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SCHAUM'S OUTLINES

Engineering Mechanics
Statics and Dynamics
(Fifth Edition)

工程力学理论与习题
静力学与动力学 （第5版）

E. W. Nelson C. L. Best W. G. Mclean

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Engineering Mechanics Statics and Dynamics

(Fifth Edition)

影 印 版 序

“沙姆纲要集”(SCHAUM'S OUTLINES)是一套品种齐全的大学生学习指导丛书，对各门理工类基础课以及部分专业课都单独出版一册，风行于欧美各高等院校，帮助学生节省学习时间、训练解题能力和提高考试成绩，是大学生们爱不释手的课外辅导书。

本书是美国著名的工程力学学习指导书和习题集，自 1972 年以来至今已经出了 5 版，在世界各国发行，销售量已超过 30 万册。

作为工程力学正式教材的补充，本书的宗旨是训练学生应用工程力学基本原理解题的思路和技巧，培养学生的逻辑思维能力、严谨科学作风以及敢于求解更难问题的自信心。

书中每章前先对基本概念、基本理论和基本公式作提纲挈领的总结，重点突出，简明扼要。接着是本书的核心内容——通过精选的解例来讲授应用工程力学基本原理进行解题的思路和技巧。共有 460 道例题，都是学生在考试中常会遇到的题型，按由易至难的顺序安排，逐步地深入提高。最后是供学生自学和训练的园地，共有 860 道习题，通过“边学边做”的方式让学生真正学到手。

全书共分 19 章和 4 个附录。第 1 章简述矢量运算。第 2 章讲力的运算。第 3 章和第 4 章分别讨论平面力系和非平面（空间）力系的合成。第 5 章和第 6 章分别介绍平面力系和非平面（空间）力系的平衡。第 7 章讲桁架与索。第 8 章研究梁内的力，包括剪力图和弯矩图。第 9 章讨论摩擦。第 10 章讲述一次（静）矩与形心。第 11 章引入虚位移和虚功概念。第 12 章和第 13 章分别是质点运动学和质点动力学。第 14 章和第 16 章分别讲解刚体平面运动的运动学和动力学。第 15 章插入惯性矩的计算。第 17 章引入功与能的概念。第 18 章讨论动量、动量矩和冲量。第 19 章研究机械振动问题。附录 A 综述度量单位制及其转换，本书采用国际度量单位和美国常用单位的习题各占一半。附录 B 给出常用几何图形的一次（静）矩与形心。附录 C 是被选出的 5 道习题的计算机解答。McGraw-Hill 出版公司出版了相应的“沙姆电子教员”，包括理论要点综述、电子参考资料和本书中约 100 道典型习题（这些习题前都加了计算器形状的小图标）的屏幕对话求解。附录 D 给出了 5 个例子的屏幕对话内容，以帮助读者理解电子教员的功能。在新版中作者引入了工程力学在现代高科技领域中的应用问题，例如轨道卫星等。

本书主要读者对象是正在高等理工院校学习工程力学或理论力学课程的本科生和准备报考研究生的学生，同时也是力学教师、工程师和科技人员的有用参考资料。

陆明万

清华大学工程力学系

Preface

This book is designed to supplement standard texts, primarily to assist students of engineering and science in acquiring a more thorough knowledge and proficiency in analytical and applied mechanics. It is based on the authors' conviction that numerous solved problems constitute one of the best means for clarifying and fixing in mind basic principles. While this book will not mesh precisely with any one text, the authors feel that it can be a very valuable adjunct to all.

The previous editions of this book have been very favorably received. This edition incorporates the U.S. Customary units and SI units, as did the third and fourth editions. The units based in the problems are roughly 50 percent U.S. Customary and 50 percent SI; however, the units are not mixed in any one problem. The authors attempt to use the best mathematical tools now available to students at the sophomore level. Thus the vector approach is applied in those chapters where its techniques provide an elegance and simplicity in theory and problems. On the other hand, we have not hesitated to use scalar methods elsewhere, since they provide entirely adequate solutions to most of the problems. Chapter 1 is a complete review of the minimum number of vector definitions and operations necessary for the entire book, and applications of this introductory chapter are made throughout the book. Some computer solutions are given, but most problems can be readily solved using other means.

Chapter topics correspond to material usually covered in standard introductory mechanics courses. Each chapter begins with statements of pertinent definitions, principles, and theorems. The text material is followed by graded sets of solved and supplementary problems. The solved problems serve to illustrate and amplify the theory, present methods of analysis, provide practical examples, and bring into sharp focus those fine points that enable the student to apply the basic principles correctly and confidently. Numerous proofs of theorems and derivations of formulas are included among the solved problems. The many supplementary problems serve as a complete review of the material covered in each chapter.

In the first edition the authors gratefully acknowledged their indebtedness to Paul B. Eaton and J. Warren Gillon. In the second edition the authors received helpful suggestions and criticism from Charles L. Best and John W. McNabb. Also in that edition Larry Freed and Paul Gary checked the solutions to the problems. In the third edition James Schwar assisted us in preparing the computer solutions in Appendix C. For the fourth edition we extend thanks again to James Schwar and to Michael Regan, Jr., for their help with the Appendix C computer solutions. For this fifth edition the authors thank William Best for checking the solutions to the new problems and reviewing the added new material. For typing the manuscripts of the third and fourth editions we are indebted to Elizabeth Bullock.

E. W. Nelson
C. L. Best
W. G. McLean

Contents

Chapter 1

Vectors

1.1 DEFINITIONS

Scalar quantities possess only magnitude, e.g., time, volume, energy, mass, density, work. Scalars are added by ordinary algebraic methods, e.g., 2 s + 7 s = 9 s; 14 kg − 5 kg = 9 kg.

Vector quantities possess both magnitude and direction,* e.g., force, displacement, velocity, impulse. A vector is represented by an arrow at the given inclination. The head of the arrow indicates the sense, and the length represents the magnitude of the vector. The symbol for a vector is shown in print in boldface type, such as **P**. The magnitude is represented by |**P**| or *P*.

A *free vector* may be moved anywhere in space provided it maintains the same direction and magnitude.

A *sliding vector* may be applied at any point along its line of action. By the *principle of transmissibility* the external effects of a sliding vector remain the same.

A *bound* or *fixed vector* must remain at the same point of application.

A *unit vector* is a vector one unit in length.

The *negative* of a vector **P** is the vector −**P** that has the same magnitude and inclination but is of the opposite sense.

The *resultant* of a system of vectors is the least number of vectors that will replace the given system.

1.1 ADDITION OF TWO VECTORS

(*a*) The *parallelogram law* states that the resultant **R** of two vectors **P** and **Q** is the diagonal of the parallelogram for which **P** and **Q** are adjacent sides. All three vectors **P**, **Q**, and **R** are concurrent as shown in Fig. 1-1(*a*). **P** and **Q** are also called the components of **R**.

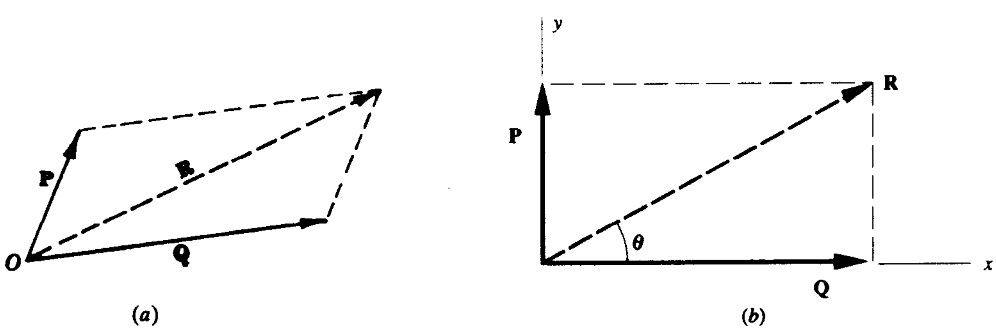

Fig. 1-1

(*b*) If the sides of the parallelogram in Fig. 1-1(*a*) are perpendicular, the vectors **P** and **Q** are said to be *rectangular* components of the vector **R**. The rectangular components are illustrated in Fig. 1-1(*b*). The magnitude of the rectangular components is given by

$$\mathbf{Q} = \mathbf{R}\cos\theta$$

and

$$\mathbf{P} = \mathbf{R}\cos(90^\circ - \theta) = \mathbf{R}\sin\theta$$

---

* Direction is understood to include both the inclination (angle) that the line of action makes with a given reference line and the sense of the vector along the line of action.

(*c*) *Triangle law.* Place the tail end of either vector at the head end of the other. The resultant is drawn from the tail end of the first vector to the head end of the other. The triangle law follows from the parallelogram law because opposite sides of the paralleloram are free vectors as shown in Fig. 1-2.

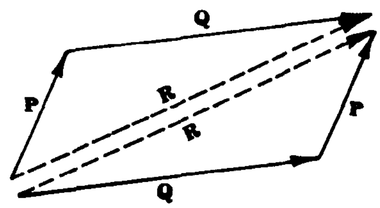

Fig. 1-2

(*d*) Vector addition is commutative; i.e., $\mathbf{P} + \mathbf{Q} = \mathbf{Q} + \mathbf{P}$.

1.3 SUBTRACTION OF A VECTOR

Subtraction of a vector is accomplished by adding the negative of the vector; i.e.,

$$\mathbf{P} - \mathbf{Q} = \mathbf{P} + (-\mathbf{Q})$$

Note also that

$$-(\mathbf{P} + \mathbf{Q}) = -\mathbf{P} - \mathbf{Q}$$

1.4 ZERO VECTOR

A *zero vector* is obtained when a vector is subtracted from itself; i.e., $\mathbf{P} - \mathbf{P} = 0$. This is also called a *null vector*.

1.5 COMPOSITION OF VECTORS

Composition of vectors is the process of determining the resultant of a system of vectors. A vector polygon is drawn placing the tail end of each vector in turn at the head end of the preceding vector as shown in Fig. 1-3. The resultant is drawn from the tail end of the first vector to the head end (terminus) of the last vector. As will be shown later, not all vector systems reduce to a single vector. Since the order in which the vectors are drawn is immaterial, it can be seen that for three given vectors **P**, **Q**, and **S**,

$$\begin{aligned} \mathbf{R} &= \mathbf{P} + \mathbf{Q} + \mathbf{S} = (\mathbf{P} + \mathbf{Q}) + \mathbf{S} \\ &= \mathbf{P} + (\mathbf{Q} + \mathbf{S}) = (\mathbf{P} + \mathbf{S}) + \mathbf{Q} \end{aligned}$$

The above equation may be extended to any number of vectors.

1.6 MULTIPLICATION OF VECTORS BY SCALARS

(*a*) The product of vector **P** and scalar m is a vector $m\mathbf{P}$ whose magnitude is $|m|$ times as great as the magnitude of **P** and that is similarly or oppositely directed to **P**, depending on whether m is positive or negative.

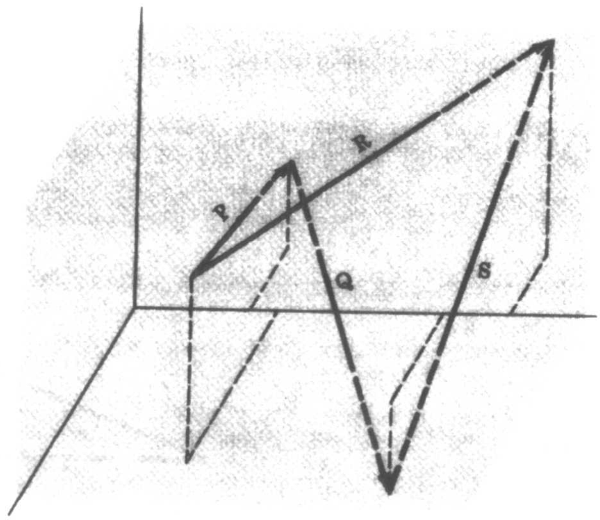

Fig. 1-3

(*b*) Other operations with scalars *m* and *n* are

$$(m+n)\mathbf{P} = m\mathbf{P} + n\mathbf{P}$$
$$m(\mathbf{P}+\mathbf{Q}) = m\mathbf{P} + m\mathbf{Q}$$
$$m(n\mathbf{P}) = n(m\mathbf{P}) = (mn)\mathbf{P}$$

1.7 ORTHOGONAL TRIAD OF UNIT VECTORS

An *orthogonal triad* of unit vectors **i**, **j**, and **k** is formed by drawing unit vectors along the *x*, *y*, and *z* axes respectively. A right-handed set of axes is shown in Fig. 1–4.

A vector **P** is written as

$$\mathbf{P} = P_x\mathbf{i} + P_y\mathbf{j} + P_z\mathbf{k}$$

where $P_x\mathbf{i}$, $P_y\mathbf{j}$, and $P_z\mathbf{k}$ are the vector components of **P** along the *x*, *y*, and *z* axes respectively as shown in Fig. 1-5.

Note that $P_x = P\cos\theta_x$, $P_y = P\cos\theta_y$, and $P_z = P\cos\theta_z$.

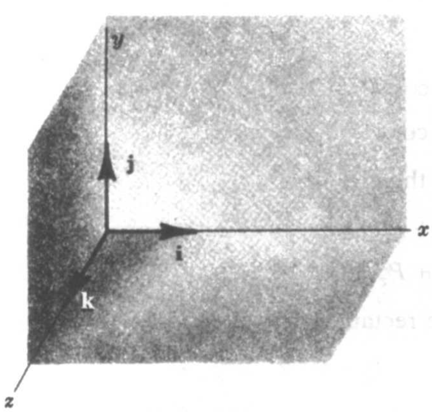

Fig. 1-4

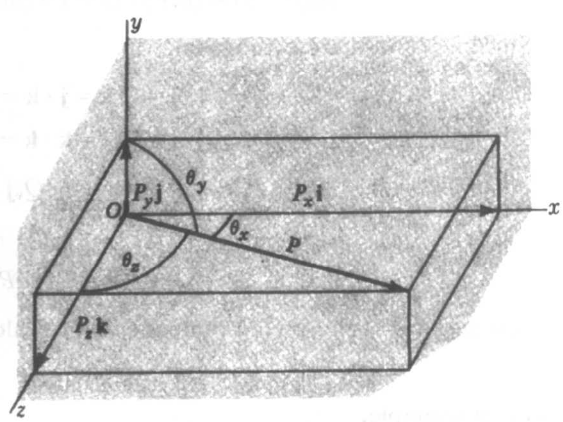

Fig. 1-5

1.8 POSITION VECTOR

The *position vector* $\mathbf{r}$ of a point (x, y, z) in space is written

$$\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$$

where $r = \sqrt{x^2 + y^2 + z^2}$. See Fig. 1–6.

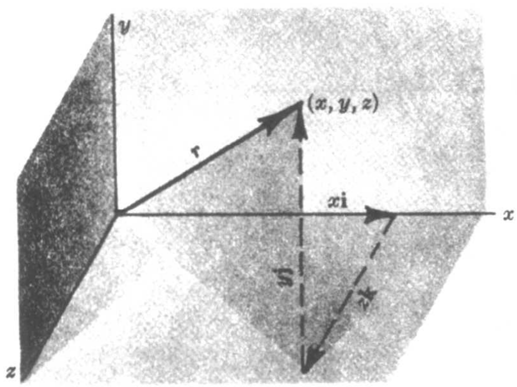

Fig. 1-6

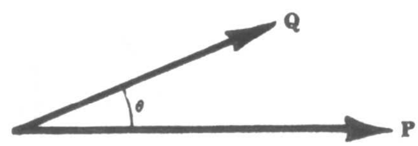

Fig. 1-7

1.9 DOT OR SCALAR PRODUCT

The *dot* or *scalar product* of two vectors $\mathbf{P}$ and $\mathbf{Q}$, written $\mathbf{P} \cdot \mathbf{Q}$, is a scalar quantity and is defined as the product of the magnitudes of the two vectors and the cosine of their included angle θ (see Fig. 1-7). Thus

$$\mathbf{P} \cdot \mathbf{Q} = PQ \cos \theta$$

The following laws hold for dot products, where m is a scalar:

$$\mathbf{P} \cdot \mathbf{Q} = \mathbf{Q} \cdot \mathbf{P}$$
$$\mathbf{P} \cdot (\mathbf{Q} + \mathbf{S}) = \mathbf{P} \cdot \mathbf{Q} + \mathbf{P} \cdot \mathbf{S}$$
$$(\mathbf{P} + \mathbf{Q}) \cdot (\mathbf{S} + \mathbf{T}) = \mathbf{P} \cdot (\mathbf{S} + \mathbf{T}) + \mathbf{Q} \cdot (\mathbf{S} + \mathbf{T}) = \mathbf{P} \cdot \mathbf{S} + \mathbf{P} \cdot \mathbf{T} + \mathbf{Q} \cdot \mathbf{S} + \mathbf{Q} \cdot \mathbf{T}$$
$$m(\mathbf{P} \cdot \mathbf{Q}) = (m\mathbf{P}) \cdot \mathbf{Q} = \mathbf{P} \cdot (m\mathbf{Q})$$

Since $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ are orthogonal,

$$\mathbf{i} \cdot \mathbf{j} = \mathbf{i} \cdot \mathbf{k} = \mathbf{j} \cdot \mathbf{k} = (1)(1) \cos 90° = 0$$
$$\mathbf{i} \cdot \mathbf{i} = \mathbf{j} \cdot \mathbf{j} = \mathbf{k} \cdot \mathbf{k} = (1)(1) \cos 0° = 1$$

Also, if $\mathbf{P} = P_x\mathbf{i} + P_y\mathbf{j} + P_x\mathbf{k}$ and $\mathbf{Q} = Q_x\mathbf{i} + Q_y\mathbf{j} + Q_z\mathbf{k}$ then

$$\mathbf{P} \cdot \mathbf{Q} = P_xQ_x + P_yQ_y + P_zQ_z$$
$$\mathbf{P} \cdot \mathbf{P} = P^2 = P_x^2 + P_y^2 + P_z^2$$

The magnitudes of the vector components of $\mathbf{P}$ along the rectangular axes can be written

$$P_x = \mathbf{P} \cdot \mathbf{i} \qquad P_y = \mathbf{P} \cdot \mathbf{j} \qquad P_z = \mathbf{P} \cdot \mathbf{k}$$

since, for example,

$$\mathbf{P} \cdot \mathbf{i} = (P_x\mathbf{i} + P_y\mathbf{j} + P_z\mathbf{k}) \cdot \mathbf{i} = P_x + 0 + 0 = P_x$$

Similarly, the magnitude of the vector component of **P** along any line L can be written $\mathbf{P} \cdot \mathbf{e}_L$, where $\mathbf{e}_L$ is the unit vector along the line L. (Some authors use **u** as unit vector.) Figure 1-8 shows a plane through the tail end A of vector **P** and a plane through the head B, both planes being perpendicular to line L. The planes intersect line L at points C and D. The vector **CD** is the component of **P** along L, and its magnitude equals $\mathbf{P} \cdot \mathbf{e}_L = Pe_L \cos \theta$.

Applications of these principles can be found in Problems 1.15 and 1.16.

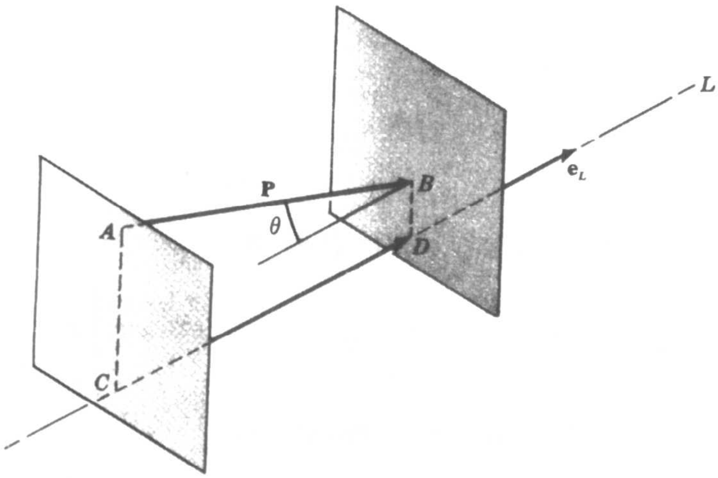

Fig. 1-8

1.10 THE CROSS OR VECTOR PRODUCT

The *cross* or *vector product* of two vectors **P** and **Q**, written $\mathbf{P} \times \mathbf{Q}$, is a vector **R** whose magnitude is the product of the magnitudes of the two vectors and the sine of their included angle. The vector $\mathbf{R} = \mathbf{P} \times \mathbf{Q}$ is normal to the plane of **P** and **Q** and points in the direction of advance of a right-handed screw when turned in the direction from **P** to **Q** through the smaller included angle θ. Thus if **e** is the unit vector that gives the direction of $\mathbf{R} = \mathbf{P} \times \mathbf{Q}$, the cross product can be written

$$\mathbf{R} = \mathbf{P} \times \mathbf{Q} = (PQ \sin \theta)\mathbf{e} \qquad 0 \le \theta \le 180°$$

Figure 1-9 indicates that $\mathbf{P} \times \mathbf{Q} = -\mathbf{Q} \times \mathbf{P}$ (not commutative).

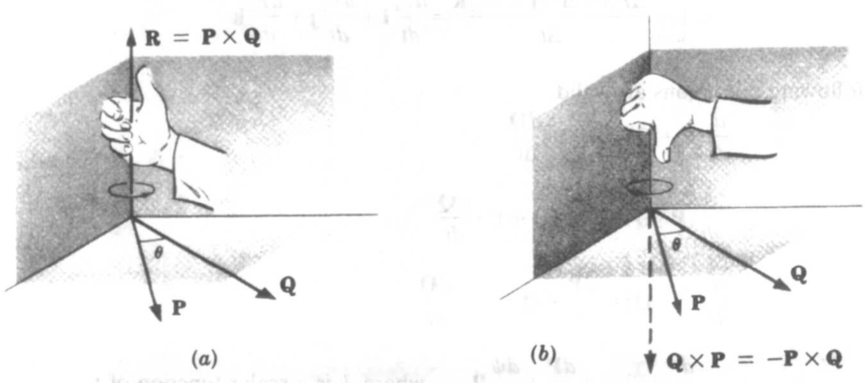

Fig. 1-9

The following laws hold for cross products, where m is a scalar:

$$\mathbf{P}\times(\mathbf{Q}+\mathbf{S})=\mathbf{P}\times\mathbf{Q}+\mathbf{P}\times\mathbf{S}$$

$$\begin{aligned}(\mathbf{P}+\mathbf{Q})\times(\mathbf{S}+\mathbf{T})&=\mathbf{P}\times(\mathbf{S}+\mathbf{T})+\mathbf{Q}\times(\mathbf{S}+\mathbf{T})\\&=\mathbf{P}\times\mathbf{S}+\mathbf{P}\times\mathbf{T}+\mathbf{Q}\times\mathbf{S}+\mathbf{Q}\times\mathbf{T}\end{aligned}$$

$$m(\mathbf{P}\times\mathbf{Q})=(m\mathbf{P})\times\mathbf{Q}=\mathbf{P}\times(m\mathbf{Q})$$

Since $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ are orthogonal,

$$\mathbf{i}\times\mathbf{i}=\mathbf{j}\times\mathbf{j}=\mathbf{k}\times\mathbf{k}=0$$

$$\mathbf{i}\times\mathbf{j}=\mathbf{k}\qquad\mathbf{j}\times\mathbf{k}=\mathbf{i}\qquad\mathbf{k}\times\mathbf{i}=\mathbf{j}$$

Also, if $\mathbf{P}=P_x\mathbf{i}+P_y\mathbf{j}+P_z\mathbf{k}$ and $\mathbf{Q}=Q_x\mathbf{i}+Q_y\mathbf{j}+Q_z\mathbf{k}$ then

$$\mathbf{P}\times\mathbf{Q}=(P_yQ_z-P_zQ_y)\mathbf{i}+(P_zQ_x-\mathbf{P}_xQ_z)\mathbf{j}+(P_xQ_y-P_yQ_x)\mathbf{k}=\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\P_x&P_y&P_z\\Q_x&Q_y&Q_z\end{vmatrix}$$

For proof of this cross-product determination see Problem 1.12.

1.11 VECTOR CALCULUS

(*a*) *Differentiation* of a vector $\mathbf{P}$ that varies with respect to a scalar quantity such as time t is performed as follows.

Let $\mathbf{P}=\mathbf{P}(t)$; that is, $\mathbf{P}$ is a function of time t. A change $\Delta\mathbf{P}$ in $\mathbf{P}$ as time changes from t to $(t+\Delta t)$ is

$$\Delta\mathbf{P}=\mathbf{P}(t+\Delta t)-\mathbf{P}(t)$$

Then $$\frac{d\mathbf{P}}{dt}=\lim_{\Delta t\to 0}\frac{\Delta\mathbf{P}}{\Delta t}=\lim_{\Delta t\to 0}\frac{\mathbf{P}(t+\Delta t)-\mathbf{P}(t)}{\Delta t}$$

If $\mathbf{P}(t)=P_x\mathbf{i}+P_y\mathbf{j}+P_z\mathbf{k}$, where P_x, P_y, and P_z are functions of time t, we have

$$\begin{aligned}\frac{d\mathbf{P}}{dt}&=\lim_{\Delta t\to 0}\frac{(P_x+\Delta P_x)\mathbf{i}+(P_y+\Delta P_y)\mathbf{j}+(P_z+\Delta P_z)\mathbf{k}-P_x\mathbf{i}-P_y\mathbf{j}-P_z\mathbf{k}}{\Delta t}\\&=\lim_{\Delta t\to 0}\frac{\Delta P_x\mathbf{i}+\Delta P_y\mathbf{j}+\Delta P_z\mathbf{k}}{\Delta t}=\frac{dP_x}{dt}\mathbf{i}+\frac{dP_y}{dt}\mathbf{j}+\frac{dP_z}{dt}\mathbf{k}\end{aligned}$$

The following operations are valid:

$$\frac{d}{dt}(\mathbf{P}+\mathbf{Q})=\frac{d\mathbf{P}}{dt}+\frac{d\mathbf{Q}}{\mathrm{dt}}$$

$$\frac{d}{dt}(\mathbf{P}\cdot\mathbf{Q})=\frac{d\mathbf{P}}{dt}\cdot\mathbf{Q}+\mathbf{P}\cdot\frac{d\mathbf{Q}}{dt}$$

$$\frac{d}{dt}(\mathbf{P}\times\mathbf{Q})=\frac{d\mathbf{P}}{dt}\times\mathbf{Q}+\mathbf{P}\times\frac{d\mathbf{Q}}{dt}$$

$$\frac{d}{dt}(\psi\mathbf{P})=\psi\frac{d\mathbf{P}}{dt}+\frac{d\psi}{dt}\mathbf{P}\qquad\text{where } \psi \text{ is a scalar function of } t$$

(*b*) *Integration* of a vector **P** that varies with respect to a scalar quantity such as time t is performed as follows. Let $\mathbf{P} = \mathbf{P}(t)$; that is, **P** is a function of time t. Then

$$\int_{t_0}^{t_1} \mathbf{P}(t)\,dt = \int_{t_0}^{t_1} (P_x\mathbf{i} + P_y\mathbf{j} + P_z\mathbf{k})\,dt$$

$$= \mathbf{i}\int_{t_0}^{t_1} P_x\,dt + \mathbf{j}\int_{t_0}^{t_1} P_y\,dt + \mathbf{k}\int_{t_0}^{t_1} P_z\,dt$$

1.12 DIMENSIONS AND UNITS

In the study of mechanics, the characteristics of a body and its motion can be described in terms of a set of fundamental quantities called dimensions. In the United States, engineers have been accustomed to a gravitational system using the dimensions of force, length, and time. Most countries throughout the world use an absolute system in which the selected dimensions are mass, length, and time. There is a growing trend to use this second system in the United States.

Both systems derive from Newton's second law of motion, which is often written as

$$\mathbf{R} = M\mathbf{a}$$

where **R** is the resultant of all forces acting on a particle, **a** is the acceleration of the particle, and M is the constant of proportionality called the mass.

U.S. Customary System

In this engineering system, the unit of length is the foot (ft), the unit of time is the second (s), and the unit of force is the pound (lb). A mass M falling freely near the earth's surface is pulled toward the earth's centre by a force W with an acceleration of gravity g. The force W is the weight measured in pounds and the acceleration g is in ft/s^2. Hence Newton's second law becomes, in scalar form,

$$W = Mg$$

The value of the acceleration of gravity g varies with the observer's location on the surface of the earth. In this book the value of 32.2 ft/s^2 will be used. An object that weighs 1 lb at or near the earth's surface will have a free-fall acceleration g of 32.2 ft/s^2. The above equation yields

$$M = \frac{W}{g} = \frac{1\text{ lb}}{32.2\text{ ft/s}^2} = \frac{1}{32.2}\,\frac{\text{lb s}^2}{\text{ft}} = \frac{1}{32.2}\text{ slug}$$

In solving statics problems, the mass is not mentioned. It is important to realize that the mass in slugs is a constant for a given body. On the surface of the moon, this same given mass will have acting on it a force of gravity approximately one-sixth of that on the earth.

The International System (SI)

In the International System (SI),* the unit of mass is the kilogram (kg), the unit of length is the meter (m), and the unit of time is the second (s). The unit of force is the newton (N) and is defined

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* SI is the acronym for Système International d'Unités (modernized international metric system).

as the force that will accelerate a mass of one kilogram one meter per second squared (m/s^2). Thus

$$1\ N = (1\ kg)(1\ m/s^2) = 1\ kg \cdot m/s^2 = 1\ kg \cdot m \cdot s^{-2}$$

A mass of 1 kg falling freely near the surface of the earth has an acceleration of gravity g that varies from place to place. In this book we shall assume an average value of 9.80 m/s^2. Thus the force of gravity acting on a 1-kg mass becomes

$$W = Mg = (1\ kg)(9.80\ m/s^2) = 9.80\ kg \cdot m/s^2 = 9.80\ N$$

Of course, problems in statics involve forces; but, in a problem, a mass given in kilograms is not a force. The gravitational force acting on the mass must be used. In all work involving mass, the student must remember to multiply the mass in kilograms by 9.80 m/s^2 to obtain the gravitational force in newtons. A 5-kg mass has a gravitational force of $5 \times 9.8 = 49$ N acting on it.

The student should further note that, in SI, the millimeter (mm) is the standard linear dimension unit for engineering drawings. Therefore, all engineering drawing dimensions must be in millimeters ($1\ mm = 10^{-3}\ m$). Further, a space should be left between the number and unit symbol; e.g., 2.85 mm, not 2.85mm. When using five or more figures, space them in groups of three starting at the decimal point as 12 832.325. Do not use commas in SI. A number with four figures can be written without the space unless it is in a column of quantities involving five or more figures.

Tables of SI units, SI prefixes, and conversion factors for the modern metric system (SI) are included in Appendix A. In this text about 50 percent of the problems are in U.S. Customary units and 50 percent in SI units.

Solved Problems

1.1. In a plane, add 120-lb force at 30° and a −100-lb force at 90° using the parallelogram method. Refer to Fig. 1-10(a).

SOLUTION

Draw a sketch of the problem, not necessarily to scale. The negative sign indicates that the 100-lb force acts along the 90° line downward toward the origin. This is equivalent to a positive 100-lb force along the 270° line, according to the principle of transmissibility.

As in Fig. 1-10(b), place the tail ends of the two vectors at a common point and draw the vectors to a suitable scale. Complete the parallelogram. The resulting R measures to the chosen scale 111 lb. By protractor, it is at an angle with the x axis of $\theta_x = 339°$.

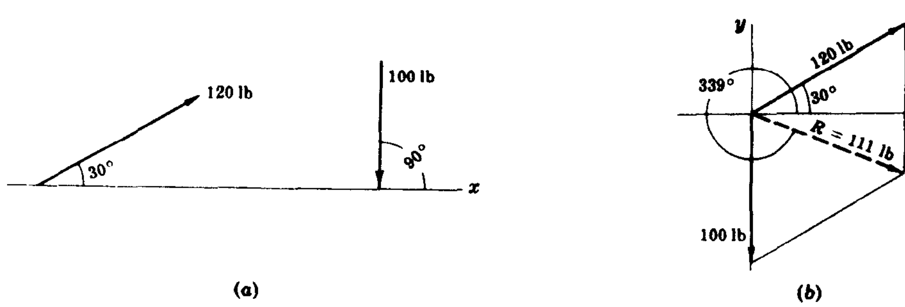

Fig. 1-10

Consider the triangle, one side of which is the y axis, in Fig. 1-10(b). The sides of this triangle are R, 100, and 200. The angle between the 100 and 120 sides is 60°. Applying the Law of Cosines,

$$R^2 = \overline{120}^2 + \overline{100}^2 - 2(120)(100)\cos 60° \qquad R = 111 \text{ lb}$$

Now applying the Law of Sines,

$$\frac{120}{\sin\alpha} = \frac{111}{\sin 60} \qquad \alpha = 69°$$

The angle of 60° added to 270° yields the measured angle of 339°.

1.2. Use the triangle law for Problem 1.1. See Fig. 1-11.

SOLUTION

It is immaterial which vector is chosen first. Take the 120-lb force. To the head of this vector attach the tail end of the 100-lb force. Draw the resultant from the tail end of the 120-lb force to the head end of the 100-lb force. When measured to the chosen scale and the direction determined, the results are the same as in Problem 1.1.

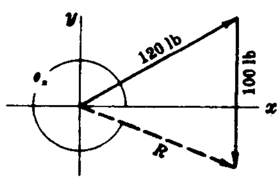

Fig. 1-11

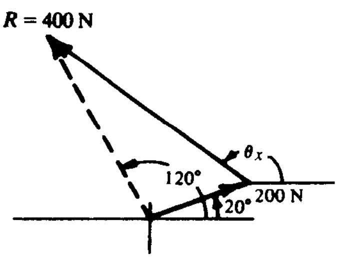

Fig. 1-12

1.3. The resultant of two forces in a plane is 400 N at 120°. One of the forces is 200 N at 20°. Determine the missing force. See Fig. 1-12.

SOLUTION

Select a point through which to draw the resultant and the given force to a convenient scale.

Draw the line connecting the head ends of the given force and the resultant. Place a head on the end of this line near the resultant. This line represents the missing force. When measured to scale, the desired force is 477 N with $\theta_x = 144°$.

This result is also obtained analytically by the laws of trigonometry. The angle between **R** and the 200-N force is 100°, and hence, by the Law of Cosines, the unknown force F is

$$F^2 = \overline{400}^2 + \overline{200}^2 - 2(400)(200)\cos 100° \qquad F = 477 \text{ N}$$

Call the angle between **F** and the 200-N force α. Then, by the Law of Sines,

$$\frac{477}{\sin 100} = \frac{400}{\sin\alpha} \qquad \alpha = 55.7° \qquad \theta_x = 144°$$

1.4. In a plane, subtract 130 N, 60° from 280 N, 320°. See Fig. 1-13.

SOLUTION

To the 280-N, 320° force add the negative of the 130-N, 60° force, obtaining a resultant force of 330 N, 297°. All angles are measured with respect to the x axis.