METHODS OF

Experimental Physics

VOLUME 1

CLASSICAL METHODS

Volume 1

Classical Methods

Edited by

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1959

ACADEMIC PRESS · New York and London

United Kingdom edition published by
ACADEMIC PRESS INC. (London) Ltd.
40 Pall Mall, London, S. W. 1

Library of Congress Catalog Card Number: 59-7686

PRINTED IN THE UNITED STATES OF AMERICA

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FOREWORD

The *Methods of Experimental Physics* is the outgrowth of a discussion, several years ago, of the need in the American literature of physics for a handbook type volume giving the experimental physicist a convenient guide in his work. In the course of this discussion the conclusion was reached that, with the present development of physics, the material would have to be subdivided into several volumes, each volume covering a sufficiently large field without too much overlapping. The scope and basic philosophies of this publishing venture were outlined in the original editorial policy statement:

"The experimental physicist—the person for whom this book should be written—is normally a specialist, working in a relatively narrow domain. He is presumed to know his own specialty very well. In the course of his work, however, he is often confronted with the necessity of using methods borrowed from neighboring fields, with which he is less familiar. In such cases he may have to make a literature search or ask advice from a specialist in the neighboring field. The existence of a concise presentation of the most important methods used in experimental physics would considerably simplify the task.

"The methods used may be purely experimental, but very often they are partly theoretical or computational. They may be qualitative, or may require the ultimate in accuracy. In an unfamiliar field, the experimenter will need a guide to the ways and means best adapted to his own investigation.

"Thus the book should be a concise, well illustrated presentation of the most important methods, or general principles, needed by the experimenter, complete with basic references for further reading. Indication of limitations of both applicability and accuracy is an important part of the presentation. Information about the interpretation of experiments, about the evaluation of errors, and about the validity of approximations should also be given. The book should not be merely a description of laboratory techniques, nor should it be a catalog of instruments.

"These volumes should be written so as to be of value to all research workers who use physical methods. Finally, the volumes should be organized in such a way that they will provide essential tools for graduate students in physics."

Having outlined the policy and scope of this work, the next task was to find editors who could bring the series to fruitful completion. We were successful in securing the collaboration of outstanding scientists to serve as editors for the volumes, and it is a pleasure to be able to list their names on the title page of this first volume of the series.

It remains to be judged by the reader how far we have achieved the goal set in our editorial policy. The volume editors and I sincerely hope that we have achieved it to a reasonable extent. Together with them I would like to invite the opinions of all our colleagues to let us know what omissions there are and what errors there may be, so that in future editions we may correct them. Uppermost in our minds has been the desire to provide the research worker and the graduate student with a good and useful tool to aid in their research, because we firmly believe that the conception of emphasizing the *method*, with a concurrent neglect of gadgetry, is an untried idea in the American literature of physics.

Finally, it is my pleasant duty to thank publicly all those who helped in the realization of these goals. The officials and staff of Academic Press Inc. were tireless in ironing out difficulties and providing a stimulating collaboration. The greatest part of the work on this first volume was on the shoulders of Dr. Immanuel Estermann; no amount of thanks can express my appreciation for the devotion and knowledge which he brought to the task. Editorial details on the home front were handled skillfully by Mrs. Claire Marton. Last, and most important, I extend my best thanks to the authors, whose contributions have created this volume.

L. Marton

Washington, D.C.
February, 1959

PREFACE TO VOLUME I

The rational subdivision of the science of physics into components is becoming increasingly difficult. In addition to the "classical" fields, such as mechanics, heat, sound, light, electricity, and magnetism, there have developed in the last decades a number of functional specialties, such as nuclear and solid state physics, which cut across the classical subdivisions and have added their own experimental methods for the pursuit of their objectives. In the determination of the material to be included in this volume, the editor was guided by the following considerations: The content should be of interest to physicists working in all the specialties, and beyond that, to researchers in other scientific fields for which physical methods are becoming more and more important. On the other hand, methods and techniques developed primarily for the various functional branches of physics have been reserved for the other volumes in the series.

It is obvious that the volume title "Classical Methods" is not to be interpreted as an antonym to modern methods. The term refers to those methods which are related to the general subdivisions of classical physics, but the editor and the authors have endeavored to include the most recent developments in experimental technology and theoretical justification, leaving out those techniques which have only historical interest. It was not intended to produce a "cook book" which gives a detailed description of favorite recipes but rather a "guide book" which points out the advantages, capabilities and limitations of the various methods and thus enables the user to select those which appear to be appropriate for his particular problem. Numerous references leading to more detailed descriptions of the different techniques are expected to provide additional information where needed. Methods and techniques customarily taught in elementary physics courses are either passed over or treated very briefly.

Completing a measurement is only one part of a research objective; the correct interpretation of the result is at least of equal importance. We have, therefore, included relatively detailed discussions of the theoretical significance of the various concepts and their relationships to the quantities that are actually measured. Thus a relatively large portion of this volume contains theoretical material which the authors felt to be necessary for the precise clarification of the concepts and for the proper interpretation of the measurements.

While the arrangement of the various parts and chapters follows a uniform style, no attempt has been made to suppress the individuality of the authors. Each part or chapter represents the approach of a different person to a common problem. Some authors are putting more emphasis on the experimental, others more on the theoretical side of their subject. It is the editor's hope that this divergence of attitude, which extends even to the preference of different systems of units by different authors, will make the book more readable and enjoyable.

The volume editor wishes to express his gratitude to the general editor and to the authors for their excellent cooperation and to the publisher and his staff for their patient assistance in coping with the many problems connected with the preparation of this volume.

I. Estermann

February, 1959

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1. EVALUATION OF MEASUREMENT*

1.1. General Rules

In a concise expression of the results of the measurement of a physical quantity, three pieces of information should be given: a number, a numerical statement of reliability, and an appropriate set of units.† The number is generally an estimate expressed in a finite set of digits (the exceptions are numbers which are exact by arbitrary definition, or mathematical constants such as the base of natural logarithms) reflecting the limited accuracy of physical measurement. The statement of reliability is usually written as plus or minus one, or at *most* two digits in units of the last digit of the number, together with a sufficient explanation to allow interpretation. In particular, one should state how many measurements were employed in the determination of the number and of its reliability. As will be seen below, this is of great value in the critical comparison of the results of different experiments, and in their combination with results of previous work. The number of digits that can be read is indicated by the smallest scale division, the *least count* of an instrument. Usually, one additional digit can be estimated between scale divisions. No more—and no less—digits should be recorded than can be read reproducibly.

To remove ambiguities, a standard form may be used for the recording of data: the decimal point is put just after the first nonzero digit, and the number is multiplied by the appropriate power of ten. Every digit is then understood to be significant. The magnitude of error is automatically indicated by the number of significant digits. The final result will generally have one more significant digit than the individual readings. This procedure implies that one should not round off readings. Any round-off increases the error. In the course of computation, round-off may be inevitable. A brief discussion of errors so introduced, with further references, is given in Chapter 1.6.

For the estimation of the best value of the desired quantity and of the significance of the result, statistical techniques are used. The terms "best" and "significant" should be understood in a technical sense: i.e., "best" and "significant" according to some statistical criterion. The criteria applied depend on assumptions which may or may not be true: attention should be paid to their validity. In the following, only a prescription of the techniques can be given. For this reason, a word of warning is in order:

† Units are discussed in Part 2 of this volume.

* Part 1 is by **Sidney Reed.**

these techniques, properly used, can improve the understanding of the results and the judgment of their worth—but they are not a substitute for thought.[1] It should be emphasized that work in certain fields, e.g., cosmic ray or high energy physics, requires more complete attention to statistical techniques in the planning and interpretation of experiments than can be discussed here.[2] The assistance of a statistician in such work will often be indispensable.

1.2. Errors

1.2.1. Systematic Errors, Accuracy

Statements about reliability of a measurement require assessment of the accuracy and of the precision of the work. Accuracy implies precision and, beyond that, freedom from what long usage has termed systematic errors. Examples best indicate the nature of systematic errors: (1) The torsion constant of a quartz fiber depends on the weight suspended by it (Bearden). This was overlooked in the course of the work aimed at the determination of the charge of the electron by Millikan. Consequently the value of air viscosity, which was used in this determination and which in turn was obtained using a quartz fiber to provide torque, probably included a systematic error because the suspended mass used in the experiment differed from that with which the torsion constant was determined.[3] (2) Miller performed a long series of experiments at Mount Wilson to detect a dependence of light speed on the direction of the earth's motion. A small dependence seemed to be present. A recent analysis of his data indicated a probable correlation of his results with heating of the building due to sunlight.[4] Here the systematic error was periodic with a long period. (3) A systematic misinterpretation which is not a gross mistake may be termed a systematic error. Michelson apparently omitted to correct some of his work on the velocity of light for the difference between group and phase velocity.[3]

In general, systematic errors are definite functions of experimental

[1] Further discussion and references are given by E. B. Wilson, Jr., "An Introduction to Scientific Research." McGraw-Hill, New York, 1952. Chapters on errors of physical measurement are given also by H. Cramer, "Elements of the Theory of Probability and its Applications." Wiley, New York, 1955; and in N. Arley and K. R. Buch, "Probability and Statistics." Wiley, New York, 1950.

[2] See, for example, L. Jánossy, "Cosmic Rays." Oxford Univ. Press, London and New York, 1953.

[3] R. T. Birge, *Nuovo cimento* **6**, Suppl. No. 1, 44 (1957).

[4] R. S. Shankland, S. W. McCuskey, F. C. Leone, and G. Kuerti, *Revs. Modern Phys.* **27**, 167 (1955). Miller was aware of this possibility, but could not pin it down.

method, instruments, or environmental conditions. If detected they can usually be corrected. Sometimes a single correction will be adequate for the entire work and can be applied at the end. The caliber of investigators cited in the above examples should be a warning that constant, or slowly varying systematic errors are hard to detect. The crucial test is the comparison of measurements of the same quantity obtained from different experiments, using different principles.

1.2.2. Accidental Errors, Precision

Precision implies close reproducibility of the results of successive individual measurements. It is assumed that, in general, there is a variation from measurement to measurement. This scatter of data is usually considered due to accidental errors; it is imagined that the experiment is aimed at a constant quantity, superimposed on which there is a random sum of small effects, independent of each other and of the quantity itself, and which are responsible for the variation of the results. Absence of variation is not necessarily an indication of precision; it may be due simply to an excessively large least count of the instrument used (see Section 1.4.1). Indefinite refinement of scale, which is possible in imagination, is limited by the fluctuation phenomena of microphysics. The ultimate degree of attainable precision is set by the probabilistic features of these fluctuations.[5]

1.3. Statistical Methods

To analyze accidental errors, the actual data are imagined to be a random selection, one for each measurement, of values from a large reference distribution which could be generated by infinite repetition of the experiment. In statistical terms, this is a finite sample from a "parent distribution" (p.d.). For reasons of mathematical convenience, it is usual to assume that the p.d. can be approximated satisfactorily by an analytic function (p.d.f.) having two or three parameters. Our finite data sample permits, at most, to estimate the p.d.f. parameters representing the true value and the precision of the measurement.

In most cases, a reasonable, explicit assumption of a definite form of the p.d.f. is desirable. Which form should be taken depends on a preliminary assessment of the probabilistic features of the experiment. If the errors are accidental in the sense described in Section 1.2.2 above, a normal (see

[5] R. B. Barnes and S. Silverman, *Revs. Modern Phys.* **6**, 162 (1934). A recent summary is given by C. W. McCrombie, *Repts. Progr. in Phys.* **16**, 266 (1953).

Section 1.3.1) distribution function (n.d.f.) is appropriate. If the experiment is directly concerned with probabilistic phenomena, e.g., counting experiments in nuclear physics, the Poisson or some other discrete probability distribution function may be chosen. In this case the measurements are generally indirect[6] (see Chapter 1.5).

To obtain an estimate of the best value one does not need to assume any particular p.d.f.; e.g., an estimation using least squares can be made.[7] A sharp quantitative statement of the statistical significance of a difference between two "best" estimates of the same quantity cannot be made, however, without assuming a definite form for the p.d.f.

1.3.1. Mean Value and Variance

The fraction of readings $dN(x)/N$ drawn from the p.d.f. $f(x)$ lying in the range between x and $x + dx$ is

$$dN(x)/N = f(x)\,dx. \tag{1.3.1}$$

The function $f(x)$ is normalized: $\int f(x)\,dx = 1$. The average of any function $g(x)$, denoted by $\langle g(x) \rangle$, is defined by $\langle g(x) \rangle = \int g(x)f(x)\,dx$. The range of integration may, for mathematical convenience, extend in both directions to infinity. Of special importance are the average of x called the *mean*

$$\langle x \rangle = \int x f(x)\,dx \tag{1.3.2}$$

and the average of $(x - \langle x \rangle)^2$, called the *dispersion* or *variance* of x

$$\sigma^2(x) = \int (x - \langle x \rangle)^2 f(x)\,dx. \tag{1.3.3}$$

The square root of the variance $\sigma(x)$ is called *standard deviation* or sometimes *standard error*. It is a measure of the spread of the data and thus of the precision. An important example of a p.d.f., often assumed to apply to accidental errors, is the Gaussian or *normal distribution* (n.d.f.):

$$f_1(x) = [\sqrt{2\pi\sigma^2(x)}]^{-1}\exp[-(x - \langle x \rangle)^2/2\sigma^2(x)] \tag{1.3.4}$$

characterized by two parameters, the mean $\langle x \rangle$ and the variance $\sigma^2(x)$. N measurements x_i allow the formation of the *sample mean*

$$\bar{x} = N^{-1} \sum_{i=1}^{N} x_i \tag{1.3.5}$$

and the *sample variance*

$$s^2(x) = (N - 1)^{-1} \sum_{i=1}^{N} (x_i - \bar{x})^2. \tag{1.3.6}$$

[6] M. Annis, W. Cheston, and H. Primakoff, *Revs. Modern Phys.* **25**, 818 (1953).
[7] E. R. Cohen, *Revs. Modern Phys.* **25**, 709 (1953).

If our sample is regarded as one of many equally reliable, independent samples of N measurements each, the means of such samples will fluctuate. The variance of the means, $\sigma^2(\bar{x})$ is related to the variance of individual measurements by

$$\sigma^2(\bar{x}) = \sigma^2(x) \cdot N^{-1}.$$

The mean has the property of being the value of a parameter a which minimizes $\Sigma_{i=1}^{N}(x_i - a)^2$. On the grounds of consistency, one expects that in some sense $\bar{x}$ converges to $<x>$ as $N \to \infty$.* For computation, it is useful to subtract a constant A of the order of size of x_i, so that

$$\bar{x} - A = N^{-1} \sum_{i=1}^{N} (x_i - A) \tag{1.3.7}$$

and

$$s^2(x) = (N - 1)^{-1} \left[\sum_{i=1}^{N} (x_i - A)^2 - N(\bar{x} - A)^2 \right]. \tag{1.3.8}$$

1.3.2. Statistical Control of Measurements

The use of any p.d. implies that the data may be regarded as drawn at random from it. There are statistical tests for this implication,[1] but in the case of data scatter because of accidental errors, a rough "control chart" can assist in detecting systematic departures which are functions of time. Such a chart may be made by plotting, on the abscissa, the order (in time) of the reading, and on the ordinate, the reading itself. If there is previous information on the scatter of the data using the same instrument under similar conditions, so that $\sigma(x)$ is known, one can, at least tentatively, draw lines on the chart at $\bar{x} \pm 3\sigma$ which should, if the data are in control, bracket practically all the points. It is quite valuable to have such a chart associated with a precise instrument.

If no previous information is available, one should take a number of points, draw lines at $\bar{x} \pm 3s$ and continue for a few more readings in order to see whether the additional data fall between these lines. If it appears that randomness is a fair assumption, one can use the function "chi-square"[8] to test the fit of an assumed p.d.f. Chi-square, or χ^2, is defined as

$$\chi^2 = \text{sum of} \left\{ \frac{(\text{observed value} - \text{value expected from p.d.f.})^2}{\text{value expected from p.d.f.}} \right\}$$

* This is so in the technical sense of convergence in probability; see, e.g., Cramer, reference 1.

[8] Examples of the use of χ^2 are given by Wilson, reference 1, p. 200; by Arley and Buch, reference 1, p. 209. A critical discussion is given by W. G. Cochran, *Ann. Math. Statistics* **23,** 315 (1952).

and is tabulated as function of the number of degrees of freedom. Here the number of degrees of freedom equals the number of terms in the sum minus one plus the number of p.d.f. parameters which must be estimated from the data; in the case of a n.d.f., the number of degrees of freedom is the number of terms minus three. It is generally necessary to group the observed data and the corresponding values from the p.d.f. into cells.[8] For moderate numbers of readings, say 20 or so, χ^2 will only show substantial discrepancies between the data and the proposed p.d.f. From the table of χ^2 one can find the probability that a value of χ^2 at least as large as that computed could have arisen by chance. If the computed value has a low probability, this is a signal to look for systematic errors.[9]

1.4. Direct Measurements

It is useful to distinguish between direct measurements, such as can be made of length, time, or electrical current; and indirect measurements, in which the quantity in question can be calculated from measurement of other quantities. In the latter case the law of connection between the quantities measured and sought may also be in question. In such a case, one has first to decide whether the proposed relation holds for any values of the quantities (establishment of the law of connection), and if so, to make as good an estimate as possible of the quantity desired.[6] In the case of direct measurements only the latter problem needs to be solved. This simple situation will be discussed first.[10] There are several cases, depending on what information is available at the start.

1.4.1. Errors of Direct Measurements

If one has information at the start of the experiment regarding the variance of readings of the measuring instrument under similar conditions, the following procedure can be employed: One can draw up a control chart, using the previous $\sigma(x)$ together with the mean $\bar{x}$ of a short pre-

[9] A brief table (I) of χ^2 is given in the Appendix to this Part. More extensive tables are readily found, e.g. in recent editions of the "Handbook of Chemistry and Physics," 39th ed. Chemical Rubber, Cleveland, Ohio, 1957–1958. Most tables are abridged versions of those due to R. A. Fisher and F. Yates "Statistical Tables for Biological, Agricultural and Medical Research." Oliver and Boyd, Edinburgh and London, 1953.

[10] A valuable, readable discussion is given by W. E. Deming and R. T. Birge, *Revs. Modern Phys.* **6**, 119 (1934).

liminary run. If subsequent readings appear to be in statistical control, i.e. if the points fall between the lines at $\bar{x} \pm 3\sigma$, one can terminate the process at a definite number of readings which depends on the precision desired. One can then say that the most likely value of $<x>$ is given by the mean $\bar{x}$, and that the reliability of this estimate is such that the probability is one-half that the interval between $\bar{x} - 0.67\sigma/\sqrt{N}$ and $\bar{x} + 0.67\sigma/\sqrt{N}$ contains $<x>$. The precision increases with N in the sense that the interval having a definite probability of containing $<x>$ narrows proportional to $N^{-\frac{1}{2}}$.* In this case, the interval length is sharply defined for fixed N and probability. But this is only so if $\sigma(x)$ is known *a priori*, or if the situation discussed in one of the next two cases prevails:

(a) If there is a long string of readings (several hundred) at hand, all taken, as far as can be judged, under statistical control, one can from this data alone determine $s(x)$ as a good approximation to $\sigma(x)$. One can then proceed as described in the preceding paragraph; the influence of previous data will be small.

(b) In most practical situations, one has probably only small numbers of readings. The case just mentioned can sometimes be approached by combining a reasonable number of readings with previously obtained data. To do this, one regards these small sets of data as drawn at random from a set of n.d.f.'s having possibly different means, but the same σ. It is here assumed that the previous work is summarized into m sets and that the numbers of measurements in each set, $n_1, \ldots n_m$, and the corresponding sample variances $s_1, \ldots s_m$ are available. An estimate of the value of σ^2 is then:

$$\sigma^2(x) \approx \left\{ \sum_{i=1}^{m} [(n_i - 1)s_i^2(x)] \right\} \Big/ (N - m) \tag{1.4.1}$$

and its dispersion can be estimated as $\sigma(s) \approx \sigma(x)/\sqrt{2(N-m)}$. In such cases the data of an additional short run should be combined with the preceding data in estimating the new over-all variance $\sigma^2(x)$ by simply adding the appropriate values of $(n_{m+1} - 1)s^2_{m+1}$. The standard deviation so obtained applies now to *any* of the samples, and as the number of these increases, provided the hypothesis of the data being drawn from a common population with a given value of σ is well founded, the value of σ will become more and more reliable.

In the cases just described, it is important to know whether or not it is reasonable to use all or only some of the different sets of data. A way of

* An interval of this type is called a *confidence interval*. It should be distinguished from a *tolerance interval* which will contain a definite fraction of the population, e.g., a single observation. (See Arley and Buch, reference 1, p. 168.)