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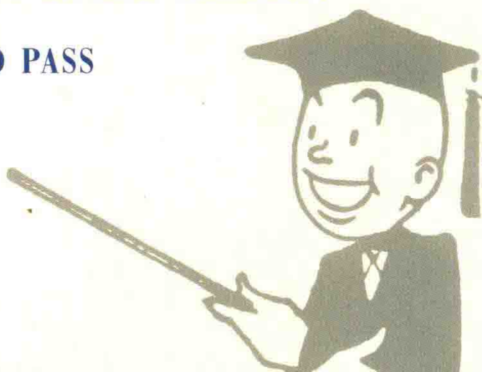

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BY SEYMOUR LIPSCHUTZ, Ph.D.
AND MARC LARS LIPSON, Ph.D.

ABRIDGEMENT EDITOR:
KIMBERLY S. KIRKPATRICK, Ph.D.

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SEYMOUR LIPSCHUTZ is on the faculty of Temple University and formerly taught at the Polytechnic Institute of Brooklyn. He received his Ph.D. in 1960 at Courant Institute of Mathematical Sciences of New York University. He is one of Schaum's most prolific authors and has written, among others, Schaum's Outlines of Beginning Linear Algebra, Probability, Discrete Mathematics, Set Theory, Finite Mathematics, and General Topology.

MARC LARS LIPSON teaches at the University of Georgia. He received his Ph.D. in finance in 1994 from the University of Michigan, and he is the co-author with Seymour Lipschutz of Schaum's Outlines of Discrete Mathematics and Probability.

KIMBERLY S. KIRKPATRICK teaches mathematics at Transylvania University in Lexington, Kentucky. She earned a B.S. in Mathematics Education, a Master of Applied Mathematics, and a Ph.D. in Mathematics, all from Auburn University. She is the co-author of professional papers and previously taught at the University of Evansville. She was the abridgement editor of *Schaum's Easy Outline: Precalculus*.

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5 6 7 8 9 DOC DOC 0 9 8 7

Library of Congress Cataloging-in-Publication Data applied for.

Sponsoring Editor: Barbara Gilson
Production Supervisors: Tama Harris and Clara Stanley
Editing Supervisor: Maureen B. Walker

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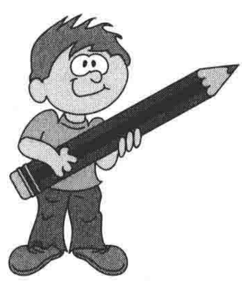
Contents

Chapter 1
VECTORS IN $\mathbf{R}^n$

IN THIS CHAPTER:

- ✔ *Vectors in* $\mathbf{R}^n$
- ✔ *Vector Addition and Scalar Multiplication*
- ✔ *Dot Product*

Vectors in $\mathbf{R}^n$



Although we will restrict ourselves in this chapter to vectors whose elements come from the field of real numbers, denoted by **R**, many of our operations also apply to vectors whose entries come from some arbitrary field K. In the context of vectors, the elements of our number fields are called *scalars*.

Lists of Numbers

Suppose the weights (in pounds) of eight students are listed as follows:

$$156 \quad 125 \quad 145 \quad 134 \quad 178 \quad 145 \quad 162 \quad 193$$

One can denote all the values in the list using only one symbol, say w, but with different subscripts; that is

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$$w_1 \quad w_2 \quad w_3 \quad w_4 \quad w_5 \quad w_6 \quad w_7 \quad w_8$$

Observe that each subscript denotes the position of the value in the list. For example,

$$w_1 = 156, \text{ the first number}, \ w_2 = 125, \text{ the second number}, \ldots$$

Such a list of values, $w = (w_1, w_2, w_3, \ldots, w_8)$ is called a *linear array* or *vector*.

The set of all n-tuples of real numbers, denoted by $\mathbf{R}^n$, is called *n-space*. A particular n-tuple in $\mathbf{R}^n$, say $u = (a_1, a_2, \ldots, a_n)$ is called a *point* or *vector*. The numbers a_i are called the *coordinates*, *components*, *entries*, or *elements* of u. Moreover, when discussing the space $\mathbf{R}^n$, we use the term *scalar* for the elements of $\mathbf{R}$.

Two vectors, u and v, are *equal*, written $u = v$, if they have the same number of components and if the corresponding components are equal. Although the vectors (1, 2, 3) and (2, 3, 1) contain the same three numbers, these vectors are not equal since corresponding entries are not equal.

The vector (0, 0, …, 0) whose entries are all 0 is called the *zero vector*, and is usually denoted by 0.

Example 1.1.

(*a*) The following are vectors:

$$(2, -5) \quad (7, 9) \quad (0, 0, 0) \quad (3, 4, 5)$$

The first two belong to $\mathbf{R}^2$ whereas the last two belong to $\mathbf{R}^3$. The third is the zero vector in $\mathbf{R}^3$.

(*b*) Find x, y, z such that $(x - y, x + y, z - 1) = (4, 2, 3)$.

By definition of equality of vectors, corresponding entries must be equal. Thus,

$$x - y = 4 \quad x + y = 2 \quad z - 1 = 3$$

Solving this system of equations yields $x = 3$, $y = -1$, $z = 4$.

Column Vectors

Sometimes a vector in n-space $\mathbf{R}^n$ is written vertically, rather than horizontally. Such a vector is called a *column vector*, and, in this context, the above horizontally written vectors are called *row vectors*. For example, the following are column vectors with 2, 2, 3, and 3 components, respectively:

$$\begin{bmatrix} 1 \\ 2 \end{bmatrix}, \quad \begin{bmatrix} 3 \\ -4 \end{bmatrix} \quad \begin{bmatrix} 1 \\ 5 \\ -6 \end{bmatrix} \quad \begin{bmatrix} 1.5 \\ \frac{2}{3} \\ -15 \end{bmatrix}$$

We also note that any operation defined for row vectors is defined analogously for column vectors.

Vector Addition and Scalar Multiplication

Consider two vectors u and v in $\mathbf{R}^n$, say

$$u = (a_1, a_2, \ldots, a_n) \text{ and } v = (b_1, b_2, \ldots, b_n)$$

Their *sum*, written $u + v$, is the vector obtained by adding corresponding components from u and v. That is,

$$u + v = (a_1 + b_1, a_2 + b_2, \ldots, a_n + b_n)$$

The *scalar product* or, simply, *product*, of the vector u by a real number k, written ku, is the vector obtained by multiplying each component of u by k. That is,

$$ku = k(a_1, a_2, \ldots, a_n) = (ka_1, ka_2, \ldots, ka_n)$$

Observe that $u + v$ and ku are also vectors in $\mathbf{R}^n$. The sum of vectors with different numbers of components is not defined.

Negatives and subtraction are defined in $\mathbf{R}^n$ as follows:

$$-u = (-1)\,u \quad \text{and} \quad u - v = u + (-v)$$

The vector $-u$ is called the negative of u, and $u - v$ is called the *difference* of u and v.

Now suppose we are given vectors $u_1, u_2, \ldots, u_m$ in $\mathbf{R}^n$ and scalars $k_1, k_2, \ldots, k_m$ in $\mathbf{R}$. We can multiply the vectors by the corresponding scalars and then add the resultant scalar products to form the vector

$$v = k_1u_1 + k_2u_2 + k_3u_3 + \ldots + k_mu_m$$

Such a vector v is called a *linear combination* of the vectors $u_1, u_2, \ldots, u_m$.

Example 1.2.
(*a*) Let $u = (2, 4, -5)$ and $v = (1, -6, 9)$. Then

$$\begin{aligned} u + v &= (2 + 1, 4 + (-5), -5 + 9) = (3, -1, 4) \\ 7u &= (7(2), 7(4), 7(-5)) = (14, 28, -35) \\ -v &= (-1)(1, -6, 9) = (-1, 6, -9) \\ 3u - 5v &= (6, 12, -15) + (-5, 30, -45) = (1, 42, -60) \end{aligned}$$

(*b*) Let $u = \begin{bmatrix} 2 \\ 3 \\ -4 \end{bmatrix}$, $v = \begin{bmatrix} 3 \\ -1 \\ -2 \end{bmatrix}$. Then $2u - 3v = \begin{bmatrix} 4 \\ 6 \\ -8 \end{bmatrix} + \begin{bmatrix} -9 \\ 3 \\ 6 \end{bmatrix} = \begin{bmatrix} -5 \\ 9 \\ -2 \end{bmatrix}$

Basic properties of vectors under the operations of vector addition and scalar multiplication are described in the following theorem.

Theorem 1.1: For any vectors u, v, w in $\mathbf{R}^n$ and any scalars k, k' in $\mathbf{R}$,

(i) $(u + v) + w = u + (v + w)$,
(ii) $u + 0 = u$,
(iii) $u + (-u) = 0$,
(iv) $u + v = v + u$,
(v) $k(u + v) = ku + kv$,
(vi) $(k + k')u = ku + k'u$,
(vii) $(k\,k')u = k(k'u)$,
(viii) $1u = u$.

Suppose u and v are vectors in $\mathbf{R}^n$ for which $u = kv$ for some nonzero scalar k in $\mathbf{R}$. Then u is called a *multiple* of v. Also, u is said to be the *same* or *opposite direction* as v accordingly as $k > 0$ or $k < 0$.

Dot Product

Consider arbitrary vectors u and v in $\mathbf{R}^n$; say,

$$u = (a_1, a_2, \ldots, a_n) \text{ and } v = (b_1, b_2, \ldots, b_n)$$

The *dot product* or *inner product* or *scalar product* of u and v is denoted and defined by $u \cdot v = a_1b_1 + a_2b_2 + \ldots + a_nb_n$. That is $u \cdot v$ is obtained by multiplying corresponding components and adding the resulting products. The vectors u and v are said to be *orthogonal* (or *perpendicular*) if their dot product is zero, that is, if $u \cdot v = 0$.

Example 1.3. Let $u = (1, -2, 3)$, $v = (4, 5, -1)$, $w = (2, 7, 4)$. Then:

$$u \cdot v = 1(4) - 2(5) + 3(-1) = 4 - 10 - 3 = -9$$

and

$$u \cdot w = 1(2) - 2(7) + 3(4) = 2 - 14 + 12 = 0$$

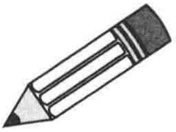
Thus u and w are orthogonal.

Basic properties of the dot product in $\mathbf{R}^n$ follow:

Theorem 1.2: For any vectors u, v, w in $\mathbf{R}^n$ and any scalar k in $\mathbf{R}$:

(i) $(u + v) \cdot w = u \cdot w + v \cdot w$
(ii) $(ku) \cdot v = k(u \cdot v)$
(iii) $u \cdot v = v \cdot u$
(iv) $u \cdot u \geq 0$ and $u \cdot u = 0$ if $u = 0$.

Note that (ii) says that we can "take k out" from the first position in an inner product. By (iii) and (ii), $u \cdot (kv) = (kv) \cdot u = k(v \cdot u) = k(u \cdot v)$ That is, we can also "take k out" from the second position in an inner product.



The space $\mathbf{R}^n$ with the above operations of vector addition, scalar multiplication, and dot product is usually called *Euclidean n-space*.

Norm (Length) of a Vector

The *norm* or *length* of a vector u in $\mathbf{R}^n$, denoted by $\|u\|$, is defined to be the nonnegative square root of $u \cdot u$. In particular, if $u = (a_1, a_2, \ldots, a_n)$, then

$\|u\| = \sqrt{u \cdot u} = \sqrt{a_1^2 + a_2^2 + \cdots + a_n^2}$. That is, $\|u\|$ is the square root of the sum of the squares of the components of u. Thus $\|u\| \geq 0$, and $\|u\| = 0$ if and only if $u = 0$.

A vector u is called a *unit vector* if $\|u\| = 1$ or, equivalently, if $u \cdot u = 1$.

For any nonzero vector v in $\mathbf{R}^n$, the vector $\hat{v} = \frac{1}{\|v\|} v = \frac{v}{\|v\|}$ is the unique unit vector in the same direction as v. The process of finding $\hat{v}$ from v is called *normalizing* v.

Example 1.4. Suppose $u = (1, -2, -4, 5, 3)$. To find $\|u\|$, we can first find $\|u\|^2 = u \cdot u$ by squaring each component of u and adding, as follows:
$\|u\|^2 = 1^2 + (-2)^2 + (-4)^2 + 5^2 + 3^2 = 1 + 4 + 16 + 25 + 9 = 55$
Then $\|u\| = \sqrt{55}$.

We can normalize u as follows:

$$\hat{u} = \frac{u}{\|u\|} = \left(\frac{1}{\sqrt{55}}, \frac{-2}{\sqrt{55}}, \frac{-4}{\sqrt{55}}, \frac{5}{\sqrt{55}}, \frac{3}{\sqrt{55}} \right)$$

This is the unique unit vector in the same direction as u.

Note

The following formula is known as the *Schwarz inequality* or *Cauchy-Schwarz inequality*. It is used in many branches of mathematics.

Theorem 1.3 (Schwarz): For any vectors u, v in $\mathbf{R}^n$, $|u \cdot v| \leq \|u\| \|v\|$.

The following result is known as the *triangle inequality* or *Minkowski's inequality*.

Theorem 1.4 (Minkowski): For any vectors u, v in $\mathbf{R}^n$,

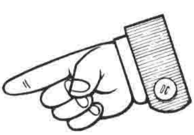
$$\|u + v\| \leq \|u\| + \|v\|.$$

Chapter 2
ALGEBRA OF MATRICES

IN THIS CHAPTER:

- ✔ *Matrices*
- ✔ *Matrix Addition and Scalar Multiplication*
- ✔ *Matrix Multiplication*
- ✔ *Transpose of a Matrix*
- ✔ *Square Matrices*
- ✔ *Powers of Matrices; Polynomials in Matrices*
- ✔ *Invertible (Nonsingular) Matrices*
- ✔ *Special Types of Square Matrices*
- ✔ *Block Matrices*

Matrices



This chapter investigates matrices and algebraic operations defined on them. These matrices may be viewed as rectangular arrays of elements where each entry depends on two subscripts (as compared with

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vectors, where each entry depends on only one subscript). Systems of linear equations and their solutions (Chapter 3) may be efficiently investigated using the language of matrices. The entries in our matrices will come from some arbitrary, but fixed, field K. The elements of K are called *numbers* or *scalars*. Nothing essential is lost if the reader assumes that K is the real field **R**.

A *matrix A over a field K* or, simply, a *matrix A* (when K is implicit) is a rectangular array of scalars usually presented in the following form:

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

The rows of such a matrix A are the m horizontal lists of scalars:

$$(a_{11}, a_{12}, \ldots, a_{1n}), (a_{21}, a_{22}, \ldots, a_{2n}), \ldots, (a_{m1}, a_{m2}, \ldots, a_{mn})$$

A matrix with m rows and n columns is called an *m by n* matrix, written $m \times n$. The pair of numbers m and n is called the *size* of the matrix. Two matrices A and B are *equal*, written $A = B$, if they have the same size and if corresponding elements are equal. Thus the equality of two $m \times n$ matrices is equivalent to a system of mn equalities, one for each corresponding pair of elements.

A matrix with only one row is called a *row matrix* or *row vector*, and a matrix with only one column is called a *column matrix* or *column vector*. A matrix whose entries are all zero is called a *zero matrix* and will usually be denoted by 0.

Example 2.1.

(*a*) The rectangular array $A = \begin{bmatrix} 1 & -4 & 5 \\ 0 & 3 & -2 \end{bmatrix}$ is a 2×3 matrix. Its rows are $(1, -4, 5)$ and $(0, 3, -2)$, and its columns are

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -4 \\ 3 \end{bmatrix}, \begin{bmatrix} 5 \\ -2 \end{bmatrix}$$

(*b*) The 2×4 zero matrix is the matrix $0 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.

(*c*) Find *x*, *y*, *z*, *t* such that $\begin{bmatrix} x+y & 2z+t \\ x-y & z-t \end{bmatrix} = \begin{bmatrix} 3 & 7 \\ 1 & 5 \end{bmatrix}$

By definition of equality of matrices, the four corresponding entries must be equal. Thus:

$$x+y=3 \quad x-y=1 \quad 2z+t=7 \quad z-t=5$$

Solving the above system of equations yields

$$x=2,\ y=1,\ z=4,\ t=-1.$$

Matrix Addition and Scalar Multiplication

Let $A=[a_{ij}]$ and $B=[b_{ij}]$ be two matrices with the same size, say $m \times n$ matrices. The *sum* of A and B, written $A+B$, is the matrix obtained by adding corresponding elements from A and B. That is,

$$A+B=\begin{bmatrix} a_{11}+b_{11} & a_{12}+b_{12} & \cdots & a_{1n}+b_{1n} \\ a_{21}+b_{21} & a_{22}+b_{22} & \cdots & a_{2n}+b_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{m1}+b_{m1} & a_{m2}+b_{m2} & \cdots & a_{mn}+b_{mn} \end{bmatrix}$$

The *product* of the matrix A by a scalar k, written $k \cdot A$ or simply kA, is the matrix obtained by multiplying each element of A by k. That is,

$$kA=\begin{bmatrix} ka_{11} & ka_{12} & \cdots & ka_{1n} \\ ka_{21} & ka_{22} & \cdots & ka_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ ka_{m1} & ka_{m2} & \cdots & ka_{mn} \end{bmatrix}$$

Observe that $A+B$ and kA are also $m \times n$ matrices.

We also define $-A=(-1)A$ and $A-B=A+(-B)$. The matrix $-A$ is called the *negative* of the matrix A, and the matrix $A-B$ is called the *difference* of A and B. The sum of matrices with different sizes is not defined.

Example 2.2. Let $A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & 4 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 6 & 8 \\ 1 & -3 & -7 \end{bmatrix}$. Then

$$A + B = \begin{bmatrix} 1+4 & -2+6 & 3+8 \\ 0+1 & 4+(-3) & 5+(-7) \end{bmatrix} = \begin{bmatrix} 5 & 4 & 11 \\ 1 & 1 & -2 \end{bmatrix}$$

$$3A = \begin{bmatrix} 3(1) & 3(-2) & 3(3) \\ 3(0) & 3(4) & 3(5) \end{bmatrix} = \begin{bmatrix} 3 & -6 & 9 \\ 0 & 12 & 15 \end{bmatrix}$$

$$2A - 3B = \begin{bmatrix} 2 & -4 & 6 \\ 0 & 8 & 10 \end{bmatrix} + \begin{bmatrix} -12 & -18 & -24 \\ -3 & 9 & 21 \end{bmatrix} = \begin{bmatrix} -10 & -22 & -18 \\ -3 & 17 & 31 \end{bmatrix}$$

The matrix $2A - 3B$ is called a *linear combination* of A and B.

Basic properties of matrices under the operations of matrix addition and scalar multiplication follow.

Theorem 2.1: Consider any matrices A, B, C (with the same size) and any scalars k and k'. Then:

(i) $(A + B) + C = A + (B + C)$,
(ii) $A + 0 = 0 + A = A$,
(iii) $A + (-A) = (-A) + A = 0$,
(iv) $A + B = B + A$,
(v) $k(A + B) = kA + kB$,
(vi) $(k + k')A = kA + k'A$,
(vii) $(kk')A = k(k'A)$,
(viii) $1 \cdot A = A$.

Note first that the 0 in (ii) and (iii) refers to the zero matrix. Also, by (i) and (iv), any sum of matrices $A_1 + A_2 + \ldots + A_n$ requires no parentheses, and the sum does not depend on the order of the matrices.

Observe the similarity between Theorem 2.1 for matrices and Theorem 1.1 for vectors. In fact, the above operations for matrices may be viewed as generalizations of the corresponding operations for vectors.

Matrix Multiplication

Before we define matrix multiplication, it will be instructive to first introduce the *summation symbol* Σ (the Greek capital letter sigma).

Suppose $f(k)$ is an algebraic expression involving the letter k. Then the expression $\sum_{k=1}^{n} f(k)$ has the following meaning. First we set $k = 1$ in

$f(k)$, obtaining $f(1)$. Then we set $k = 2$ in $f(k)$, obtaining $f(2)$, and add this to $f(1)$, obtaining $f(1) + f(2)$. Then we set $k = 3$ in $f(k)$, obtaining $f(3)$, and add this to the previous sum, obtaining $f(1) + f(2) + f(3)$. We continue this process until we obtain the sum $f(1) + f(2) + \ldots + f(n)$. Observe that at each step we increase the value of k by 1 until we reach n. The letter k is called the *index*, and 1 and n are called, respectively, the *lower* and *upper* limits. Other letters frequently used as indices are i and j.

We also generalize our definition by allowing the sum to range from any integer n_1 to any integer n_2. That is, we define

$$\sum_{k=n_1}^{n_2} f(k) = f(n_1) + f(n_1 + 1) + f(n_1 + 2) + \ldots + f(n_2)$$

Example 2.3.

$$(a)\quad \sum_{k=1}^{5} x_k = x_1 + x_2 + x_3 + x_4 + x_5 \quad \text{and} \quad \sum_{i=1}^{n} a_i b_i = a_1 b_1 + a_2 b_2 + \ldots + a_n b_n$$

$$(b)\quad \sum_{j=2}^{5} j^2 = 2^2 + 3^2 + 4^2 + 5^2 = 54 \quad \text{and} \quad \sum_{i=0}^{n} a_i x^i = a_0 + a_1 x + a_2 x^2 + \ldots + a_n x^n$$

The product of matrices A and B, written AB, is somewhat complicated. For this reason, we first begin with a special case.

The product AB of a row matrix $A = [a_i]$ and a column matrix $B = [b_i]$ with the same number of elements is defined to be the scalar (or 1×1 matrix) obtained by multiplying corresponding entries and adding; that is,

$$AB = [a_1, a_2, \ldots, a_n] \begin{bmatrix} b_1 \\ b_2 \\ \ldots \\ b_n \end{bmatrix} = a_1 b_1 + a_2 b_2 + \ldots + a_n b_n = \sum_{k=1}^{n} a_k b_k$$

We emphasize that AB is a scalar (or a 1×1 matrix). The product AB is not defined when A and B have different numbers of elements.

Example 2.4.

(*a*) $[7, -4, 5]\begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix} = 7(3) + (-4)(2) + 5(-1) = 21 - 8 - 5 = 8$

(*b*) $[6, -1, 8, 3]\begin{bmatrix} 4 \\ -9 \\ -2 \\ 5 \end{bmatrix} = 24 + 9 - 16 + 15 = 32$

We are now ready to define matrix multiplication in general. Suppose $A = [a_{jk}]$ and $B = [b_{kj}]$ are matrices such that the number of columns of A is equal to the number of rows of B; say, A is an $m \times p$ matrix and B is a $p \times n$ matrix. Then the product AB is the $m \times n$ matrix whose ij-entry is obtained by multiplying the ith row of A by the jth column of B. That is,

$$\begin{bmatrix} a_{11} & \dots & a_{1p} \\ \cdot & \dots & \cdot \\ a_{i1} & \dots & a_{ip} \\ \cdot & \dots & \cdot \\ a_{m1} & \dots & a_{mp} \end{bmatrix} \begin{bmatrix} b_{11} & \dots & b_{1j} & \dots & b_{1n} \\ \cdot & \dots & \cdot & \dots & \cdot \\ \cdot & \dots & \cdot & \dots & \cdot \\ \cdot & \dots & \cdot & \dots & \cdot \\ b_{p1} & \dots & b_{pj} & \dots & b_{pn} \end{bmatrix} = \begin{bmatrix} c_{11} & \dots & c_{1n} \\ \cdot & \dots & \cdot \\ \cdot & c_{ij} & \cdot \\ \cdot & \dots & \cdot \\ c_{m1} & \dots & c_{mn} \end{bmatrix}$$

where $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \ldots + a_{ip}b_{pj} = \sum_{k=1}^{p} a_{ik}b_{kj}$.

The product AB is not defined if A is an $m \times p$ matrix and B is a $q \times n$ matrix, where $p \neq q$.

Example 2.5.

(*a*) Find AB where $A = \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 0 & -4 \\ 5 & -2 & 6 \end{bmatrix}$.

Since A is 2×2 and B is 2×3, the product AB is defined and AB is a 2×3 matrix. To obtain the first row of the product matrix AB, multiply the first row [1, 3] of A by each column of B,

$$\begin{bmatrix} 2 \\ 5 \end{bmatrix}, \begin{bmatrix} 0 \\ -2 \end{bmatrix}, \begin{bmatrix} -4 \\ 6 \end{bmatrix}$$

respectively. That is,

$$AB = \begin{bmatrix} 2+15 & 0-6 & -4+18 \\ & & \end{bmatrix} = \begin{bmatrix} 17 & -6 & 14 \\ & & \end{bmatrix}$$

To obtain the second row of AB, multiply the second row $[2, -1]$ of A by each column of B. Thus

$$AB = \begin{bmatrix} 17 & -6 & 14 \\ 4-5 & 0+2 & -8-6 \end{bmatrix} = \begin{bmatrix} 17 & -6 & 14 \\ -1 & 2 & -14 \end{bmatrix}$$

(*b*) Suppose $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 6 \\ 0 & -2 \end{bmatrix}$. Then

$$AB = \begin{bmatrix} 5+0 & 6-4 \\ 15+0 & 18-8 \end{bmatrix} = \begin{bmatrix} 5 & 2 \\ 15 & 10 \end{bmatrix} \text{ and}$$

$$BA = \begin{bmatrix} 5+18 & 10+24 \\ 0-6 & 0-8 \end{bmatrix} = \begin{bmatrix} 23 & 34 \\ -6 & -8 \end{bmatrix}$$

The above example shows that matrix multiplication is not commutative, i.e., the products AB and BA of matrices need not be equal. However, matrix multiplication does satisfy the following properties.

Theorem 2.2: Let A, B, C be matrices. Then, whenever the products and sums are defined:

(i) $(AB)C = A(BC)$ (associative law),
(ii) $A(B + C) = AB + AC$ (left distributive law),
(iii) $(B + C)A = BA + CA$ (right distributive law),
(iv) $k(AB) = (kA)B = A(kB)$, where k is a scalar.

We note that $0A = 0$ and $B0 = 0$, where 0 is the zero matrix.

Transpose of a Matrix

The *transpose* of a matrix A, written A^T, is the matrix obtained by writing the columns of A, in order, as rows. For example,

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