j_x = Component of neutron current in x direction
k = Boltzmann's constant
k_{eff} = Effective reproduction constant
k_∞ = Infinite medium reproduction constant
L = Thermal neutron diffusion length
L_s = Neutron slowing down length
l = Neutron mean free path
M = Neutron migration length
MWD = Megawatt day of energy
N = Number of atoms/cm^3
N_f = Average number of collisions made by neutron in slowing down
n = Neutron density (neutrons/cm^3)
P = Collision probability for fission neutron in fuel rod
p = Resonance escape probability
r = Radial distance in polar co-ordinates
S = Neutron source strength (neutrons/cm^3 sec)
T = Neutron temperature in °K
T_M = Moderator temperature in °K
T_e = Extrapolated reflector thickness
V = Volume
v = Neutron velocity
z = Axial distance in cylindrical coordinates

$\alpha = (A-1)^2/(A+1)^2$
δ = Reflector saving
ϵ = Fast fission factor
η = Average number of neutrons produced per neutron absorbed in fissile material
θ = Neutron scattering angle (in laboratory frame of reference)
κ = Inverse neutron diffusion length (i.e. L^{-1})
Λ = Neutron non-leakage probability ($=\Lambda_f\,\Lambda_t$)
Λ_f = Fast neutron non-leakage probability
Λ_t = Thermal neutron non-leakage probability
λ = Linear extrapolation length
λ = Ratio of epithermal to thermal neutron flux
ξ = Mean logarithmic energy decrement per collision
ρ = Density (gm/cm^3)
ρ = Reactivity
Σ = Macroscopic cross-section ($=N\sigma$)
Σ_{12} = Two-group theory macroscopic slowing down cross-section
σ = Microscopic cross-section
$\hat{\sigma}$ = Microscopic cross-section reduced to 2200 m/s (Westcott convention)
ϕ = Neutron flux
$\bar{\phi}$ = Average neutron flux in a region
$\hat{\phi}$ = Neutron flux reduced to 2200 m/s (Westcott convention)
ψ = Neutron scattering angle (centre of mass frame of reference)

Subscripts

a	Absorption	f	Fission	tr	Transport
C	Core	M	Moderator	U	Uranium
C	Canning	R	Reflector	0	2200 m/s
c	Radiative capture	s	Scattering	1, 2	Fast, thermal groups
e	Epithermal	th	Thermal	5, 8	^{235}U, ^{238}U
eff	Effective	tot	Total		

Contents

Chapter One

NUCLEAR CHAIN REACTIONS

Nuclear Power

A nuclear reactor is a device in which atomic nuclei are caused to take part in a self-sustaining nuclear reaction and in doing so to produce energy. In this book we consider only *fission* reactors, in which energy is produced by breaking up heavy nuclei, although a *fusion* (or *thermonuclear*) reactor, in which energy is produced by building up very light nuclei into heavier ones, is also possible in principle. Such fusion reactors may become feasible in the future.

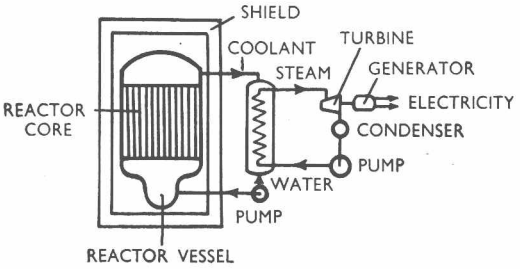

FIG. 1
A power reactor

Most of the energy produced by nuclear reactions appears as heat, and in a typical power-producing reactor this heat is carried away by a coolant which passes on to a *heat exchanger* where it gives up its heat to water. This water is converted to steam and fed to a conventional type of steam turbine to produce electricity. This is shown schematically in Fig. 1. It is seen that the nuclear reactor replaces the coal- or oil-burning furnace of a conventional electrical generating station.

Nuclear Fission

Hahn and Strassman showed in 1938 that a heavy nucleus can absorb a neutron and undergo fission, that is, it splits up into two lighter nuclei (fission products). The fission process has two very important features. First, there is a loss of mass in this reaction (i.e. the end-products have a smaller mass than the initial nucleus and neutron). It is a consequence of Einstein's Special Theory of Relativity that mass and energy are equivalent, and consequently this mass deficit appears as energy. The release of energy is not unusual in nuclear reactions, but fission

differs from other reactions in that, on the average, each fission produces more than one new neutron. Under certain conditions these secondary neutrons can cause further fissions, and a chain reaction is set up. Fission differs from most other nuclear reactions in that it makes possible *continuous* production of energy.

All heavy nuclei can undergo fission when bombarded by neutrons that are sufficiently fast, but in 1939, Bohr and Wheeler predicted that only very heavy nuclei containing an odd number of neutrons would be fissile to neutrons of all kinetic energies down to zero. The only naturally occurring nucleus which satisfies this condition is ^{235}U, containing 92 protons and 143 neutrons. ^{235}U occurs in nature only in natural uranium, where it is mixed with the chemically identical material ^{238}U, in the proportions of 1 atom of ^{235}U to about 138 atoms of ^{238}U (i.e. 0·71 per cent by weight). Thus, only a small fraction of natural uranium can be fissioned by low energy neutrons. Since ^{235}U and ^{238}U are chemically identical, the more reactive fraction of natural uranium (^{235}U) can be separated only by very elaborate physical processes (e.g. based on the very small difference in diffusion rates of gaseous compounds of ^{235}U and ^{238}U, due to their slight difference in density) requiring expensive plants and large power consumption.

Although ^{235}U is the only naturally occurring material which is fissile to slow neutrons, other such materials can be made artificially in nuclear reactors, e.g. ^{239}Pu (made from ^{238}U) and ^{233}U (made from ^{232}Th).

When a ^{235}U nucleus is fissioned, on the average about 200 MeV* of energy and about 2·5 neutrons are produced. Thus, one fission releases $3{\cdot}2 \times 10^{-11}$ watt sec of energy, i.e. $3{\cdot}1 \times 10^{10}$ fissions/sec produce 1 watt of power. Since a ^{235}U atom weighs about $3{\cdot}90 \times 10^{-22}$ g, it follows that if 1 g of ^{235}U is completely fissioned, then $8{\cdot}2 \times 10^{10}$ watt sec = 0·95 MW days (MWD) of energy are produced. Thus approximately:

$$1 \text{ g of material fissioned} \equiv 1 \text{ MWD of heat.}$$

For comparison, the combustion of 1 ton of coal produces about 0·36 MWD of heat, i.e. the heat produced by the complete fission of 1 ton of uranium is equivalent to that from $2{\cdot}7 \times 10^6$ tons of coal.

The neutrons produced in fission have a spread of energies, which by experiment is found to fit the empirical expression plotted in Fig. 2:

$$n(E)\,dE = Ae^{-E} \sinh \sqrt{(2E)}\,dE \quad (E \text{ in MeV})$$

where $n(E)\,dE$ = the fraction of neutrons with energies in the range

* An eV (electron-volt) is a unit of energy used in nuclear physics. 1 MeV ($=10^6$ eV) is equivalent to $1{\cdot}60 \times 10^{-6}$ erg $= 1{\cdot}60 \times 10^{-13}$ watt sec. or $1{\cdot}52 \times 10^{-16}$ B.t.u.

E to $E+dE$ (MeV). Integration of this expression gives:

$$A = \sqrt{(2/\pi e)} = 0{\cdot}484$$

and the average energy of a neutron produced in fission, $E_f = 2{\cdot}0$ MeV.

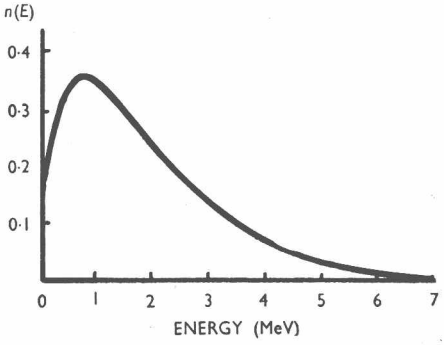

FIG. 2
Energy distribution of fission neutrons

THE FISSION CHAIN REACTION

One neutron can cause a fission which produces more than one secondary neutron; these secondary neutrons also have a chance of causing a fission and so a chain reaction is possible. If, on the average, each neutron from fission causes exactly one further fission, then a stable chain reaction is established and there will be a steady production of power. This is known as a *critical* system. If, on average, each neutron produces more or less than one further fission, however, then the power will either increase indefinitely (a *supercritical* or *divergent* reaction), or decrease to zero (a *subcritical* or *convergent* reaction).

The possible fates of neutrons from fission are:

(1) causing further fission
(2) absorption in fissile material without causing fission (*radiative capture*)
(3) absorption in other materials (*radiative capture*)
(4) leakage from the system boundaries.

The probability of leakage depends on the size of the system, and is zero for an infinitely large system. Thus, if a divergent chain reaction is possible in an infinite system then, for a given geometrical shape, there will be a particular size for which leakage is just sufficient to maintain a stable chain reaction. This is known as the *critical size* of the system.

The fundamental task of reactor physics is to determine under what conditions a critical chain reaction can occur, and what the

appropriate critical size is. We will first consider under what conditions a divergent chain reaction is possible in an infinite system, since only then will it be possible for a finite system to be critical.

Fast Neutron Reaction

First consider a mass of pure uranium, and for the moment consider only ^{238}U. This has a cross-section* for fission which varies with the energy of the incident neutron as shown in Fig. 3. We see that neutrons of energy below about 1 MeV cannot cause fission in it.

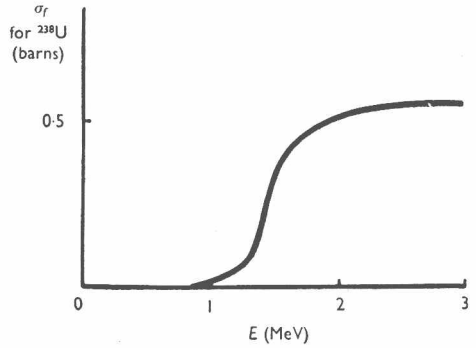

FIG. 3
Fission cross-section of ^{238}U

Consider a fission neutron, with an energy of about 2 MeV. Possible fates are:

(1) inelastic scattering—causing loss of energy
(2) radiative capture—absorption without fission
(3) fission.

At energies of about 2 MeV, the fission cross-section exceeds that for radiative capture and one might therefore expect a fast neutron chain reaction to be maintained in ^{238}U. The probability of inelastic scattering is much greater than that for fission, however, and, in consequence, neutrons are rapidly degraded in energy to below the fission threshold of ^{238}U, with only a slight increase in number due to fission. These neutrons are then captured in ^{238}U and a divergent chain reaction is not achieved.

In natural uranium, there is some ^{235}U present, and this has a higher fission cross-section than ^{238}U at all energies. The greater fission cross-section, however, is insufficient to compensate for the very low

* The cross-section of a nucleus is a measure of the probability of a particular nuclear reaction occurring. It is discussed in Chapter Two. It is usually represented by the symbol σ and measured in units of 10^{-24} cm^2, or *barns*.

concentration of ^{235}U, and neutrons are again degraded by inelastic scattering to *resonance* energies. Here the probability of capture in ^{238}U is very high—much greater than that of causing fission in the small amount of ^{235}U present—and as in ^{238}U, a divergent chain reaction is not achieved.

Although a divergent chain reaction is not possible in pure ^{238}U or natural uranium, it can be achieved in pure fissile material, or in highly enriched uranium (containing more than about 10 per cent ^{235}U), since then the probability of fission for fast neutrons is comparable with that for inelastic scattering and much greater than the probability of radiative capture. Such a fast neutron chain reaction is utilized in the atomic bomb, also in a *fast reactor* which consists essentially of enriched uranium, structural materials, and coolant.

Thermal Neutron Reaction

In natural uranium, no chain reaction is possible at high or intermediate energies, due to resonance absorption in ^{238}U. At very low neutron energies, however, below the resonance region of ^{238}U, the fission cross-section of ^{235}U is very large—about 200 times the capture cross-section of ^{238}U—and even in natural uranium each low energy neutron absorbed produces, on the average, more than one secondary neutron. If, then, it could be arranged that neutrons produced in fission were slowed down to low energies, below the resonance region of ^{238}U, without appreciable absorption in ^{238}U during the slowing down, then a divergent chain reaction would be possible with natural uranium, using the ^{235}U as fuel.

This can be achieved by having present with the uranium a material of low atomic weight and low neutron absorption. Such a material is known as a *moderator*. Neutrons make elastic collisions with moderator atoms and share their energy with the nearly stationary atoms, causing the neutrons to be slowed down until their energy is comparable with the energy of thermal vibration of the moderator atoms. The neutrons then reach equilibrium with the thermal energy of the moderator atoms and are known as *thermal neutrons*. If the moderator atoms are light enough, and if there is a sufficient quantity of moderator present, then neutrons will be rapidly slowed down through the resonance region with only a small chance of meeting a ^{238}U atom and undergoing capture. The thermal neutrons then can cause fission and so set up a chain reaction. A nuclear reactor working in this way is known as a *thermal reactor*.

In practice it is found, with almost all moderators, that if natural uranium is mixed uniformly with the moderator, then a divergent chain reaction cannot be achieved, because resonance absorption by ^{238}U is

still too high. The addition of more moderator would decrease resonance absorption, but this would be offset by the additional loss of neutrons to the moderator itself. Fermi and Szilard made the suggestion that resonance absorption could be decreased by arranging the uranium in discrete lumps, instead of mixing it homogeneously with the moderator. The interior of each uranium lump would then be screened from neutrons of resonance energy which would be mostly absorbed by the outer layers of each lump.

Thus, almost the only type of natural uranium reactor possible is a *heterogeneous thermal reactor*, i.e. a reactor in which discrete lumps of uranium are embedded in the moderator. (A homogeneous thermal reactor is possible with enriched uranium, and it is just possible with natural uranium if heavy water is used as the moderator.) For convenience of handling and cooling, the lumps of uranium are actually rods in a lattice arrangement in a matrix of moderator.

Classification of Reactors

Fission reactors are usually classified in two ways:

1. *Type of Chain Reaction*

   (*a*) Fast reactors: most fissions are caused by neutrons with near-fission energies (of the order of 1 MeV).

   (*b*) Thermal reactors: most fissions are caused by neutrons with thermal energies (of the order of 0·1 eV).

   (*c*) Intermediate reactors: here there is some moderator present but not enough to thermalize most of the neutrons. Most fissions are caused by neutrons with energies in the range 0·1 eV to 0·1 MeV, depending on the design.

Most of the reactors at present in existence, or in course of design or construction, are thermal reactors, and this book is devoted mainly to the study of such reactors. In particular, natural uranium graphite-moderated thermal reactors, of the type that are being built in the United Kingdom for power generation, are taken as an example. The methods used are, however, applicable to other types of reactor.

2. *Purpose*

   (*a*) Power: fission produces heat which, in a suitably designed system, can be removed and converted into useful power.

   (*b*) Production of Fissile Material: in a reactor containing ^{238}U an appreciable fraction of the neutrons produced is absorbed in ^{238}U. These neutrons are lost, so far as the maintenance of a chain reaction is concerned, but they are not wasted since they convert ^{238}U into

plutonium by the reaction:

$$^{238}\mathrm{U} + n \rightarrow {}^{239}\mathrm{U} \xrightarrow[\text{decay}]{\beta^-} {}^{239}\mathrm{Np} \xrightarrow[\text{decay}]{\beta^-} {}^{239}\mathrm{Pu}.$$

^{239}Pu is stable and fissile to slow neutrons, and since it is chemically different from uranium it can be separated from the irradiated fuel. Thus, pure fissile material can be made from natural uranium and used to fuel reactors which cannot be made critical with natural uranium, or for the manufacture of atomic bombs. Fissile material can similarly be produced in a reactor by absorbing neutrons in thorium to make ^{233}U.

(*c*) Research: reactors, being high intensity sources of neutrons, can be used for research on nuclear physics and the effects of neutron irradiation on materials; also for the manufacture of radioactive isotopes.

Natural Uranium Thermal Reactor

A typical natural uranium thermal reactor consists of a lattice of rods of uranium in a matrix of moderating material (Fig. 4). Cooling of the

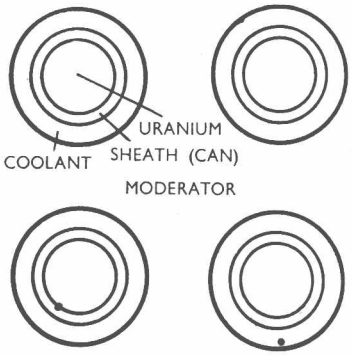

FIG. 4. *A thermal reactor lattice*

uranium is necessary to remove the heat produced by fission and thus the uranium is surrounded by a coolant annulus. The uranium rod is contained in a *can* (or *sheath*) to prevent: (1) contamination of the coolant by radioactive fission products; (2) corrosion of the uranium by the coolant.

Much of this book is devoted to demonstrating how the conditions for a chain reaction in such a system can be calculated.

Energy of Neutrons in a Thermal Reactor

Neutrons produced in fission are slowed down by collision with moderator atoms until they become thermal neutrons, i.e. until they reach equilibrium with the thermal vibration of the moderator atoms. Further collisions with moderator atoms cause no further energy loss, and thermal neutrons diffuse through the moderator like molecules in a gas. Thus, the distribution of thermal neutron energies is determined by the moderator temperature, and is given by the same expression as that for the energy distribution of molecules in a gas. This is the Maxwell-Boltzmann distribution (Fig. 5), which is derived in the kinetic theory of gases by statistical mechanics:

$$n(E)\,dE = \{2\pi/(\pi kT)^{3/2}\} \exp(-E/kT).E^{1/2}\,dE \quad . \quad (1.1)$$

where $n(E)\,dE$ = the fraction of neutrons in the range E to $E+dE$
k = Boltzmann's constant ($=1{\cdot}38\times10^{-16}$ erg /°C)
T = the moderator temperature (in °K).

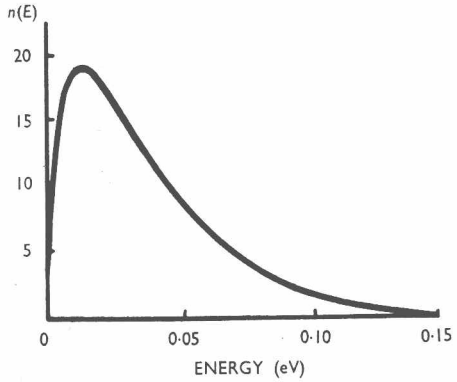

FIG. 5
Energy distribution of thermal neutrons at 20°C

This expression is strictly correct only if there is no neutron absorption. Low energy neutrons tend to be preferentially absorbed which causes a distortion of this distribution, but the distorted distribution can still be approximately represented by an expression of this form with a somewhat higher value of T. Thus, for thermal neutrons in a reactor, the neutron temperature (i.e. the value of T for which the Maxwell-Boltzmann expression fits the actual neutron energy distribution) is slightly greater than the moderator temperature, due to neutron absorption in the fuel.

From the Maxwell-Boltzmann expression it can be shown that the most probable velocity has the value $1{\cdot}28\times10^{4}\sqrt{T}$ cm/sec, and the corresponding kinetic energy is kT, i.e. $(8{\cdot}61\times10^{-5}T)$ eV. (This is *not*

the same as the most probable energy, which is $\frac{1}{2}kT$, or the average energy, which is $\frac{3}{2}kT$.) It has become conventional to tabulate values of neutron cross-sections corresponding to a neutron velocity of 2200 m/sec (i.e. an energy of 0·0253 eV), that is, for neutrons with the most probable velocity in a thermal neutron distribution at a temperature of 20·4°C (i.e. $T=293{\cdot}6$°K).

In a thermal reactor, in addition to thermal neutrons, neutrons in the process of slowing down are present. These neutrons have higher energies than the thermal neutrons, and are called *epithermal* neutrons.

Life Cycle of Neutrons in a Thermal Reactor

The life cycle of a neutron in a thermal reactor is shown schematically in Fig. 6.

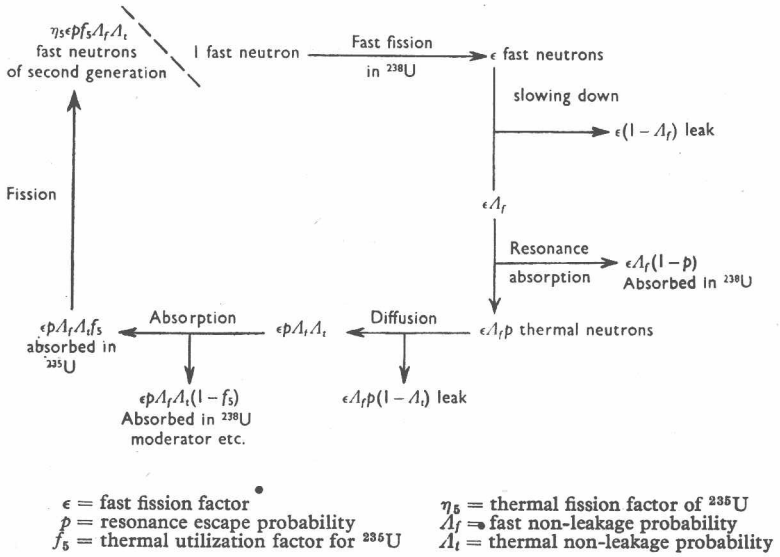

FIG. 6. *The life cycle of a neutron in a thermal reactor*

Three simplifying assumptions are made:

(1) It is assumed that thermal neutrons can be represented by a single group of neutrons, all with the same average energy.

(2) It is assumed that no absorption of neutrons occurs while slowing down, except in ^{238}U. Epithermal absorption in other materials is small, mostly occurs at energies just above thermal, and

is included in the absorption of thermal neutrons. Thus in thermal reactor calculations, the thermal cross-sections used should contain a correction for epithermal absorption.

(3) It is assumed that resonance absorption of neutrons in ^{238}U occurs in a narrow energy band just above the thermal region.

We will consider one fast neutron from thermal fission of ^{235}U. This has a small probability of causing fast fission in uranium, and on the average results in enhancement of fast neutrons by a factor ϵ. These ϵ fast neutrons are slowed down by the moderator and while being slowed down, some of them leak from the boundaries of the system. A fraction Λ_f reach the resonance region of ^{238}U without leakage. A fraction p of these $\epsilon\Lambda_f$ neutrons, on the average, escape resonance capture in ^{238}U and $\epsilon p\Lambda_f$ neutrons become thermalized. These thermal neutrons diffuse through the moderator until they leak from the system boundaries, are absorbed in ^{235}U, or are absorbed in some other material, e.g. ^{238}U, moderator, etc. If the probability of escaping thermal leakage is Λ_t, then on the average $\epsilon p\Lambda_f\Lambda_t$ neutrons are absorbed at thermal energies. We define f_5 as the average fraction of thermal neutrons absorbed which are absorbed in ^{235}U, and η_5 as the average number of fission neutrons produced for each thermal neutron absorbed in ^{235}U. Then the average number of second generation fast neutrons produced from thermal fission of ^{235}U is:

$$\eta_5\epsilon p f_5\Lambda_f\Lambda_t.$$

This is defined as the effective reproduction constant of the reactor $(=k_{\text{eff}})$.

Therefore

$$k_{\text{eff}} = \eta_5\epsilon p f_5\Lambda_f\Lambda_t \quad . \quad . \quad . \quad . \quad (1.2)$$

We also define a quantity called the infinite medium reproduction constant of the reactor (k_∞) equal to the number of second generation neutrons produced by one neutron in an infinite system with the same properties as the finite reactor.

Now ϵ, p, η_5, f_5 depend only on the properties and configuration of the reactor materials and do not depend on the overall size and shape of the reactor. On the other hand, Λ_f and Λ_t depend both on the internal reactor configuration and on the overall size and shape of the reactor. In particular, for an infinite system (for which $k_{\text{eff}}=k_\infty$)

$$\Lambda_f = \Lambda_t = 1.$$

Thus, equation (1.2) becomes:

$$k_\infty = \eta_5\epsilon p f_5 \quad \text{(known as the 4 } \textit{Factor Formula}\text{)} \quad . \quad . \quad (1.3)$$

$$k_{\text{eff}} = k_{\infty}\Lambda \quad . \quad . \quad . \quad . \quad . \quad . \quad (1.4)$$

where $\Lambda = \Lambda_f \Lambda_t$ = the probability that a fission neutron will slow down and be absorbed without leakage.

Critical Size

In a reactor operating at steady power (i.e. in a critical system), the number of neutrons does not change with time and the value of k_{eff} is therefore unity, i.e.

Neutron production = Neutron absorption + Neutron leakage.

Now for any fuel configuration with a value of k_{∞} greater than unity, there exists a particular size of reactor (for a given geometrical shape) which will make $k_{\infty}\Lambda$ equal to unity. This is the critical size of the reactor.

The calculation of the critical size of a reactor can be divided into two parts:

(1) Determination of k_{∞}—this depends on the reactor materials and internal configuration but is independent of the size and shape of the reactor.

(2) Determination of Λ—this depends both on the average distance a neutron travels before slowing down or absorption (the *characteristic lengths*) and on the size and shape of the reactor.

Chapter Two

DIFFUSION OF THERMAL NEUTRONS

Neutron Flux and Cross-sections

It was shown in Chapter One that the critical size of a reactor depends on the average distance that a fission neutron travels before it is absorbed. In a thermal reactor it is convenient to divide this distance into two parts: the average distance travelled by a neutron in slowing down to thermal energies; and the average distance travelled by a thermal neutron before absorption. Before considering these two processes in more detail, we will define some terms that will be used.

Consider a pure material containing N nuclei/cm^3. For a pure material of atomic weight A and density ρ g/cm^3, the number of atoms in A g is given by Avogadro's number ($6{\cdot}025 \times 10^{23}$). Hence:

$$N = 0{\cdot}6025 \frac{\rho}{A} \times 10^{24} \text{ atoms/cm}^3 \text{ sec} \quad . \quad . \quad . \quad (2.1)$$

Suppose that in the material there are n neutrons/cm^3, each moving with the same velocity v cm/sec and energy E. Then the number of neutrons absorbed by the material per cm^3/sec is proportional to the total distance travelled by all the neutrons present in one cm^3 in one second; also to the number of nuclei present in one cm^3. Thus the number of absorptions can be written:

$$Nnv\sigma_a \text{ absorptions/cm}^3 \text{ sec}$$

where σ_a is a function of v that is characteristic of the material and independent of n. It has the units of area, and is known as the *absorption cross-section* of the material; it is usually measured in units of *barns* (1 barn $= 10^{-24}$ cm^2).

The number of absorptions/cm^3 sec can be written:

$$\text{Reaction rate} = Nnv\sigma_a = \phi\Sigma_a \quad . \quad . \quad . \quad (2.2)$$

where $\Sigma_a = N\sigma_a$ is the *macroscopic absorption cross-section* of the material and is measured in units of cm^{-1}

$\phi = nv$ is the *neutron flux* and is measured in units of neutrons/cm^2 sec. It is the total track length executed by the neutrons in 1 cm^3/sec.

Thus:

$$\text{Reaction rate} = \text{Macroscopic cross-section} \times \text{Neutron flux}$$

The absorption mean free path (l_a) is the average distance that a neutron travels before absorption, and it is equal to the total distance