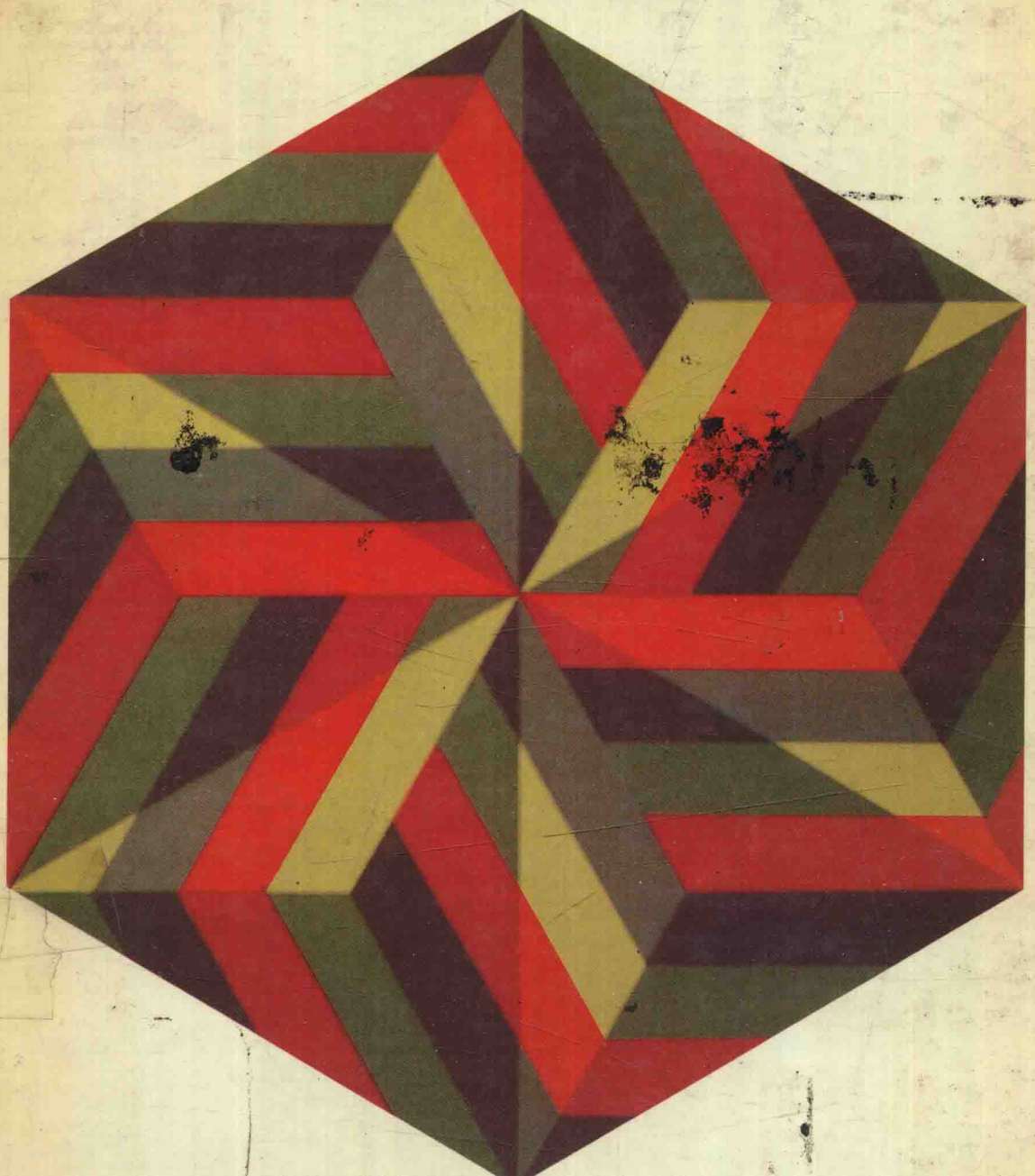
MATHEMATICS

CONTEMPORARY TOPICS AND APPLICATIONS

Howard A. Silver



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Howard A. Silver

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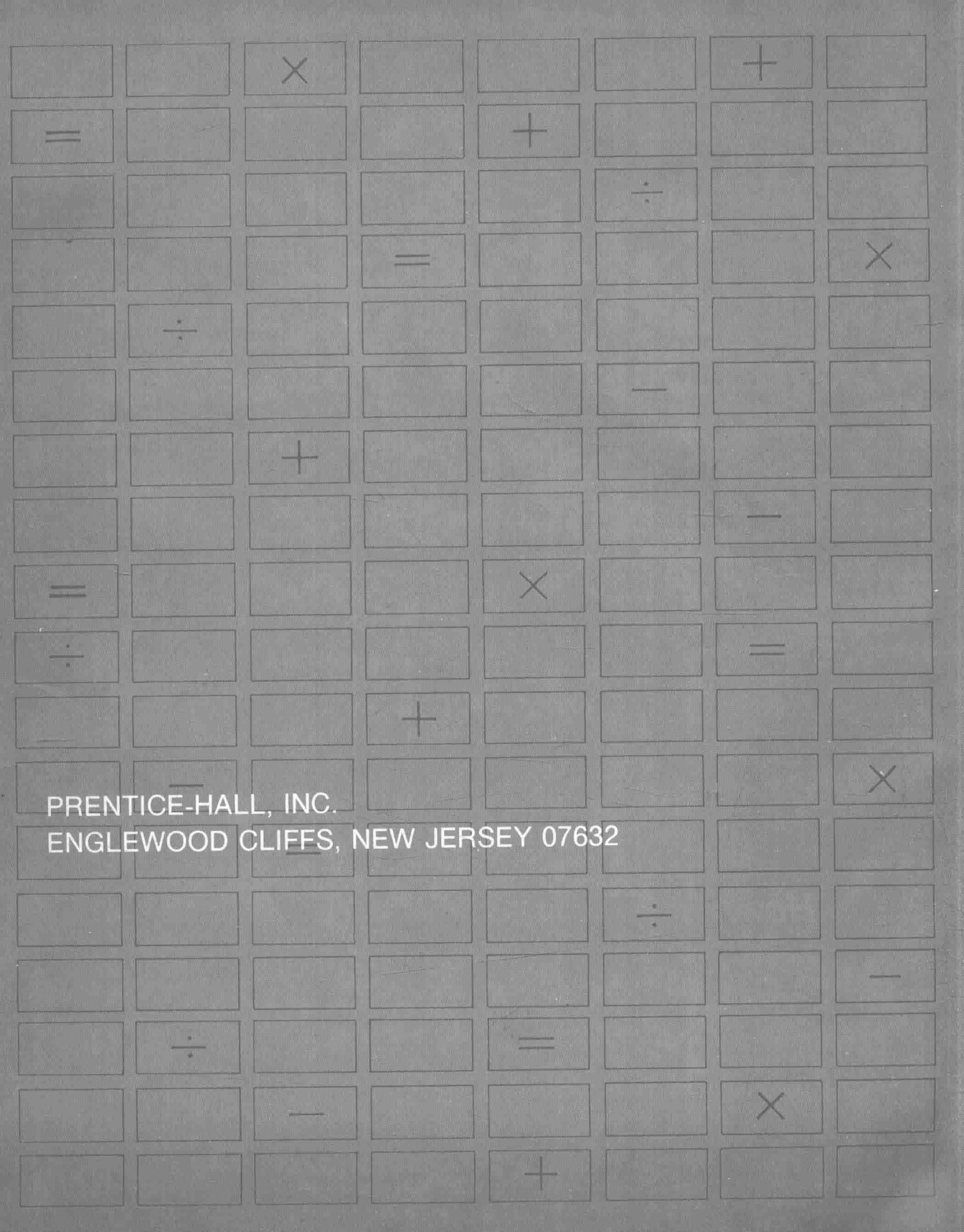

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PREFACE

Virtually no one would deny that mathematics is a vital part of our daily life. It is almost impossible to go through a normal twenty-four hour day without meeting situations involving discount percentages, interest, tax, inflation; or the dimensions of rooms, cars, food portions; or the chances of rain, a team's winning, our getting into graduate school; or a computerized billing; and so on.

This book is intended for the general (non-physical science major) college student. The course may have various titles, such as Liberal Arts Math, Introductory Mathematics, Finite Mathematics. My goal in writing this book is not only to expose the student to the beauty of mathematics but, primarily, to equip him or her with the skills and confidence needed to cope with mathematics in the real world. With this purpose in mind I have, by and large, selected topics of a concrete nature. Students will *not* ask the question, "Where will I ever use this stuff?" They will use much of it every day of their lives.

The writing has been kept simple and concise. Every idea is followed by a clearly explained example. Rules and procedures are summarized in boxes. "Hand Calculator Instant Replays" appear throughout the text to familiarize the student with the operation and capabilities of calculators.

Problem sets are also included as a learning tool: there are over 2500 problems, many involving real-life applications. Answers to more than half of the exercises are given at the end of the book. A list of "important words" and a set of review exercises appear at the ends of chapters to help the student integrate the material and prepare for tests.

Prerequisites are minimal: knowing some arithmetic and a little common sense. Chapter 1 is almost all review of arithmetic (except for Section 6 on the hand calculator). This chapter could be skipped. Chapter 2 is on problem solving and may also seem like a review, but it probably should not be skipped. The other chapters are largely independent.

The entire book makes a two-semester sequence. In a one-semester course the instructor would be free to choose the topics that appealed to him or her.

My hope is that instructors will enjoy teaching from this book and that students will enjoy learning from it.

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Park Forest, Illinois HOWARD A. SILVER

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Massive crowd at rock concert. (Christopher Little/ Camera 5)

1

REVIEW OF REAL NUMBERS

Question What are the real numbers?

ANSWER **Real numbers** are all the numbers we use to express measurements we make in the real world. Real numbers come in different shapes and sizes. We have the following types within the real numbers.

1. **Integers**, such as 10, 215, -31, -27, and 5, are real numbers.
2. **Fractions** (or rational numbers), such as $\frac{1}{2}$, $\frac{17}{20}$, $-\frac{13}{50}$, $\frac{61}{5}$, and $-\frac{21}{4}$, are real numbers.
3. **Decimals**, such as 5.1, 18.75, -4.94, 3.14159 . . . , and -21.063, are real numbers.

Real numbers can also be pictured on a number line. Every point on the number line stands for a real number.

8:09 a.m. on John Street, New York City. (© 1977 by Jan Lukas from Rapho/Photo Researchers)

The purpose of this chapter is to review quickly some of the fundamental operations with real numbers. We will not stress the theory of real numbers at all.

1–1 FRACTIONS

Addition and Subtraction

Question What is the rule for adding and subtracting fractions?

ANSWER This is the rule that we use.

To add or subtract fractions:

1. Find a common denominator, if necessary.
2. Rewrite all fractions to have this common denominator.
3. Add (or subtract) the numerators; put the answer over the common denominator.

We use the following formulas.

Formula 1-1-1

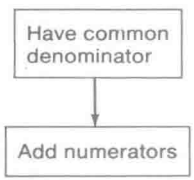
$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

Formula 1-1-2

$$\frac{a}{c} - \frac{b}{c} = \frac{a-b}{c}$$

Problem Add $\frac{4}{10} + \frac{3}{10}$.

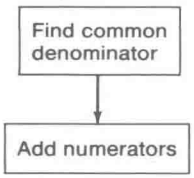
ANSWER



$$\begin{array}{r} \frac{4}{10} \\ +\frac{3}{10} \\ \hline \frac{7}{10} \end{array}$$

Problem Add $\frac{1}{2} + \frac{2}{3}$.

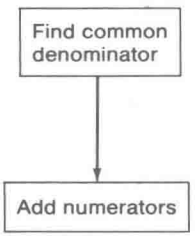
ANSWER



$$\begin{array}{r} \frac{1}{2} = \frac{3}{6} \\ +\frac{2}{3} = +\frac{4}{6} \\ \hline \frac{7}{6} \end{array}$$

Problem Add $\frac{1}{2} + \frac{3}{4} + \frac{5}{6}$.

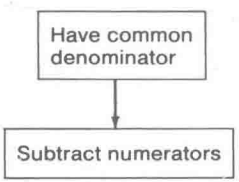
ANSWER We can add three or more fractions by finding a common denominator for all of them.



$$\begin{array}{r} \frac{1}{2} = \frac{6}{12} \\ \frac{3}{4} = \frac{9}{12} \\ +\frac{5}{6} = +\frac{10}{12} \\ \hline \frac{25}{12} \end{array}$$

Problem Subtract $\frac{7}{9} - \frac{2}{9}$.

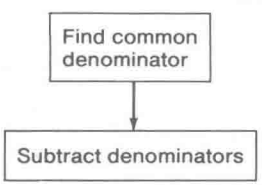
ANSWER



$$\begin{array}{r} \frac{7}{9} \\ -\frac{2}{9} \\ \hline \frac{5}{9} \end{array}$$

Problem Subtract $\frac{7}{4} - \frac{1}{6}$.

ANSWER



$$\begin{array}{r} \frac{7}{4} = \frac{21}{12} \\ -\frac{1}{6} = -\frac{2}{12} \\ \hline \frac{19}{12} \end{array}$$

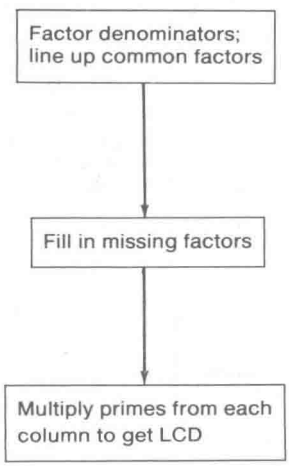
Question What is the least common denominator and how do we find it?

ANSWER The **least common denominator (LCD)** is the smallest number that can be divided by all the denominators evenly. For example, given a problem such as $\frac{1}{4} + \frac{1}{6}$, it is easy to see that the LCD is 12, since 12 is the smallest number that can be divided by both 4 and 6 evenly.

Sometimes, the LCD isn't quite that easy to find. In such cases, we have a special rule for finding the LCD and then adding (or subtracting) the fractions.

Problem Add $\frac{1}{6} + \frac{3}{10} + \frac{7}{18}$.

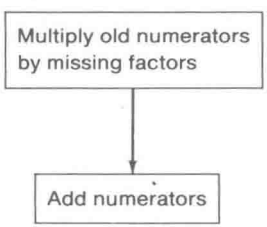
ANSWER First, we find the LCD by factoring the denominators. Remember, a **prime** is a number that is divisible only by 1 and itself, such as 2, 3, 5, 7, 11, 13, 17, and 19. We can factor any whole number into a product of primes.



$$6 = 2 \cdot 3$$
$$10 = 2 \cdot \quad 5$$
$$18 = 2 \cdot 3 \cdot \quad \cdot 3$$

$$6 \rightarrow 2 \cdot 3 \cdot \textcircled{5} \cdot \textcircled{3}$$
$$10 \rightarrow 2 \cdot \textcircled{3} \cdot 5 \cdot \textcircled{3}$$
$$18 \rightarrow 2 \cdot 3 \cdot \textcircled{5} \cdot 3$$
$$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$$
$$\text{LCD} = 2 \cdot 3 \cdot 5 \cdot 3 = 90$$

Now we have that the LCD = 90. We can also use the numbers above to help us add the fractions.



$$\frac{1}{6} = \frac{1 \cdot \textcircled{5} \cdot \textcircled{3}}{90} = \frac{15}{90}$$
$$\frac{3}{10} = \frac{3 \cdot \textcircled{3} \cdot \textcircled{3}}{90} = \frac{27}{90}$$
$$\frac{7}{18} = \frac{7 \cdot \textcircled{5}}{90} = \frac{35}{90}$$
$$\frac{77}{90}$$

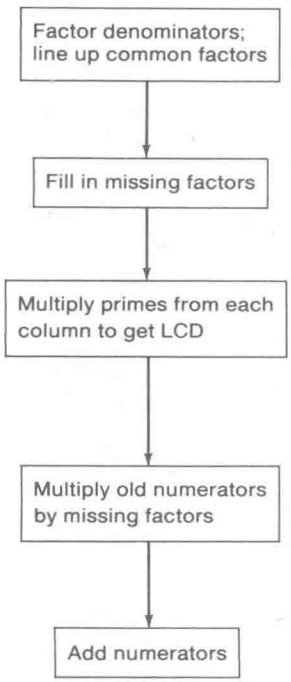
Let us summarize this procedure in the following rule.

To find the LCD and add (or subtract) fractions:

1. Factor all the denominators into primes.
2. Line up the common primes in columns, with unlike primes in different columns.
3. Fill in the missing factors in each column. (Circle these factors for later use.)
4. Bring down a prime from each column. The product is the LCD.
5. For each of the fractions, multiply the old numerator by the missing (circled) factors for that denominator.
6. Add (or subtract) the numerators.

Problem Add $\frac{3}{8} + \frac{1}{12} + \frac{7}{20}$.

ANSWER We use the rule given above.



$$\begin{aligned} 8 &= 2 \cdot 2 \cdot 2 \\ 12 &= 2 \cdot 2 \cdot \quad \cdot 3 \\ 20 &= 2 \cdot 2 \cdot \quad\quad \cdot 5 \end{aligned}$$

$$\begin{aligned} 8 &\to 2 \cdot 2 \cdot 2 \cdot ③ \cdot ⑤ \\ 12 &\to 2 \cdot 2 \cdot ② \cdot 3 \cdot ⑤ \\ 20 &\to 2 \cdot 2 \cdot ② \cdot ③ \cdot 5 \end{aligned}$$

$$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow$$

$$\text{LCD} = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 5 = 120$$

$$\frac{3}{8} = \frac{3 \cdot ③ \cdot ⑤}{120} = \frac{45}{120}$$

$$\frac{1}{12} = \frac{1 \cdot ② \cdot ⑤}{120} = \frac{10}{120}$$

$$\frac{7}{20} = \frac{7 \cdot ② \cdot ③}{120} = \frac{42}{120}$$

$$\frac{97}{120}$$