

QUANTUM FIELD THEORY

Second edition

LEWIS H. RYDER

量子场论

第2版

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REPUBLIC OF CHINA ONLY, EXCLUDING TAIWAN, HONG KONG AND MACAO
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This book is a modern pedagogic introduction to the ideas and techniques of quantum field theory.

After a brief overview of particle physics and a survey of relativistic wave equations and Lagrangian methods, the quantum theory of scalar and spinor fields, and then of gauge fields, is developed. The emphasis throughout is on functional methods, which have played a large part in modern field theory. The book concludes with a brief survey of 'topological' objects in field theory and, new to this edition, a chapter devoted to supersymmetry.

Comment on the first edition:

'It is very strongly recommended to anyone seeking an elementary introduction to modern approaches to quantum field theory.' *Physics Bulletin*

QUANTUM FIELD THEORY

*Yet nature is made better by no mean
But nature makes that mean: so, over that art
Which you say adds to nature, is an art
That nature makes.*

William Shakespeare, *A Winter's Tale*

*Omnia disce, videbis postea nihil esse superfluum.
(Learn everything, you will find nothing superfluous.)*

Hugh of St Victor

Preface to the first edition

This book is designed for those students of elementary particle physics who have no previous knowledge of quantum field theory. It assumes a knowledge of quantum mechanics and special relativity, and so could be read by beginning graduate students, and even advanced third year undergraduates in theoretical physics.

I have tried to keep the treatment as simple as the subject allows, showing most calculations in explicit detail. Reflecting current trends and beliefs, functional methods are used almost throughout the book (though there is a chapter on canonical quantisation), and several chapters are devoted to the study of gauge theories, which at present play such a crucial role in our understanding of elementary particles. While I felt it important to make contact with particle physics, I have avoided straying into particle physics proper. The book is pedagogic rather than encyclopaedic, and many topics are not treated; for example current algebra and PCAC, discrete symmetries, and supersymmetry. Important as these topics are, I felt their omission to be justifiable in an introductory text.

I acknowledge my indebtedness to many people. Professors P.W. Higgs, FRS, and J.C. Taylor, FRS, offered me much valuable advice on early drafts of some chapters, and I have benefited (though doubtless insufficiently) from their deep understanding of field theory. I was lucky to have the opportunity of attending Professor J. Wess's lectures on field theory in 1974, and I thank him and the Deutscher Akademischer Austauschdienst for making that visit to Karlsruhe possible. I am also very grateful to Dr I.T. Drummond, Dr I.J.R. Aitchison, Professor G. Rickayzen and Dr W.A.B. Evans for reading various sections and making helpful suggestions. I wish to thank Miss Mary Watts for making a difficult and unattractive manuscript a handsome typescript, and this with constant good humour and cheerfulness. I also thank Mr Bernard Doolin for drawing the diagrams with such great care. I am grateful to the late Fr Eric Doyle, OFM, for unearthing the quotation by Hugh of St Victor. I am grateful to the organisers of the first and second UK Theory Institutes in High Energy Physics for the opportunity they provided for stimulating discussions. I wish to record my special thanks to Dr Simon Mitton of Cambridge University Press for his constant encouragement and his indulgence over my failure to meet deadlines. Finally, and most of all, my thanks go to my wife for her unfailing encouragement and support over a long period.

I am grateful to a number of people, but particularly to Mr M.D. Cahill and Dr S.R. Huggins, for pointing out many errors and misprints in the first printing of this book.

Canterbury, Kent
August, 1984

Lewis Ryder

Preface to the second edition

The most important change that has been made for this edition is the addition of a chapter on supersymmetry. It was approximately twenty years ago that supersymmetry burst on the scene of high energy physics. Despite the fact that there is still almost no experimental evidence for this symmetry, its mathematical formulation continues to have appeal to many theoretical physicists justifying, I think, the inclusion of a chapter on supersymmetry in an introductory text. Beyond this, I have rewritten a few sections of the book and incorporated a large number of corrections. I am particularly grateful to Messrs Chris Chambers, Halvard Fausk, Stephen Lyle, Michael Ody, John Smith and Gerhard Soff for pointing out errors and misconceptions in the first edition. The impetus to prepare this second edition owes a lot to the encouragement and friendly advice of Rufus Neal of Cambridge University Press, to whom I should like to express my thanks. Finally, I should like to express my gratitude to Mrs Janet Pitcher for so expertly typing the new material for this edition.

Lewis Ryder

Canterbury, Kent
January, 1996

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Introduction: synopsis of particle physics

1.1 Quantum field theory

Quantum field theory has traditionally been a pursuit of particle physicists. In recent years, some condensed matter physicists have also succumbed to its charms, but the rationale adopted in this book is the traditional one: that the reason for studying field theory lies in the hope that it will shed light on the fundamental particles of matter and their interactions. Surely (the argument goes), a structure that incorporates quantum theory – which was so amazingly successful in resolving the many problems of atomic physics in the early part of this century – and field theory – the language in which was cast the equally amazing picture of reality uncovered by Faraday, Maxwell and Hertz – surely, a structure built on these twin foundations should provide some insight into the fundamental nature of matter.

And indeed it has done. Quantum electrodynamics, the first child of this marriage, predicted (to name only one of its successes) the anomalous magnetic moment of the electron correctly to six decimal places; what more could one want of a physical theory? Quantum electrodynamics was formulated in about 1950, many years after quantum mechanics. Planck's original quantum hypothesis (1901), however, was indeed that the electromagnetic *field* be quantised; the quanta we call photons. In the years leading up to 1925, the quantum idea was applied to the *mechanics* of atomic motion, and this resulted in particle-wave duality and the Schrödinger wave equation for electrons. It was only after this that a proper, systematic treatment of the quantised electromagnetic field was devised, thus coming, as it were, full circle back to Planck and completing the quantisation of a major area of classical physics.

Now, in a sense, quantisation blurs the distinction between particles and fields; 'point' particles become fuzzy and subject to a wave equation, and the (electromagnetic) field, classically represented as a continuum, takes on a particlelike nature (the photon). It may then very well be asked: if we have charged particles (electrons, say) interacting with each other through the electromagnetic field, then in view of quantisation, which renders the particle and the field rather similar, is there an *essential* distinction between them? The answer to this question takes us into elementary particle physics. The salient point is that photons are the quanta of the field *which describes the interaction* between the particles of matter. The electrons 'happen to be there' and because they interact (if they did not we would not know they were there!) the

electromagnetic field and, therefore, photons *become compulsory*! But this is not all. Muons and protons and all sorts of other charged particles also happen to exist, and to interact in the same way, through the electromagnetic field. The reason for the existence of all these particles is so far unknown, but we may summarise by saying that we have a *spectrum of particle states* (e , μ , p , Σ , Ω , etc.) and a *field through which these particles interact* – an interaction, in short. This treatment and mode of comprehension of electrodynamics provides the paradigm, I believe, for a complete understanding of particle interactions. The idea is simply to apply the same methods and concepts to the other interactions known in nature. The only other interaction known in classical physics is the gravitational one, so let us first consider that.

1.2 Gravitation

The gravitational field is described by the general theory of relativity. It turns out, however, that the quantisation of this theory is beset by great problems. First, there are mathematical ones. Einstein's field equations are much more complicated than Maxwell's equations, and in fact are non-linear, so consistency with the superposition principle, the mathematical expression of wave-particle duality, which requires the existence of a linear vector space, would seem to be threatened. Second, there are conceptual problems. In Einstein's theory the gravitational field is manifested as a curvature of space-time. In electrodynamics, the field is, as it were, an actor on the space-time stage, whereas in gravity the actor becomes the space-time stage itself. In some sense, then, we are faced with quantisation of space-time; what is the meaning of this? Finally, there are practical problems. Maxwell's equations predict electromagnetic radiation, and this was first observed by Hertz. Quantisation of the field results in the possibility of observing individual photons; these were first seen in the photoelectric effect, in Einstein's classic analysis. Similarly, Einstein's equations for the gravitational field predict gravitational radiation, so there should, in principle, be a possibility of observing individual *gravitons*, quanta of the field. However, although some claims have been made that gravitational radiation has been observed, these are not unanimously accepted, and the observation of *individual gravitons*, a much more difficult enterprise, must be a next-generation problem! The basic reason for this is that gravity is so much weaker than the other forces in nature. In view of this, the particle physicist is justified in ignoring it; and, because of the difficulties mentioned above, is happy to! On the other hand, the methods that have been recently developed for the quantisation of non-Abelian gauge fields, relevant for an understanding of the strong and weak nuclear forces, do seem to have relevance to gravity, and these will be briefly described in the book, where appropriate.