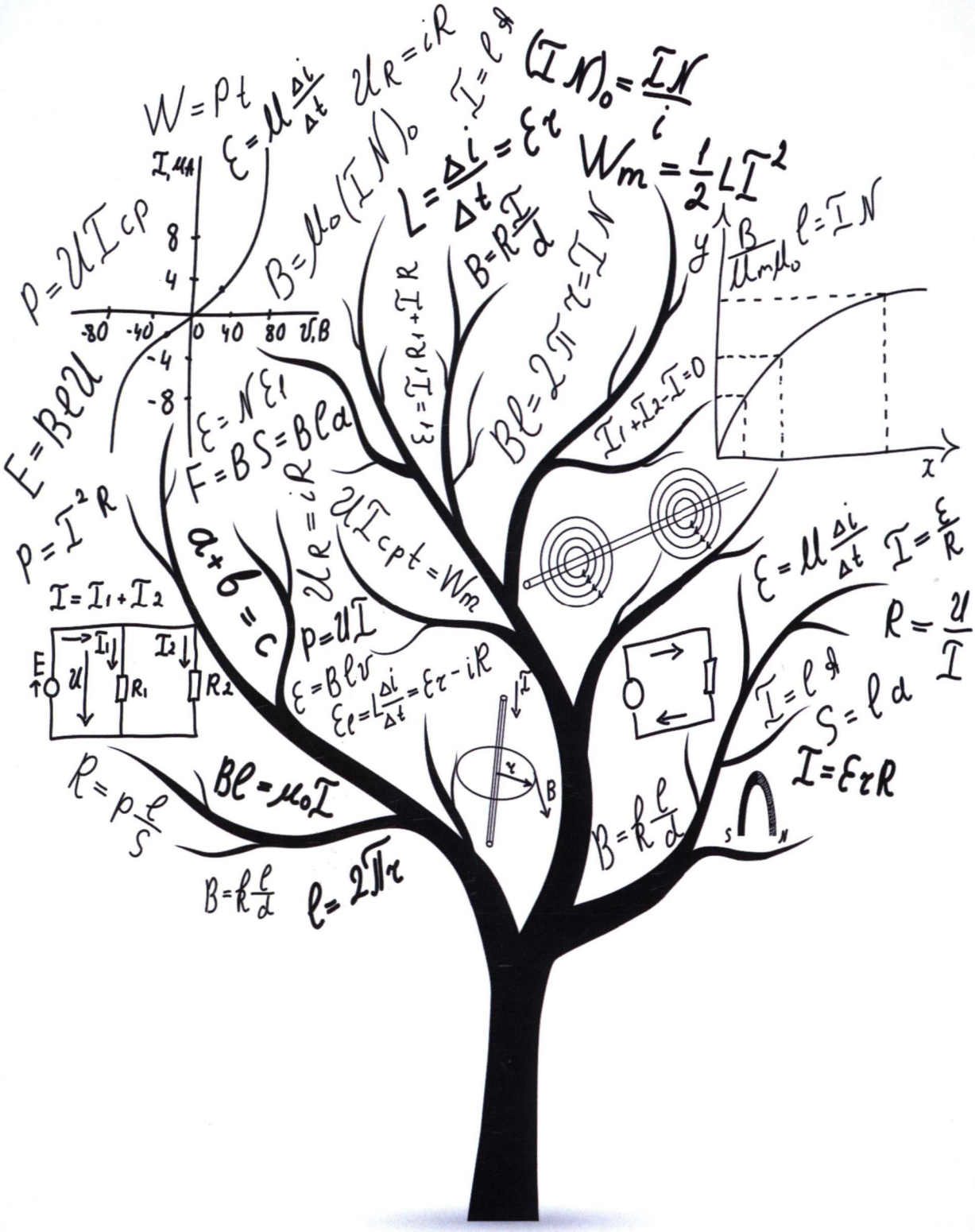
Trends in Linear Algebra and Matrix Theory

Contributors: Oliver J. D. Barrowclough , Tor Dokken, et al.



About the Book

Many difficult problems can be handled easily once relevant information is organized in a certain way Linear algebra is the branch of mathematics concerning vector spaces and linear mappings between such spaces. It includes the study of lines, planes, and subspaces, but is also concerned with properties common to all vector spaces. In broad terms, vectors are things you can add and linear functions are functions of vectors that respect vector addition. Again, in analogue with theories of other algebraic objects, linear algebra is interested in subsets of vector spaces that are themselves vector spaces; these subsets are called linear subspaces. For example, both the range and kernel of a linear mapping are subspaces, and are thus often called the range space and the nullspace; these are important examples of subspaces.

Trends in Linear Algebra and Matrix Theory is intended to provide various new issues and developments in different areas of Linear Algebra. Emphasis is given to topics that will be useful in other disciplines, including systems of equations, vector spaces, determinants, eigenvalues, similarity, and positive definite matrices. Linear algebra is contributing to a number of post-forward problems, including data assimilation, inverse and parameter estimation problems, PDE-constrained optimization and control, and uncertainty quantification (UQ). The multiparameter equations associated with stochastic PDEs, which are often a bottleneck in computations, can generally be formulated as tensor equations or huge matrix equations. Advances in these fields could significantly speed up UQ analysis. As for new applications, many of these continue to be provided by industrial problems in science and engineering. Two other topics that are particularly prominent at present are network analysis and big data. In network problems, linear algebra has helped develop and compare measures of important graph properties, such as, centrality and communicability. These advances are having an impact in many areas, including physics, chemistry, biology, engineering and the social sciences. Looking to the future, there are many open questions in matrix analysis and numerical linear algebra for the techniques and applications.

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Barrowclough ,
Dokken

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List of Contributors

Oliver J. D. Barrowclough
Applied Mathematics, SINTEF, ICT, P.O. Box 124, Blindern, 0314 Oslo, Norway

Tor Dokken
Applied Mathematics, SINTEF, ICT, P.O. Box 124, Blindern, 0314 Oslo, Norway

Victoria E. Howle
Department of Mathematics and Statistics, Texas Tech University, Lubbock, TX, USA

Robert C. Kirby
Department of Mathematics and Statistics, Texas Tech University, Lubbock, TX, USA

Kevin Long
Department of Mathematics and Statistics, Texas Tech University, Lubbock, TX, USA

Brian Brennan
Department of Mathematics and Statistics, Texas Tech University, Lubbock, TX, USA

Kimberly Kennedy
Department of Mathematics and Statistics, Texas Tech University, Lubbock, TX, USA

Chun'e Huang
College of Biochemical Engineering, Beijing Union University, Beijing, China

Yan Song
Mudanjiang Normal University, Mudanjiang, China

Xiruo Wang
Mudanjiang Normal University, Mudanjiang, China

R. O. Akinola
Department of Mathematics, Faculty of Natural Sciences, University of Jos, Jos, Nigeria

S. Y. Kutchin
Department of Mathematics, Faculty of Natural Sciences, University of Jos, Jos, Nigeria

I. A. Nyam
Department of Mathematics, Faculty of Natural Sciences, University of Jos, Jos, Nigeria

O. Adeyanju
Department of Chemistry, Faculty of Natural Sciences, University of Jos, Jos, Nigeria

George P. H. Styan
Department of Mathematics and Statistics, McGill University, 805 Ouest, Rue Sherbrooke, Montréal, Québec, Canada H3A 2K6

Gotz¨Trenkler
Fachbereich Statistik, Universität Dortmund, Dortmund 44221, Germany

Zhiping Shi
Beijing Key Laboratory of Electronic System Reliability Technology, Capital Normal University, Beijing 100048, China
Guangxi Key Laboratory of Trusted Software, Guilin University of Electronic Technology, Guilin 541004, China

Zhenke Liu
[1]Beijing Key Laboratory of Electronic System Reliability Technology, Capital Normal University, Beijing 100048, China

Yong Guan
Beijing Key Laboratory of Electronic System Reliability Technology, Capital Normal University, Beijing 100048, China

Shiwei Ye
College of Information Science and Engineering, Graduate University of Chinese Academy of Sciences, Beijing 100049, China

Jie Zhang,
College of Information Science and Technology, Beijing University of Chemical Technology, Beijing 100029, China

Hongxing Wei
School of Mechanical Engineering and Automation, Beijing University of Aeronautics and Astronautics, Beijing 100191, China

M. R. Rajesh Kanna
Department of Mathematics, Government First Grade College, Puttur, India
Department of Mathematics, Maharani's Science College for Women, Mysore, India

R. Jagadeesh
Research and Development Centre, Bharathiar University, Coimbatore, India

Alireza Ataei
Department of Mathematics, Faculty of Science, Persian Gulf University, Bushehr, Iran

Hayk Avdalyan
Department of Informatics and Applied Mathematics, Yerevan State University, Yerevan, Armenia

Hakop Hakopian
Department of Informatics and Applied Mathematics, Yerevan State University, Yerevan, Armenia

Preface

Many difficult problems can be handled easily once relevant information is organized in a certain way Linear algebra is the branch of mathematics concerning vector spaces and linear mappings between such spaces. It includes the study of lines, planes, and subspaces, but is also concerned with properties common to all vector spaces. In broad terms, vectors are things you can add and linear functions are functions of vectors that respect vector addition. Again, in analogue with theories of other algebraic objects, linear algebra is interested in subsets of vector spaces that are themselves vector spaces; these subsets are called linear subspaces. For example, both the range and kernel of a linear mapping are subspaces, and are thus often called the range space and the nullspace; these are important examples of subspaces.

Trends in Linear Algebra and Matrix Theory is intended to provide various new issues and developments in different areas of Linear Algebra. Emphasis is given to topics that will be useful in other disciplines, including systems of equations, vector spaces, determinants, eigenvalues, similarity, and positive definite matrices. Linear algebra is contributing to a number of post-forward problems, including data assimilation, inverse and parameter estimation problems, PDE-constrained optimization and control, and uncertainty quantification (UQ). The multiparameter equations associated with stochastic PDEs, which are often a bottleneck in computations, can generally be formulated as tensor equations or huge matrix equations. Advances in these fields could significantly speed up UQ analysis. As for new applications, many of these continue to be provided by industrial problems in science and engineering. Two other topics that are particularly prominent at present are network analysis and big data. In network problems, linear algebra has helped develop and compare measures of important graph properties, such as, centrality and communicability. These advances are having an impact in many areas, including physics, chemistry, biology, engineering and the social sciences. Looking to the future, there are many open questions in matrix analysis and numerical linear algebra for the techniques and applications.

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APPROXIMATE IMPLICITIZATION USING LINEAR ALGEBRA

Oliver J. D. Barrowclough and Tor Dokken

Applied Mathematics, SINTEF, ICT, P.O. Box 124, Blindern, 0314 Oslo, Norway

ABSTRACT

We consider a family of algorithms for approximate implicitization of rational parametric curves and surfaces. The main approximation tool in all of the approaches is the singular value decomposition, and they are therefore well suited to floating-point implementation in computer-aided geometric design CAGD systems. We unify the approaches under the names of commonly known polynomial basis functions and consider various theoretical and practical aspects of the algorithms. We offer new methods for a least squares approach to approximate implicitization using orthogonal polynomials, which tend to be faster and more numerically stable than some existing algorithms. We propose several simple propositions relating the properties of the polynomial bases to their implicit approximation properties.

INTRODUCTION

Implicitization algorithms have been studied in both the CAGD and algebraic geometry communities for many years. Traditional approaches to implicitization have focused on exact methods such as Gröbner bases, resultants and moving curves and surfaces, or syzygies [1]. Approximate methods that are particularly well suited to floating-point implementation have also emerged in the past 25 years [2–6]. These methods are closely related to the algorithms we present; however, those that fit most closely into the framework of this paper include [4, 7–9].

Implicitization is the conversion of parametrically defined curves and surfaces into curves and surfaces defined by the zero set of a single polynomial. Exact implicit representations of rational parametric manifolds often have very high polynomial degrees, which can cause numerical instabilities and slow floating-point calculations. In cases where the geometry of the manifold is not sufficiently complicated to justify this high degree, approximation is often desirable. Moreover, for CAGD systems based on floating point arithmetic, exact implicitization methods are often unfeasible due to performance issues. The methods we present attempt to find "best fit" implicit curves or surfaces of a given degree m (the definition of "best fit" varies with regard to the chosen method of approximation). One important property of all the algorithms is that they are guaranteed to give exact implicitizations for sufficiently high implicit degrees, up to numerical stability. In addition, some of the methods are also suitable for implementation in exact arithmetic, hence constituting alternative methods for exact implicitization.

For simplicity of notation, we proceed for the majority of the paper to describe the implicitization of curves. In Sections 2, 3, and 4, we introduce the notation and review existing methods. In Section 5, we present a new method for approximate implicitization using orthogonal polynomials and prove a theoretical relation to the previous methods. Implementations of the methods using different basis functions will be presented in Section 6 and a qualitative comparison and discussion given in Section 7. Finally, the generalization to both tensor-product and triangular surfaces will be covered in Section 8.

PRELIMINARIES

A parametric curve in $\mathbb{R}^2$ is given by $\mathbf{p}(t) = (p_1(t), p_2(t))$, where p_1 and p_2 are functions in t on some parameter domain Ω. Of particular importance both in CAGD and classical algebraic geometry are rational parametric curves i.e., where p_1 and p_2 are rational functions. In the majority of this paper, we will thus restrict our attention to planar rational curves, where the domain of interest is Ω [0, 1]. In order to use polynomial bases in our construction, we can use the representation of the curves in the projective plane P^2. For a rational parametric curve $\mathbf{p}(t) = (g_1(t)/h(t), g_2(t)/h(t))$ in $\mathbb{R}^2$, where g_1, g_2, and h are polynomials, we thus use the homogeneous description

$$\mathbf{p}(t) = (g_1(t), g_2(t), h(t)). \tag{2.1}$$

All the methods to be described require a choice of degree m and a choice of basis $(q_k(\mathbf{u}))_{k=1}^M$ for the implicit polynomial. Here, M is defined as the number of basic functions in a polynomial of total degree m. Thus, for a general bivariate polynomial, we have $M = \binom{m+2}{2}$. Any polynomial q can be written in terms of such a basis by choosing coefficients $\mathbf{b} = (b_k)_{k=1}^M$:

$$q(\mathbf{u}) = \sum_{k=1}^{M} b_k q_k(\mathbf{u}). \tag{2.2}$$

The choice of implicit basis is an important factor which has implications for both the stability of the algorithms and the quality of the approximations. However, most of the work in this paper is independent of the choice of implicit basis. In $\mathbb{R}^2$, a good choice is the Bernstein basis in a barycentric coordinate system defined over a triangle containing the parametric curve.

For curves in $\mathbb{P}^2$, we use the homogeneous Bernstein basis given by

$$q_{\mathbf{k}}(u, v, w) = \binom{m}{k_1, k_2, k_3} u^{k_1} v^{k_2} w^{k_3}, \quad \text{for } |\mathbf{k}| = k_1 + k_2 + k_3 = m, \tag{2.3}$$

where u, v, and w denote the homogeneous coordinates and $\mathbf{k} = (k_1, k_2, k_3)$ denotes a multiindex. We order the basis by letting q_k correspond to $q_{\mathbf{k}}$ for k 1,...,M, where k k(**k**)

Table 1: Convergence rates for approximate implicitization of sufficiently smooth parametric curves in $\mathbb{R}^2$ by algebraic curves of degree m, given by $k = (1/2)(m+1)(m+2) - 1$

Algebraic degree m	1	2	3	4	5	6	7	8
Convergence rate k	2	5	9	14	20	27	35	44

Table 2: Convergence rates for approximate implicitization of sufficiently smooth parametric surfaces in R^3 by algebraic surfaces of degree m, given by $k = \lfloor (1/6)\sqrt{9 + 12m^3 + 72m^2 + 132m} - 1/2 \rfloor$

Algebraic degree m	1	2	3	4	5	6	7	8
Convergence rate k	2	3	5	7	10	12	14	17

denotes lexicographical order on the indices k_1,k_2, and k_3. Unless otherwise stated, we will assume that the implicit basis $(q_k(\mathbf{u}))_{k=1}^M$ is the Bernstein basis. In particular, it forms a partition of unity

$$\sum_{k=1}^{M} q_k(\mathbf{u}) \equiv 1. \tag{2.4}$$

The choice of degree m determines the number of degrees of freedom the implicit curve will have. If the chosen degree m is sufficiently high, all the algorithms will produce exact results, up to numerical stability. If the degree is lower than necessary, approximations are produced.

Since we are searching for implicit representations and want to avoid the trivial solution $q \equiv 0$, we add a normalization constraint to q in the approximation. How best to make this choice has been discussed by several authors see 6 for an overview. However, as we use the singular value decomposition SVD as the means of approximation, our results are given with the quadratic normalization $\|\mathbf{b}\|_2 = 1$.

The techniques in this paper focus on minimization of the objective function $q \circ \mathbf{p}$ over the space of polynomials $\{q : \|\mathbf{b}\|_2 = 1\}$, where q is defined by 2.2 in a fixed implicit basis. Such a minimization, although not directly minimizing the Hausdorff distance between the implicit and parametric curves, is closely related to the geometric approximation problem. It has been shown that minimization of $q \circ \mathbf{p}$ gives excellent results in geometric space away from singularities [5].

APPROXIMATE IMPLICITIZATION: THE ORIGINAL APPROACH

In 1997, a class of techniques for approximate implicitization of rational parametric curves, surfaces, and hypersurfaces was introduced in 5. The approximation quality of the techniques was substantiated by a general proof that the methods exhibit very high convergence rates, as shown in Tables 1 and 2. Extensions of this original approach, all of which inherit these high convergence rates, form the basis of this paper.

The guiding principle behind these methods is to find a polynomial q which minimizes the maximal algebraic error in a given parameter domain Ω. That is, in the given implicit basis $(q_k)_{k=1}^{M}$, to find the coefficients $\mathbf{b} = (b_k)_{k=1}^{M}$ which solve

$$\min_{\|\mathbf{b}\|=1} \max_{t\in\Omega} |q(\mathbf{p}(t))|. \tag{3.1}$$

The solution to this problem, which we call the minimax or uniform approximation, is not easy to find exactly. However, approximations to the minimax solution can be found directly, using linear algebra.

We notice that the expression $q(\mathbf{p}(t))$ is a univariate polynomial of degree mn in t. We can thus approach the problem by first factorizing the error expression in a polynomial basis $\boldsymbol{\alpha}(t) = (\alpha_j(t))_{j=1}^{L}$, where $L = mn + 1$, as follows:

$$q(\mathbf{p}(t)) = \boldsymbol{\alpha}(t)^T \mathbf{D}_\alpha \mathbf{b}, \tag{3.2}$$

where $\mathbf{D}_\alpha$ is a matrix whose columns are the coefficients of $q_k(\mathbf{p}(t))$ expressed in the α- basis and **b** is the unknown vector of implicit coefficients. Now, we have

$$\begin{aligned}\min_{\|\mathbf{b}\|=1} \max_{t\in\Omega} |q(\mathbf{p}(t))| &= \min_{\|\mathbf{b}\|=1} \max_{t\in\Omega} \left|\boldsymbol{\alpha}(t)^T \mathbf{D}_\alpha \mathbf{b}\right| \le \max_{t\in\Omega} \|\boldsymbol{\alpha}(t)\|_2 \min_{\|\mathbf{b}\|=1} \|\mathbf{D}_\alpha \mathbf{b}\|_2 \\ &= \max_{t\in\Omega} \|\boldsymbol{\alpha}(t)\|_2 \, \sigma_{\min},\end{aligned} \tag{3.3}$$

giving an upper bound on the maximal algebraic error, dependent on the choice of basis. Here, we have used the fact that

$$\min_{\|\mathbf{b}\|_2=1} \|\mathbf{D}_\alpha \mathbf{b}\|_2 = \sigma_{\min}, \tag{3.4}$$

where σ_{min} is the smallest singular value of $\mathbf{D}_\alpha$. We thus choose $\mathbf{b} = \mathbf{v}_{min}$, the right singular vector corresponding to σ_{min}, as an approximate solution to the problem. The other singular vectors corresponding to larger singular values also give candidates for approximation that generally decrease in quality as the singular values increase 8. It is important to note that the value of σ_{min} is dependent on the choice of basis α.

In this paper, we use the following "normalization" to compare the approximation qualities of different polynomial bases:

$$\max_{t\in\Omega}\|\boldsymbol{\alpha}(t)\|_2 = 1. \tag{3.5}$$

It should be noted that bases with different scaling coefficients on the individual basis functions will produce different results. For example, the standard Legendre basis, where each basis function $(P_j)_{j=1}^{L}$ has the normalization $P_j(1) = 1$, will produce quantitatively different results to the Legendre basis normalized with respect to the L_2-inner product. This choice of scaling is somewhat arbitrary, but experience shows that small differences in the scaling result in small differences in the approximation. Thus, for the bases we study, standard choices will be made.

Given a choice of basic functions $\alpha = (\alpha_i)_{i=1}^{L}$, we summarize the general approach of this section in the following algorithm.

Algorithm 3.1. Input: a rational parametric curve p(t) of degree n on the interval [0, 1], and a degree m for the implicit polynomial:

(1) For each basis function $(q_k)_{k=1}^{M}$, compute the vector $\mathbf{d}_k = (d_{j,k})_{j=1}^{L}$ of coefficients such that $q_k(\mathbf{p}(t)) = \sum_{i=1}^{L} d_{j,k}\alpha_j(t)$,

(2) Construct a matrix $\mathbf{D}_\alpha = (\mathbf{d}_k)_{k=1}^{M}$ from the column vectors dk,

(3) Perform an SVD $\mathbf{D}_\alpha = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^T$, and select b = vmin, the right singular vector corresponding to the smallest singular value σmin as an approximate solution.

This algorithm is known as the original method in the α-basis; however, for this paper, we will refer to it simply as the α-method for an arbitrary basis α.

WEAK APPROXIMATE IMPLICITIZATION

Two approaches to approximate implicitization by continuous least squares minimization of the objective function were introduced simultaneously in 2001 in [7, 11], and further developed in [12]. These methods perform minimization in the weighted L_2-inner product:

$$\min_{\|\mathbf{b}\|_2=1}\langle q\circ\mathbf{p}\ ,\ q\circ\mathbf{p}\rangle_w = \min_{\|\mathbf{b}\|_2=1}\int_\Omega w(t)q(\mathbf{p}(t))^2\ \mathrm{d}t, \tag{4.1}$$

where w(t) is some weight function defined on the domain of approximation **Ω**.

After choosing a basis for the implicit representation we obtain a linear algebra problem as before:

$$\min_{\|\mathbf{b}\|_2=1}\mathbf{b}^T\mathbf{M}_w\mathbf{b}, \tag{4.2}$$

where $\mathbf{M}_w = (m_{k,l})_{k=1,l=1}^{M,M}$ is given by

$$m_{k,l} = \langle q_k\circ\mathbf{p}, q_l\circ\mathbf{p}\rangle_w. \tag{4.3}$$

This approach eliminates the need for a choice of basis, but a choice of weight function is necessary. The standard approach in [7, 11] has been to take w(t) ≡ 1.

Later we will discuss the benefits of choosing the Chebyshe weight function on $[0,1]$, $w(t) = 1/\sqrt{t(1-t)}$, instead.

This problem, unlike the minimax problem, can be solved directly if the parametric components are integrable. We simply take $\mathbf{b} = \mathbf{v}_{min}$, the eigenvector corresponding to the smallest eigenvalue of $\mathbf{M}_w$. Since the matrix is symmetric, it has orthonormal eigenvectors. Similarly to the previous method, eigenvectors corresponding to larger eigenvalues give gradually degenerating approximations. We summarize this algorithm, for a given weight function w, as follows:

Algorithm 4.1. Input: a parametric curve **p**(t) on the interval [0,1], and a degree m for the implicit polynomial:

1. Construct a matrix $\mathbf{M}_w$ by performing the integrals according to

$$m_{k,l} = \langle q_k\circ\mathbf{p}, q_l\circ\mathbf{p}\rangle_w \tag{4.4}$$

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