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Advanced Robust and Adaptive Control Theory and Applications

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Preface

The control groups in Sophia University, Tsinghua University and Institute of Systems Science, Chinese Academy of Sciences have a long efficient collaboration in research of control theory and its applications to power systems and some other engineering problems.

The idea of organizing a joint workshop on robust and adaptive control appeared from a salon discussion on some cross problems met in different researches. Then through some personal communications it was responded warmly from the system and control community of both Chinese and Japanese sides. Finally the *China-Japan Joint Workshop on Advanced Robust Control and Adaptive Control Theory and Applications* had been held in Fragrant Hill Hotel Beijing, China. The workshop was sponsored technically by Technical Committee on Control Theory (TCCT), Chinese Association of Automation (CAA) (China), Committee of Electric Mathematics (CEM), Chinese Association of Electrical Engineering (CAEE) (China), and Committee of Adaptive Leaning Control (CALC), The Society of Instrument and Control Engineers(SICE) (Japan). Financially, the workshop was supported by Natural Science Foundation of China(NSFC) and Japan Science Promotion Society(JSPS).

The workshop provided a forum for control scientists and engineers from China and Japan to present their new research results, control techniques etc. and exchange their creative ideas. After two days discussion most of the participants found that the topics and the results presented on workshop are interesting, and a book for selected works could be beneficial to the world control community. That is the motivation for publishing this book.

The book contains four parts of different topics.

Part one is about the nonlinear systems and robust controls, which contains Hamiltonian realization, recursive design of controller, nonlinear internal model, stabilization of stochastic nonlinear systems, and control of grazing of multi-agent systems.

Part two considers the non-smooth systems and non-smooth control. Particular interest has been focused on switched systems. Non-smooth control design, non-smooth finite-time stabilization, $\mathcal{L}_2$ robust control of switched systems and robust control of discrete-time switched systems are investigated.

Part three concerns adaptive control and modelling. It discusses robust adaptive control, adaptive control of non-holonomic systems, date-based PID controller, nonparametric estimation, decoupling control using neural networks, and nonparametric estimation.

Part four consists of various applications of robust and/or adaptive controls. The applications include transfer conductance on power systems, water control under uncertainty, neural network technique, design effective FACTS controller, coordinated design for multi-machine systems, optimal cost minimization control, sound measure via wavelet analysis.

We are facing an information rich world. High technologies and complexity have put a heavy impact on control community. It is not only a challenge but also an opportunity as well. To meet such challenge we have to develop new control technologies, particularly, advanced robust and adaptive controls based on information and theory of complexity have to be further investigated and improved. New and advanced robust and adaptive control techniques have to be developed. We are expecting that this book is a meaningful contribution to these fields.

Finally, we would like to express our sincere thanks to NSFC and JSPS for their support, and for Professor Xiaohong Jiao and Ms. Shuangling Ding for helping in organizing papers.

Daizhan Cheng, Yuanzhang Sun
Tielong Shen, Hiromitsu Ohmori
Beijing, January 31, 2005

Contents

Part 1. Nonlinear Systems and Robust Control

Part 2. Nonsmooth and Switched System Control

Part 3. Adaptive Control and Estimation

Part 4. Applications of Nonlinear and Adaptive Control

Part 1

Nonlinear Systems and Robust Control

On Port Controlled Hamiltonian Systems

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1 Introduction

The importance of the notion of passivity for analysis and control design can, nowadays, hardly be overestimated, see e.g. [13, 12, 8]. The central question of transforming a non–passive system into a passive system via state–feedback was elegantly settled in [2] where succinct, necessary and sufficient, geometric conditions are given. In spite of the unquestionable beauty of this result feedback equivalence to a (general) passive system has been used more as a conceptual framework to understand stabilization mechanisms than as an actual controller design procedure.

On the other hand, feedback equivalence to port controlled Hamiltonian (PCH) models, which are a class of passive systems, has attracted the attention of many researchers lately, in particular for stabilization objectives. A PCH system (with dissipation) is defined as [13]:

$$\begin{aligned} \dot{x} &= [J(x) - R(x)]\, \nabla H(x) + g(x)u, \quad x \in \mathbb{R}^n,\ u \in \mathbb{R}^m, \\ y &= g^\top(x)\nabla H(x), \quad y \in \mathbb{R}^m, \end{aligned} \tag{1}$$

where $H : \mathbb{R}^n \to \mathbb{R}$ is the total stored energy, $J(x) = -J^\top(x)$ is known as the interconnection matrix, $R(x) = R^\top(x) \geq 0$ represents the dissipation and $\nabla = \frac{\partial}{\partial x}$. The vector signals u and y are the conjugated port variables and their product $u^\top y$ has units of power. It is easy to see that, if the total energy function is non-negative, then PCH systems are passive.

As explained in [13] PCH systems constitute an extension of classical Hamiltonian and Euler–Lagrange models that naturally incorporate interaction with the environment (through power port variables) and capture the essential physical property of power conservation. Given this nice features of PCH models it is then natural to ask when is a general nonlinear system transformable into a PCH system? The investigation of this question is the topic of interest of the present work.

The rest of the paper is organized as follows: Section 2 formulates the problem. Two equivalent conditions are proved in Section 3. Section 4 considers the munimum number of linear PDEs for the solvability of PCH equivalence. Global equivalence is studied in Section 5. An illustrative example is given in Section 6. Section 7 gives some concluding remarks.

2 Problem Formulation

Given an affine system

$$\Sigma_{f,G}: \quad \dot{x} = f(x) + G(x)u, \quad x \in \mathbb{R}^n,\ u \in \mathbb{R}^m,\ m < n \tag{2}$$

and the matrix $J(x) - R(x)$, when does there exists a state feedback $\beta : \mathbb{R}^n \to \mathbb{R}^m$ and an energy function $H : \mathbb{R}^n \to \mathbb{R}$ such that

$$f(x) + G(x)\beta(x) = [J(x) - R(x)]\, \nabla H(x). \tag{3}$$

In this case, we say that the system $\Sigma_{f,G}$ is feedback equivalent to a PCH system (with given interconnection and damping matrices.)

Remark 2.1 Transforming a system to be controlled into a PCH system is the central idea of the Interconnection and Damping Assignment Passivity-based Control method firstly introduced in [9], where (3) is viewed as a set of partial differential equations (PDE's), parameterized in $J(x)$ and $R(x)$, to be solved—see Remark 3.3. A summary of some recent developments may be found in [9], see also [3, 14] for its applications to power systems. In [5], the problem is addressed from an alternative perspective. That is, in equation (3) we fix $H(x)$ and solve the resulting algebraic equations for $J(x)$, $R(x)$ and $\beta(x)$. See also [7, 1] for the case of feedback equivalence to a Lagrangian system.

Remark 2.2 As will become clear in the sequel the particular structure of the matrix $J(x) - R(x)$ does not play any role in the characterization of the class of feedback equivalent systems, therefore we will address the slightly more general problem of feedback equivalence to a pseudo–gradient system. That is, instead of the PCH model (1) we will consider pseudo–gradient systems

$$\Sigma_F \ : \ \dot{x} = F(x)\nabla H(x)$$

where $F(x)$ is a fixed, but otherwise arbitrary, $n \times n$ matrix. This yields, instead of (3), the matching equation

$$f(x) + G(x)\beta(x) = F(x)\nabla H(x), \tag{4}$$

If (4) holds, we will say that the system $\Sigma_{f,G}$ is feedback equivalent to Σ_F.

Before giving our solution to this problem a word on notation is in order.

- All vectors, including the gradient, are row vectors.
- For all vectors and matrices which are functions of x we will write explicitly this dependence only the first time they are defined.
- Throughout the paper we will assume that all functions are sufficiently smooth.
- Finally, no particular attention is given to the characterization of the domain of validity of our statements, to which the local qualifier should be attached. The global equivalence is discussed in Section 5. Where you can see that global rank condition is not enough to assure a global equivalence.

3 Two Equivalent Conditions for Feedback Equivalence

If the matrix F is full rank, Poincare's Lemma give us directly a necessary and sufficient condition for feedback equivalence. Indeed, the vector field $F^{-1}(f+G\beta)$ is a gradient vector field, that is, (4) is satisfied for some scalar function H, if and only if

$$\nabla[F^{-1}(f+G\beta)] = \left(\nabla[F^{-1}(f+G\beta)]\right)^{\top}. \tag{5}$$

The latter condition—for fixed F, f and G—translates into $\frac{n}{2}(n-1)$ PDE's in terms of β. This was the method proposed in [9]. One of the objectives of this note is to show that we can significantly reduce the number of PDE's to be solved, therefore simplifying the associated computational problem. Actually, we will identify the minimal number of PDE's that needs to be solved to achieve the feedback equivalence.

Before presenting our characterization we recall a basic linear algebra lemma.

Lemma 3.1 *Consider two linear subspaces $\mathcal{S}_1, \mathcal{S}_2 \subset \mathbb{R}^n$. If, $\dim \mathcal{S}_1 = \dim \mathcal{S}_2$ and $\mathcal{S}_1 \subset \mathcal{S}_2$ (or $\mathcal{S}_2 \subset \mathcal{S}_1$), then $\mathcal{S}_1 = \mathcal{S}_2$.*

We need the following standard assumption.

Assumption A.1 For all points $p \in \mathbb{R}^n$ there exists an open and simply connected neighborhood $\mathcal{N}_p$ such that for all $x \in \mathcal{N}_p$ we have $\operatorname{rank} G(x) = m$. Without loss of generality, we partition[3]

$$G(x) = \begin{bmatrix} G_1(x) \\ G_2(x) \end{bmatrix}, \quad G_2(x) \in \mathbb{R}^{m\times m}, \tag{6}$$

where $\operatorname{rank} G_2(x) = m$ for all $x \in \mathcal{N}_p$.

[3] This partition can always be locally achieved simply swapping and relabelling the state equations.

Remark 3.2 Note that under assumption A.1, the feedback control for PCH equivalence can be uniquely determined from (4) as

$$u(x) = \beta(x) = [G^\top(x)G(x)]^{-1}G^\top(x)\,[F(x)\nabla H(x) - f(x)]\,. \tag{7}$$

This control will be used through the paper.

Proposition 3.3 *Under Assumption A.1, the following statements are equivalent*

1. *$\Sigma_{f,G}$ is feedback equivalent to Σ_F.*
2. *The n PDE's*

$$[I_n - \Pi_G(x)][F(x)\nabla H(x) - f(x)] = 0 \tag{8}$$

admit a solution, where

$$\Pi_G(x) = G(x)[G^\top(x)G(x)]^{-1}G^\top(x).$$

3. *The $n-m$ PDE's*

$$[F_1(x) - G_1(x)G_2^{-1}(x)F_2(x)]\nabla H(x) = f_1(x) - G_1(x)G_2^{-1}(x)f_2(x) \tag{9}$$

admit a solution, where

$$F(x) = \begin{bmatrix} F_1(x) \\ F_2(x) \end{bmatrix}, \quad F_1(x) \in \mathbb{R}^{(n-m)\times n}, \quad f(x) = \begin{bmatrix} f_1(x) \\ f_2(x) \end{bmatrix}, \quad f_1(x) \in \mathbb{R}^{n-m}.$$

Proof. [i) $\Rightarrow$ iii)] Define the $(n-m)\times n$ matrix[4]

$$\xi(x) = \begin{bmatrix} I_{n-m} & -G_1(x)G_2^{-1}(x) \end{bmatrix}. \tag{10}$$

We have the following chain of implications

$$\begin{aligned} i) &\Leftrightarrow f + G\beta = F\nabla H \\ &\Rightarrow \xi(f + G\beta) = \xi F\nabla H \\ &\Leftrightarrow \xi(F\nabla H - f) = 0 \\ &\Leftrightarrow iii) \end{aligned} \tag{11}$$

[iii) $\Rightarrow$ i)] This will be established by contradiction. Assume iii) does not hold. From (9), or (11), we see that this is equivalent to saying that

$$F\nabla H - f \ni \operatorname{Ker}\xi, \tag{12}$$

We will prove now that $\operatorname{Ker}\xi = \operatorname{Im} G$. First, note that both spaces have the same dimension, m. Consider then the chain of implications:

[4]Clearly, ξ is a left annihilator of G, that is $\xi G = 0$.

$$
\begin{aligned}
a \in \text{Im } G \Leftrightarrow \exists b \in \mathbb{R}^n \ &: \ a = Gb \\
&\Rightarrow \xi a = \xi G b = 0 \\
&\Rightarrow a \in \text{Ker } \xi \\
&\Rightarrow \text{Im } G \subset \text{Ker } \xi
\end{aligned}
$$

Finally, we can invoke Lemma 3.1 to conclude that Ker $\xi =$ Im G. From the proof above we have that (12) is equivalent to

$$F\nabla H - f \ni \text{Im } G,$$

but the latter contradicts i), that states the existence of β such that $F\nabla H - f = G\beta$.

[iii) $\Leftrightarrow$ ii)] To prove this equivalence we will establish that

$$\text{Ker } \xi = \text{Ker } (I_n - \Pi_G),$$

which, together with (8) and (11), shows that the set of solutions of both PDE's are the same—completing the proof. Towards this end, define two $n \times n$ matrices

$$C(x) = \left[\xi^\top(x)[\xi(x)\xi^\top(x)]^{-1} \ G(x)\right], \quad D(x) = \begin{bmatrix} \xi(x) \\ (G^\top(x)G(x))^{-1}G^\top(x) \end{bmatrix}.$$

Now, using $\xi G = 0$ we have that $DC = I_n$, and consequently $D = C^{-1}$. This also implies that $CD = I_n$, which doing the computations is equivalent to

$$\Pi_{\xi^\top}(x) = I_n - \Pi_G(x). \tag{13}$$

where we have defined the projector matrix

$$\Pi_{\xi^\top}(x) = \xi^\top(x)[\xi(x)\xi^\top(x)]^{-1}\xi(x).$$

We will prove now, by contradiction, that $\operatorname{rank} \Pi_{\xi^\top} = n - m$. Assume $\operatorname{rank} \Pi_{\xi^\top} < n - m$. We have the following set of equations, that lead to a contradiction,

$$
\begin{aligned}
n - m &= \operatorname{rank} \xi^\top \\
&= \operatorname{rank} \Pi_{\xi^\top}\xi^\top \\
&\le \min\{\operatorname{rank} \Pi_{\xi^\top}, \operatorname{rank} \xi^\top\} \\
&< n - m,
\end{aligned}
$$

where we used $\Pi_{\xi^\top}\xi^\top = \xi^\top$ for the second identity, and to obtain the third line we invoked the fact that, for any pair of conformal matrices A, B,

$$\operatorname{rank} AB \le \min\{\operatorname{rank} A, \operatorname{rank} B\}.$$

Therefore, $\operatorname{rank} \Pi_{\xi^\top} = n - m$ and consequently $\dim \operatorname{Ker} \Pi_{\xi^\top} = m$.

To conclude the proof we recall that $\dim \operatorname{Ker} \xi = m$, and given that $\operatorname{Ker} \xi \subset \operatorname{Ker} \Pi_{\xi^\top}$, we have that

$$\begin{aligned}\operatorname{Ker} \xi &= \operatorname{Ker} \Pi_{\xi^\top} \\ &= \operatorname{Ker} [I_n - \Pi_G]\end{aligned}$$

where we have invoked Lemma 3.1 for the first identity and (13) for the second. □

Remark 3.4 The proposition establishes the interesting fact that the set of solutions of the n PDE's (8) exactly coincides with the set of solutions of the $n - m$ PDE's (9)—equivalently, (11). (The lack of such a formal statement was a source of some confusion on the literature, see e.g., [9].) The proposition also gives alternative parameterizations of the matching equation (4) that complements the original proposal of [9] to solve the $\frac{n}{2}(n-1)$ PDE's (5).

Remark 3.5 Although the left annihilator matrix of G is not uniquely defined the proposition remains unaffected if we choose a matrix different from (10).[5] Indeed, let us take any left annihilator, say $\bar{\xi}(x)$, of G, and partition it as

$$\bar{\xi}(x) = \left[\bar{\xi}_1(x)\ \bar{\xi}_2(x)\right], \quad \bar{\xi}_1(x) \in \mathbb{R}^{(n-m)\times(n-m)}.$$

The left annihilator condition $\bar{\xi}G = 0$ imposes the relationship $\bar{\xi}_2 = -\bar{\xi}_1 G_1 G_2^{-1}$ that leads to the factorization

$$\bar{\xi} = \bar{\xi}_1 \left[I_{n-m}\ -G_1 G_2^{-1}\right].$$

For all full–rank matrices $\bar{\xi}_1$, $\bar{\xi}$ has the same kernel as ξ, whence the proposition remains unaltered by the use of this "new" left annihilator.

Remark 3.6 The necessity of (8) for feedback equivalence can be easily established as follows. (4) implies that

$$G^\top G\beta = G^\top (F\nabla H - f),$$

which together with invertibility of[6] $G^\top G$ defines, uniquely, the control β. Equation (8) is then obtained plugging the expression of β in (4).

Remark 3.7 It is easy to prove that $\operatorname{Im} \xi^\top$ is the orthogonal complement of $\operatorname{Im} G$, that is

$$\operatorname{Im} G \oplus \operatorname{Im} \xi^\top = \mathbb{R}^n,$$

with $\oplus$ denoting direct sum. This stems from the fact that $\Pi_{\xi^\top}$ is an orthogonal projector (over the rows of ξ), whence

$$\operatorname{Im} \Pi_{\xi^\top} \oplus \operatorname{Ker} \Pi_{\xi^\top} = \mathbb{R}^n, \quad \operatorname{Im} \Pi_{\xi^\top} = \operatorname{Im} \xi^\top, \quad \operatorname{Ker} \Pi_{\xi^\top} = \operatorname{Ker} \xi$$

and that, as shown in the proof above, $\operatorname{Ker} \xi = \operatorname{Im} G$. See Section 5 of this paper or Section 5.8 of [6] for further details on projectors (also known as idempotent matrices.)

---

[5] Of course, as a space, the orthogonal complement of $\operatorname{Im} G$ is uniquely defined. See Remark 11.

[6] This follows from the fact that $G^\top G$ is the Gram matrix of a set of linearly independent vectors.