

Maths from Scratch for Biologists

Alan J. Cann



 WILEY

Maths

from Scratch for

Biologists



Alan J. Cann

Numerical ability is an essential skill for everyone studying the biological sciences; but many students are frightened by the 'perceived' difficulty of mathematics, and are nervous about applying mathematical skills in their chosen field of study. Having taught introductory maths and statistics for many years, Alan Cann understands these challenges, and just how invaluable an accessible, confidence building textbook could be to the fearful student. Unable to find a book pitched at the right level that concentrated on why numerical skills are useful to biologists, he wrote his own. The result is *Maths from Scratch for Biologists*, a highly instructive, informal text that explains step by step how and why you need to tackle maths within the biological sciences.

- An accessible, jargon-busting approach to help readers master basic mathematical, statistical and data handling techniques in biology
- Numerous end of chapter problems reinforce key concepts, and encourage students to test their newly acquired skills
- A handy, time saving glossary
- A supplementary website including self-test exercises
- Essential reading for all students within the biological sciences taking core skills and numeracy courses
- An invaluable reference for those working within academia and industry

Alan Cann has worked in both the UK and USA, and in addition to teaching undergraduate and postgraduate biologists and medical students, he runs an active research laboratory at the University of Leicester, UK, studying the molecular biology and pathogenesis of viruses.

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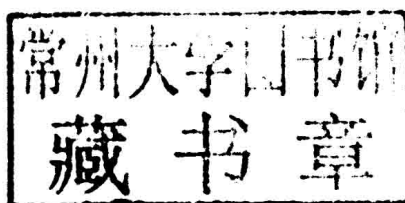
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Maths from Scratch for Biologists

To my Mum and Dad

Preface

This book arose from my own need for a text that I would be happy to recommend to my students. Although there is no particular shortage of volumes claiming to help biologists with mathematics, all those I am familiar with have one of two flaws. Either they are written by well-meaning mathematicians and pay scant attention to biology, or they are not appropriate for the level at which most of the problems lie – new college students who do not have much confidence in approaching mathematical problems, in spite of extensive prior exposure to mathematics in school.

I make no claim to be a mathematical genius. Indeed, I believe my struggle to explain the material in an easily accessible form is one of the strengths of this book, bringing me closer to the students I am trying to communicate with. I reject any charges of ‘dumbing down’ – anyone who has ever tried to help a panic-stricken student in the grip of maths phobia will know that a calming but not patronizing voice is an essential attribute in these circumstances. Throughout, my intention is to provide a highly accessible text for students who, with or without formal mathematics qualifications, are frightened by the perceived ‘difficulty’ of mathematics and unwilling, inept or inexperienced in applying mathematical skills. To accommodate these students, many of whom opt to undertake studies in biology in the belief (conscious or unconscious) that this is a way of pursuing a scientific career while avoiding mathematics, the ethos of the book is consciously informal and intended to be confidence-building.

The maths in this volume has been checked vigorously, but I cannot guarantee that the text is entirely free of numerical errors. In addition

there may be some passages where the subject matter is not expressed as clearly as I would have hoped. I rely on readers to point these out to me – as I am sure they will.

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Maths in Biology

Mathematics, from the Greek, manthano, 'to learn'

Some people opt to undertake studies in biology in the belief (conscious or unconscious) that this is a way of pursuing a scientific career while avoiding maths. This book is designed to be accessible to students who, with or without formal mathematical qualifications, are frightened by the perceived 'difficulty' of maths and hence are unwilling to apply what mathematical skills they might have. Have you ever noticed when you have been taught how to solve a mathematical problem, that you still don't know *why* you need to do a particular step? This is the root of many problems with maths, so this book will try explain the *why* of maths, in addition to the *how*. Sometimes, these explanations may seem unnecessary, but I urge you not to skip them – understanding why you need to do something is the key to remembering how to do it. The intention is to be informal and confidence-building to ensure that *all* readers will gain a general appreciation of basic mathematical, statistical and data handling techniques appropriate to biology. I will try to explain the jargon which confuses the non-numerically minded.

In subsequent chapters, we will look at manipulating numbers, units and conversions, molarities and dilutions, areas and volumes, exponents and logs and statistics. However, the basic advice in this chapter is really the most important part of the book, so please keep reading.

1.1. What can go wrong?

It is easy to make mistakes with maths. One answer looks much like another, so how can you tell if it is right or not? Look at some examples

of the sort of mistakes it is all too easy to make. Everyone knows that numbers are meaningless without the units which define what they mean (more of this in Chapter 3). Even if we avoid the elementary mistake of forgetting this and giving an answer of '33.6' (33.6 what? volts? metres? frogs?), things are not always simple. Consider the following questions:

An aquarium has internal dimensions of $100 * 45 * 45$ cm. What is its volume in litres?

This is fairly simple. Calculate the volume in cubic centimetres then convert to litres. $1 \text{ litre} = 1000 \text{ cm}^3$ so divide by 1000:

$$100 * 45 * 45 = 202\,500 \text{ cm}^3 \div 1000 = 202.5 \text{ litres}$$

However, life is not always that simple. If the same calculation is given in a different way, it is not as easy to answer:

An aquarium has internal dimensions of $39 * 18 * 18$ inches. What is its volume in litres?

This is harder because the units in which the data is given and those in which the answer is required are from different systems of measurement. In real life, this happens all too frequently.

WARNING!

Using mixed units is dangerous (see Chapter 3).

To avoid mistakes we need to convert the units so that they are consistent throughout. However, this means there are two ways to do the calculation:

1. Convert inches to centimetres ($1 \text{ inch} = 2.54 \text{ cm}$), then perform the calculation as above,

$$\begin{aligned} & (39 * 2.54) * (18 * 2.54) * (18 * 2.54) \\ &= 99.06 * 45.72 * 45.72 \\ &= 207\,066.94 \text{ cm}^3 \div 1000 = 207.067 \text{ litres} \end{aligned}$$