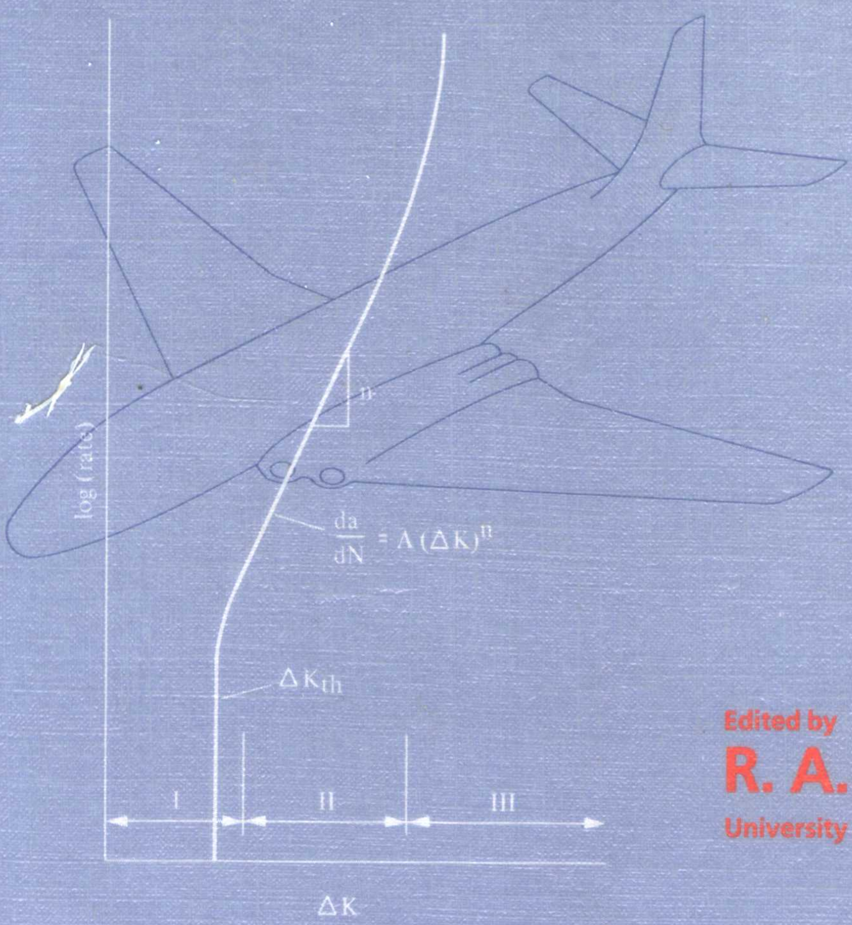
Fatigue Crack Growth

30 Years of Progress



Edited by
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FATIGUE CRACK GROWTH
30 Years of Progress

Proceedings of a Conference on Fatigue Crack Growth
Cambridge, UK, 20 September 1984

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Introduction

These papers are the proceedings of a Symposium held in September 1984 which served two purposes. Three of the outstanding figures of the fatigue world - Peter Forsyth, Norman Frost and Gerry Smith - have recently retired. Their careers spanned a period during which our understanding of the fatigue crack growth process has increased enormously. It was therefore felt that a review of this period would be of great interest and would serve to mark their contributions.

A short summary of the careers of the three principals is given, together with their own choice of their five most interesting and/or significant works. What is hard to express in print, and is perhaps inappropriate in a formal volume, is the warmth and affection with which these men are held in the fatigue, and wider, communities. This volume aims to express our appreciation of their activities.

Two of the papers by Fleck and Young, were not presented at the Symposium, but have been added to provide balance to the volume. The first two papers are historical reviews, by Smith on the development in Fatigue Crack Growth and by Marsh and Smith on Fatigue Testing Equipment - improvements of which have made possible many advances. Although there is, quite naturally, an emphasis on the British scene, a glance at the contents and references of these and the subsequent papers, should serve to dispel any charges of insularity from a wider international view. Knott deals with the modelling of the crack growth process, whilst Lindley and Nix review metallurgy - vital aspects of fatigue crack growth. Although the work of the last thirty years has seen a great simplification of the techniques of approaching fatigue crack propagation (largely through the use of the alternating stress intensity factor), research is still being conducted at an ever increasing rate. In many cases, this work is in the nature of dotting the i's, but Fleck's paper on the complications of fatigue crack growth seeks to identify which problems are significant and which are second order. The paper by Beevers and Carlson on the important topic of thresholds, might be viewed as representative of the fine detail of current research. Although the vast majority of fatigue problems occur in metals, Young's paper on non-metallics serves to remind us of the increasing use of such materials and the increasing sophistication of their design. Harrison's paper on Damage Tolerant Design reviews the approaches now possible to the design of defective (cracked) structures and how the advances in fracture mechanics over our period of review have been fed into the industrial design scene. The final paper by Tomkins attempts to draw the threads together and speculates on the future of cracks and defects.

The Comet accidents of thirty years ago involved the mysterious catastropic disintegration of aircraft in stable flight. By a curious coincidence of timing, as these papers were being prepared an Air India Jumbo disappeared into the Atlantic off the Irish coast. Strong, albeit circumstantial, evidence points to terrorist activity, but no firm proof of this theory has emerged. This accident was rapidly followed by the worst ever loss of life on a single aircraft - a Japanese Jumbo, and a serious fire on take off to a British 737. Both the latter accidents were caused by some form of mechanical and structural failure involving fatigue. A sad reminder of the technological difficulties of applying our fatigue knowledge to real components.

I have enjoyed stimulating discussions with Norman Frost, Peter Forsyth and Gerry Smith during the preparations for our Symposium and would like to record my thanks to them. Production of camera-ready copy (with its attendent defects!) does have the advantage of requiring close collaboration with the authors, which I have thoroughly enjoyed. Finally, to many people in Cambridge University Engineering Department, particularly my research students Dr Wilf Nixon, James Cooper, Chou Sheng Shin and Grant Leaity, the photographer John Read, Sheila Owen of the Drawing Office, and last, but by no means least, Rosalie Orriss for her help in producing the manuscripts, I record my grateful thanks.

R A Smith

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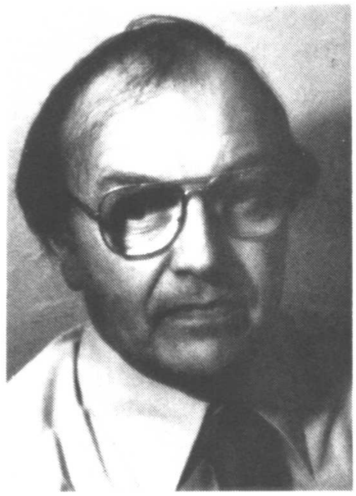
Biographical Notes



N. E. FROST, CBE

Norman Frost served an engineering apprenticeship with Rolls-Royce, Derby, obtaining a London Mechanical Engineering degree in the process. After four years in Rolls-Royce research laboratories, in 1948 he joined the National Physical Laboratory working on high temperature materials problems. In 1952 he transferred to the Mechanical Engineering Research Laboratory (latterly the National Engineering Laboratory, East Kilbride), of which, after several promotions over the years, he was appointed Deputy Director in 1977.

His technical interest has always been centred around engineering aspects of fatigue, particularly for steels, and he has published some 60 papers on notch effects, crack growth rates and threshold levels for fatigue crack growth.

"A relation between the critical alternating stress for propagation and crack length for mild steel", Proc. Instn. Mech. Engrs. 1959 173 811

"Non-propagating cracks in Vee-notched specimens subject to fatigue loading", Aero. Quart. 1957 8 1

"Significance of non-propagating cracks in the interpretation of notched fatigue data", J. Mech. Eng. Sci. 1961 3 299

"A fracture mechanics analysis of fatigue crack growth data for various materials", with L P Pook and K Denton, Eng. Fracture Mech. 1971 3 109

"Metal fatigue" with K J Marsh and L P Pook, Clarendon Press, Oxford, 1974

P. J. E. FORSYTH

After training as a Mechanical and Electrical engineer with the Post Office, Peter Forsyth joined the Royal Aircraft Establishment in 1944. His interest in materials grew from his early investigations of materials and methods of manufacture of captured German aircraft and rocket components. He started work on fatigue in 1949, his contributions causing his promotion on individual Merit to Deputy Chief Scientific Officer in 1977. His various awards include the Rosenhain Medal of the Institute of Physics (1967) and the degree of Doctor of Science (NCAA) in 1973.

His main interest has centred round aluminium alloys and the use of fractography to elucidate the physical mechanisms of fatigue. His many publications in this field include some heavily referenced classics.

"Some metallographic observations on the fatigue of metals", J. Inst. Metals 1951 80 181

"Slip band extrusion and damage", Proc. Roy. Soc. 1957 242 198

"A two stage fatigue fracture mechanism", Proc. Cranfield Symposium on Fatigue Crack Propagation 1961 1 76

"The Physical Basis of Metal Fatigue", Blackie, 1969

"A unified description of micro and macroscopic fatigue crack behaviour", Int. J. Fatigue 1983 5 3

G. C. SMITH

Gerry Smith started a shortened war-time metallurgy course at Cambridge University in October 1942 and after final examinations in 1944 undertook research work for the Ministry of Supply. In 1946 he was appointed a Departmental Demonstrator in the Department of Metallurgy at Cambridge and subsequently a University Demonstrator then Lecturer.

His research activities have included fatigue, dispersion hardening, powder metallurgy, ductile fracture and hydrogen-metal interactions. In addition to his Departmental teaching he has been a Tutor, Director of Studies and finally Senior Tutor (1966-1980) of Pembroke College. He has been a member of the Council of the Institute of Metals and Institution of Metallurgists, and a member of MOD and Aeronautical Research Council committees concerned with materials problems.

"Fatigue damage and crack formation in pure aluminium", with D R Harris, J. Inst. Metals 1959-60 88 182

"Changes occurring in the surface of mild steel specimens during fatigue stressing" with G F Modlen, J. Iron and Steel Institute 1960 194 459

"Crack propagation in high stress fatigue" with C Laird, Phil. Mag. 1962 7 847

"Fatigue strength of sintered iron compacts" with J M Wheatley, Powder Metallurgy 1963 12 141

"Crack closure and surface microcrack thresholds - some experimental observations" with M N James, Int. J. Fatigue 1983 5 75

Contents

Thirty Years of Fatigue Crack Growth — an Historical Review

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ABSTRACT

This survey of the last thirty years of fatigue, identifies as the major success the quantification of the rate at which fatigue cracks grow by the use of the alternating stress intensity factor parameter, ΔK. That mechanistic understanding has also advanced is shown by a review of major conferences which have occured in this period. The particular contributions made by P J E Forsyth, N Frost and G C Smith are emphasised where appropriate.

KEYWORDS

Fatigue; fatigue cracks; fatigue crack growth; stress intensity factor; crack growth threshold.

INTRODUCTION

It is convenient to interpret 30 years ago rather liberally, and to take 1953 as a starting point to remind readers just how much the world has changed in under half a lifetime. In 1953 Eisenhower succeeded Truman as the President of the United States, Stalin died and Churchill was Prime Minister of the United Kingdom. The Coronation of Queen Elizabeth II took place in St Paul's Cathedral which was still surrounded by blitz damage. Television broadcasting was in its infancy with black and white pictures being transmitted for four hours per day. Communications were such that America could only take a sound commentary of the Coronation; still pictures were sent across the Atlantic by radio, to be viewed by the public seven minutes after their live creation. There were some 15 working computers in America, compared with 10 in Britain. Food was still rationed in Britain, Everest was climbed, Dennis Compton scored the runs that won the Ashes at the Oval (cricket was still a national sport!). Industry in the UK boomed, with less than 1/4 million unemployed, although the average wage was only £380/year, 40% of cars produced were exported, the ship building industry was greater than that of the USA, Germany and Japan combined and, significantly for our review of the history of fatigue, the Comet had become the world's first passenger jet liner to enter service.

STATUS OF FATIGUE KNOWLEDGE PRIOR TO 1953

The gradual replacement of wooden parts of machines with cast then wrought iron, the invention of steam power and the increasing momentum of the industrial revolution must have caused many unrecognised fatigue failures. It was, however, the rapid rise of railways and failures of, in particular, axles which caused the name 'fatigue' to be coined and experimental investigations to begin in earnest. By the end of the nineteenth century Wöhler's empirical findings of stress range/life relationships and the concept of a fatigue limit were well established. Rules to deal with mean stress effects were proposed and the advice to avoid stress concentrating notches in design was proclaimed, even though quantitative assessments were lacking. However, the mechanisms of fatigue remained a mystery, and fatigue failures were not associated with progressive crack growth, despite, what with hindsight, must have been clear evidence on the fracture surfaces of failed components. It is worth noting that as far back as 1903, Ewing and Humfrey published photomicrographs which clearly showed the progressive development of slip bands on the surface of cyclically stressed iron samples, the slip bands eventually broadening to form cracks. It is indeed strange that this work passed largely unnoticed for more than half a century. By 1948, Nevil Shute, an aircraft engineer, writing in a prophetic novel, made a statement which, even today, would accurately summarise the perception of the wider public of fatigue:

> "Fatigue may be described as a disease of metal. When metals are subjected to an alternating load, after a great many reversals the whole character of the metal may alter, and this change may happen very suddenly. An aluminium alloy which has stood up quite well to many thousands of hours in flight may suddenly become crystalline and break under quite small forces, with most unpleasant consequences to the aeroplane."

THE COMET ACCIDENTS AND SUBSEQUENT ENQUIRY

After two catastrophic accidents involving the total loss of two Comet aircraft, off Elba on 10 January 1954 and, again in the Mediterranean, near Naples on 8 April 1954, the Comet aircraft were grounded and an investigation, which has become a classic of its kind, undertaken. A Comet was taken from service and the fuselage pressure cycled in a hydraulic tank at pressures equivalent to the flight-by-flight loadings. After some 1,830 flights in the tank, together with 1,230 pressurised flights before the test, making a total of 3,060, the cabin structure failed. The starting point of the failure being the corner of one of the cabin windows, see Fig. 1. The Elba accident had occured after 1,290 flights; the Naples accident after 900. In the light of examination of the Elba wreckage, a substantial quantity of which had been recovered from the sea bed, it was deduced that the accidents were caused by structural failure of the pressure cabin brought about by fatigue.

The most likely sources of fatigue initiation were found to be fastener holes located in the highly stressed regions near the corners of the remarkably square cabin windows. Some of the fastener holes were cracked during manufacture! Great difficulties were encountered in estimating accurate stress levels round these fasteners, whilst favourable test results may have been obtained for the detail under consideration because of compressive residual stresses induced by proof testing into the plastic region.

The official report (Cohen et al, 1955) contains much valuable detail, together with some classic (mis-)statements, describing this type of low cycle fatigue failure:

> "the symptoms of failure when it occurs after relatively few cycles (are) therefore less familiar, but they are also less specific. The process has not endured long, and most of the symptoms of a disease which spreads gradually are absent"

(Low cycle fatigue failure may have been thought to be new, but Braithwaite (1854) described the failure of iron girders supporting a beer vat over a century ago. It was some years after the Comet accidents that the Manson/Coffin plastic strain/ endurance work became widely accessible (Manson, 1966). No attempts of crack growth calculations were published; in the light of our knowledge thirty years on, it would have been extremely straightforward to condemn the fatal detail of the design. However, all disasters promote research, and although sadly the Comet accidents were a large nail in the coffin of the British civil aircraft industry, they promoted great interest in fatigue research. Indeed the official report went as far as saying of fatigue that,

> "Although the subject is a difficult one and rapid results cannot be expected, it is fully recognised that the best brains and resources ought to be directed to it."

WORK HARDENING AND FATIGUE IN METALS (1957)

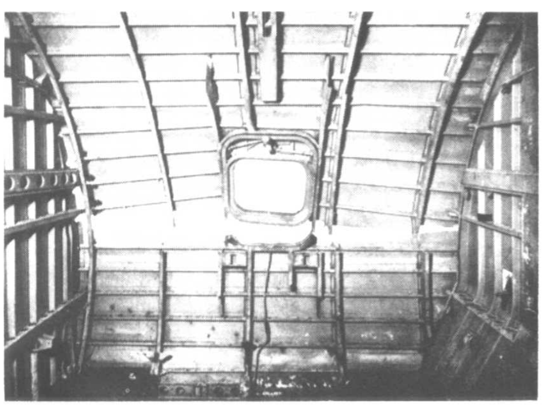
A convenient summary of the studies of fatigue knowledge in 1957, is contained in the reports of a discussion meeting held under the leadership of N F Mott at the Royal Society in February 1957. The title: 'Work-hardening and Fatigue in Metals' served to emphasise the view that fatigue was still thought of as a bulk phenomenon. Indeed in his introduction to the meeting Mott said:

> "It is hardly likely that we shall understand fracture in fatigue without

Comet G-ALYU at Khartoum shortly after entering service in 1952

The same Comet in the pressure testing tank built for the accident investigation.

(This and photograph below from Cohen et al, 1955)



View from inside of failure produced in the pressure testing tank. Note the shape of the window.

Fig. 1: The Comet accidents and subsequent investigation

first understanding work-hardening and the first few papers (in the meeting) are devoted to this subject"

The metal physicists, whose thoughts were full of the (relatively) new concepts of dislocations, seemed to many at this time to provide a way forward. However, both Smith and Forsyth, Fig. 2, discussed the initial stages of the formation of persistent slip bands and their development into fatigue cracks, albeit in a largely qualitative way, and added detail to the much earlier observations of Ewing and Humfrey. Frost, in a paper co-authored by Phillips, reviewed work undertaken over the previous five years or so, from which an understanding of the propagation phase of the fatigue process was beginning to emerge. Frost noted that more experimental information was needed concerning the extent of the crack tip plastic zone, before the theoretical crack propagation model due to Head (1953), probably the first such model, could be tested. Head had a mechanical model comprising rigid-plastic work-hardening element ahead of the crack tip and elastic elements over the remainder of the infinite sheet, which lead to a relationship of the type

$$\frac{da}{dN} = \frac{C_1 \; \Delta\sigma^3 \; a^{3/2}}{(C_2 - \sigma)\omega_o^{1/2}}$$

with a as crack length, $\Delta\sigma$ stress range and ω_o the size of the crack tip plastic zone. The problem of non-propagating cracks initiated at stress concentration was discussed by Frost and data given on the length v. cycles relationships of such cracks. However, no crack propagation 'laws' were suggested, although this, and associated works, were important milestones in the macroscopic 'engineering' approach to fatigue.

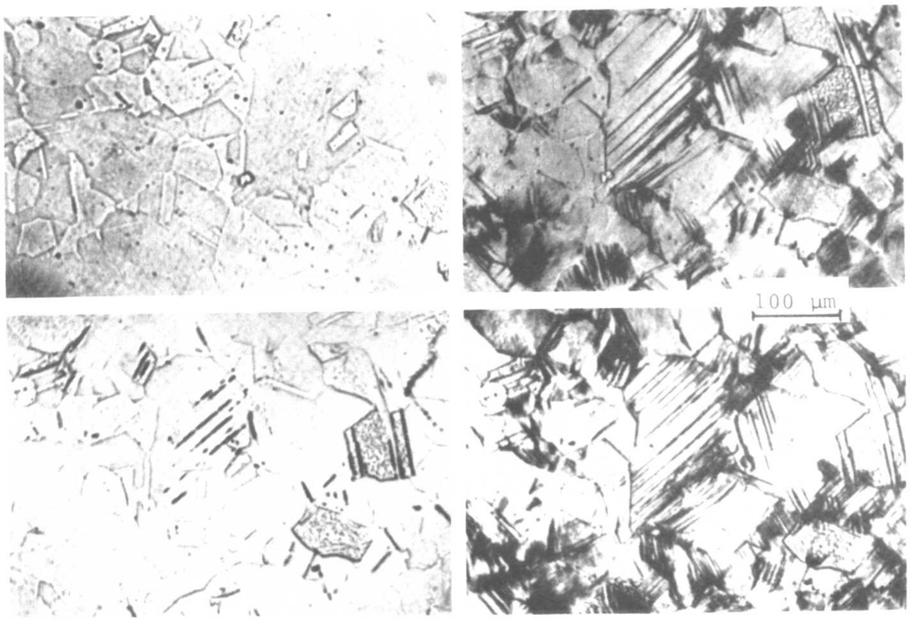
THE CRANFIELD CRACK PROPAGATION SYMPOSIUM (1961)

The changes which occurred in the four years after the meeting just described, are nowhere more evident than in the two volume proceedings of the Cranfield Crack Propagation Symposium held in 1961. It is fair to say that this meeting was a watershed in fracture research and with its passing we entered into the modern age. The range of topics was wide; crack propagation included unstable brittle and ductile fractures as well as sub-critical fatigue crack growth. Although the presentations were firmly focussed on cracks, the engineering community was clearly alarmed by their presence. Thus, Air Marshal Sir Owen Jones in his introduction to the meeting:

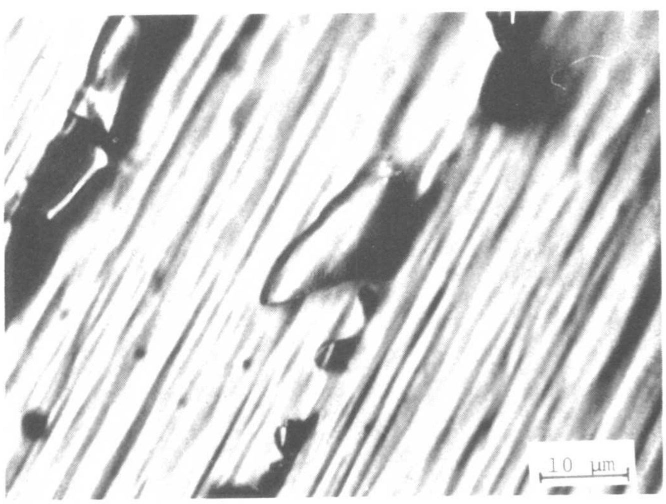
> "Perhaps our concern might have been indicated less ambiguously if the title of the Conference had been Crack Inhibition Symposium, since we are all much more interested in stopping cracks than in propagating them. However, I am sure I shall be told that it is only when we know what makes cracks propagate that we shall understand how to eliminate or control them."

This meeting further differed from the earlier Royal Society discussion, in that it was an international meeting with over half of the contributions coming from the United States; and it was from the US that the key new ideas emerged. However, two significant papers were presented by Forsyth and Frost, which will be discussed first.

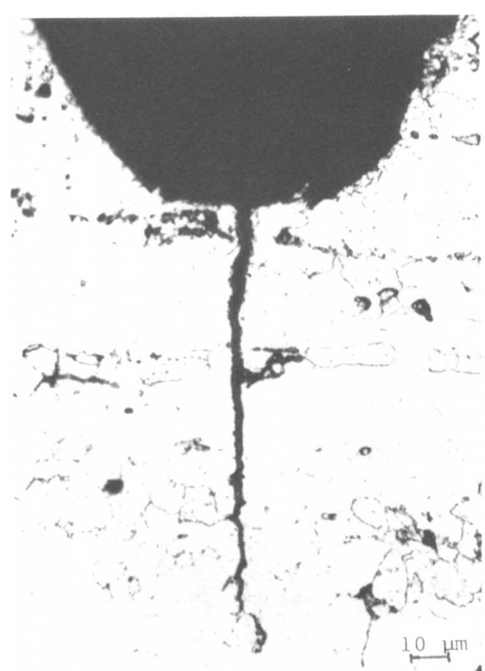
Forsyth's paper 'A Two Stage Process of Fatigue Crack Growth', must rank as one of the most heavily referenced classics of the literature. In it he divided the fatigue process into two stages; Stage I being a crystallographic, shear stress controlled slip process, Fig. 3, occurring to deepen a slip band groove, followed by Stage II growth, normal to the bulk maximum principal tensile stress, giving rise to features on the subsequent fracture surface known as striations. These striations had been reported some two years before the Cranfield meeting (Forsyth and Ryder, 1961), but the original observation was probably due to Zappfe and Worden (1951) who conducted some of the earliest reported fractographs of fatigue fracture surfaces using optical microscopy. However, Zappfe missed the essential point concerning the mode of formation of striations thinking they were caused by some substructural change in the material. Forsyth's Cranfield paper confirmed that one striation corresponded to a single load encursion by illustrating micrographs of programmed loaded specimen fracture surfaces, but he clung to his earlier view (1961), that in the precipitation aluminium alloys of his study, cleavage fracture occurred ahead of the crack tip and that the striation profile was formed by subsequent necking of the intervening material, Fig. 4. This would suggest that striation spacing would be largely governed by the inter-particle distance rather than by stress amplitude and crack length. A year later Laird and Smith (1962), published their findings on high strain growth in aluminium and nickel. They described a model involving sucessive crack blunting on the tensile stroke of the external load cycle followed by re-sharpening during the compression stroke, Fig. 4. This would lead to the peaks of the striations being formed on the compression stroke as opposed to the formation on the tensile stroke in the Forsyth model. In both cases the general form of the surface produced would be the same. Some debate ensued in the succeeding years: it is now acknowledged that the Laird/Smith model is more appropriate for low strength materials, whilst the



Above: G C Smith's illustration of the development of slip banαs into cracks in pure copper.



Left: Extrusion from a slip band in Aluminium/4% copper by P J E Forsyth



Right: A non-propagating crack at a 0.002 in. root radius in L65 Aluminium alloy, N E Frost

Fig. 2: The 1957 Royal Society Conference - A Discussioɪ on Work-Hardening and Fatigue in Metals

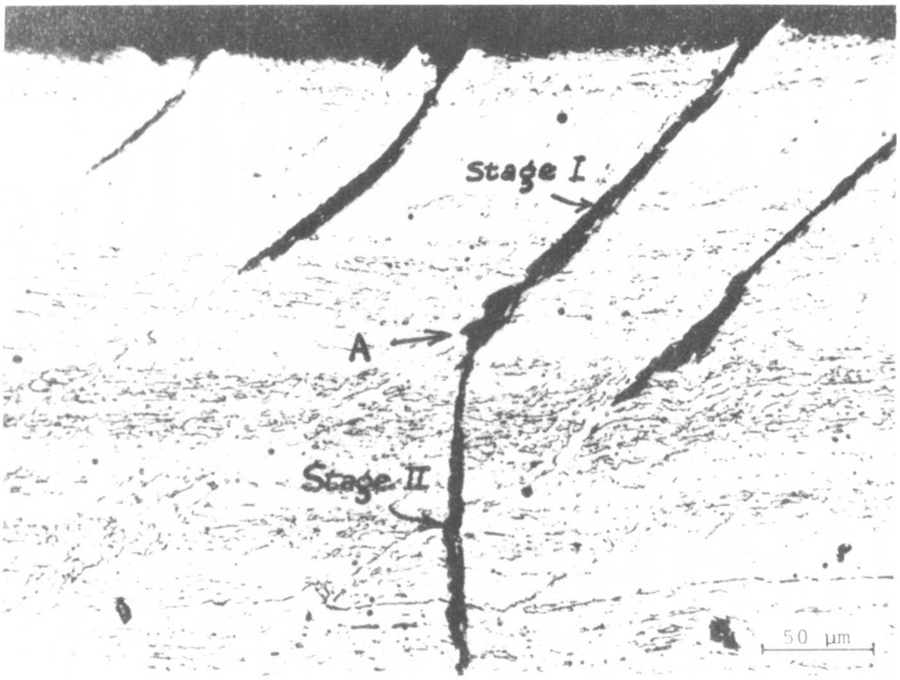


Fig. 3: The two stage process of fatigue, showing change over from Stage I (shear controlled) to Stage II (tensile controlled) growth (Forsyth, 1962).

Forsyth model better fits the observations on higher strength materials where some cleavage may take place in addition to the plastic blunting and re-sharpening of the crack tip. The quantitative aspects of this debate were to resurface when the details of crack propagation 'laws' were debated.

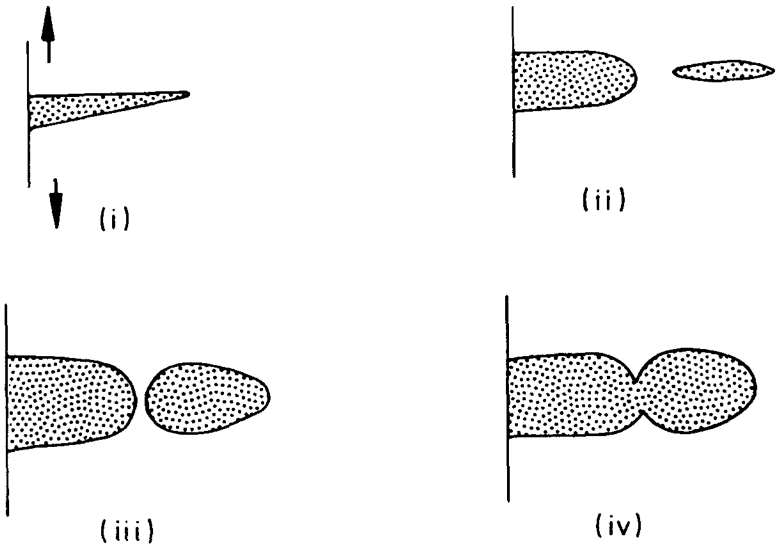
At the same meeting, Frost, with co-authors Holden and Phillips discussed some 'Experimental Studies into the Behaviour of Fatigue Cracks', undertaken with a view to determining the critical alternating stress required to propagate a crack of given length and the laws governing the rate of growth of a growing crack. These tests were made on sheet specimens 0.3 inches (7.6 mm) thick, of mild steel, nickel-chromium alloy steel, copper and a 4½% Cu-Aluminium alloy. The essential finding was 'Frost's Law', relating the rate of growth of a crack of length a with applied cycles N, to the alternating stress $\Delta\sigma$ and the crack length, by

$$\frac{da}{dN} = A\Delta\sigma^3 a$$

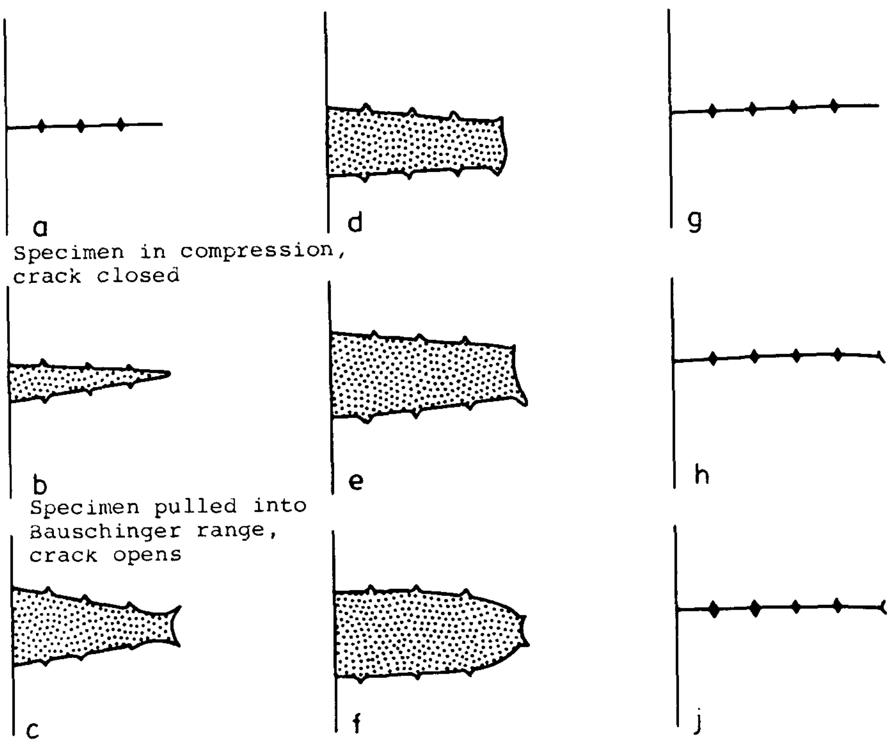
and that for crack growth to occur, $\Delta\sigma^3 a \geq C$, a critical value, corresponding to the threshold in modern usage. These findings were the result of extensive, thorough and painstaking research over a number of years (e.g. Frost, 1959(a), 1959(b), 1960; Frost and Dugdale, 1958), which went a great way to simplifying our view of the mechanics of crack growth. In particular Frost and Dugdale had noted that the crack tip plastic zone size increases in proportion to crack length. However, a more generally applicable parameter, ΔK, the alternating stress intensity factor, was aired at the Cranfield meeting by Donaldson and Anderson of the Boeing Company. This characterisation of fatigue crack growth has become so universal that it is worth considering its historical origins in some detail.

BRITTLE FRACTURE OF LIBERTY SHIPS

Losses in Allied shipping during the early stages of World War II led to a remarkable replacement programme in America, based on all welded construction of pre-fabricated ship sections. So successful was the management of this scheme, that the later ships were on the stocks in the builders yards for less than a day! Unfortunately, many of the so-called "Liberty" ships suffered from severe brittle fracture problems, some even separating into two whilst moored in still water, see Figs. 5 and 6. The problem proved perplexing; detailed records were kept, vast numbers of impact tests



Fracture ahead of the crack tip (i) end of compressive half cycle which may have fractured particles in front of the crack, (ii) tensile half cycle blunts crack and produces void in region of triaxial tension, (iii) thinning of unfractured bridge under biaxial tension, (iv) profile of striation formed. (Forsyth and Ryder, 1961)



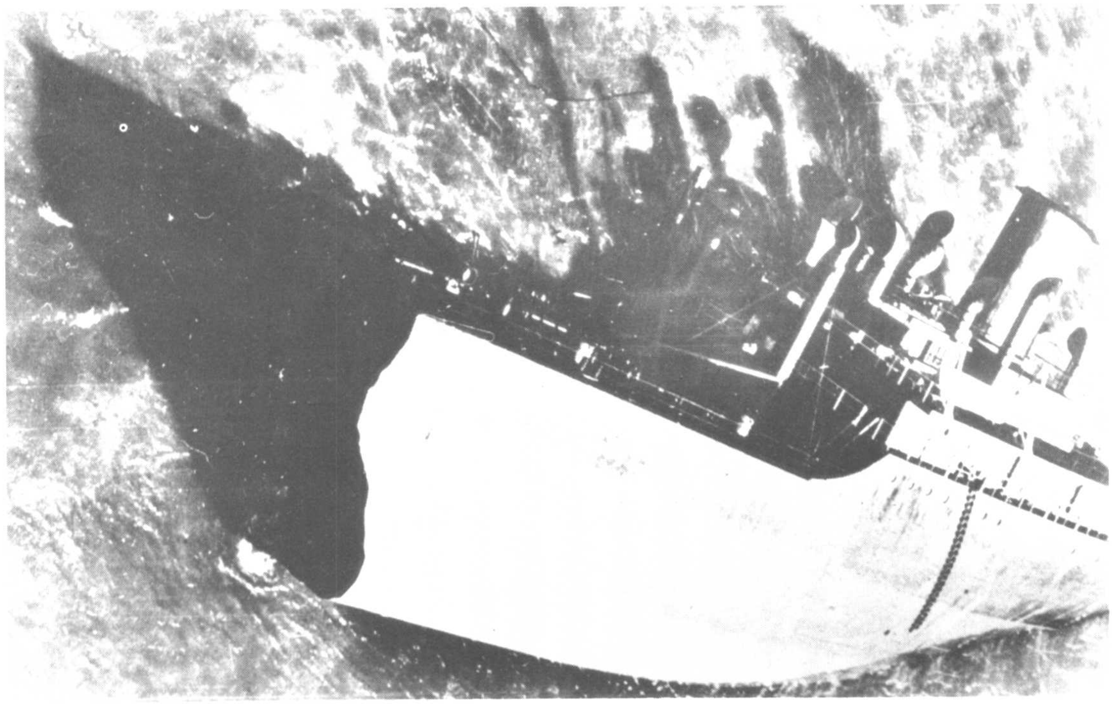
Specimen strained further, initial marked plastic deformation at the crack tip

(d) (e) (f) Alternative contours developed at the full plastic strain

Corresponding crack tip geometries on compressing the specimen

(Laird and Smith, 1962)

Fig. 4: Alternative models of striation formation



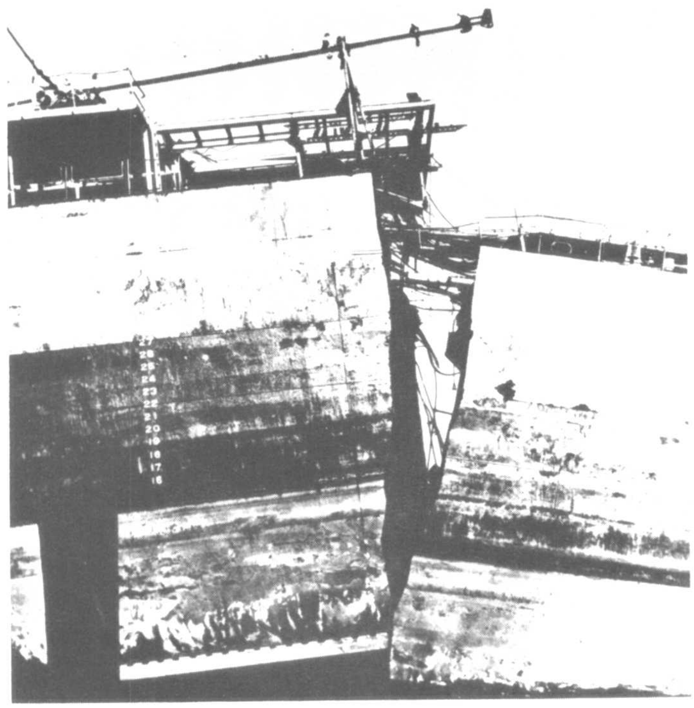


Fig. 5: Brittle fracture in 'Liberty' ships. Many fractures caused complete separation of the ships hull both whilst underway or, even more surprising, whilst alongside in calm water!

DIAGRAMMATIC ILLUSTRATION OF FRACTURES IN WELDED OR MAINLY WELDED SHIPS

DIAGRAM 'A'

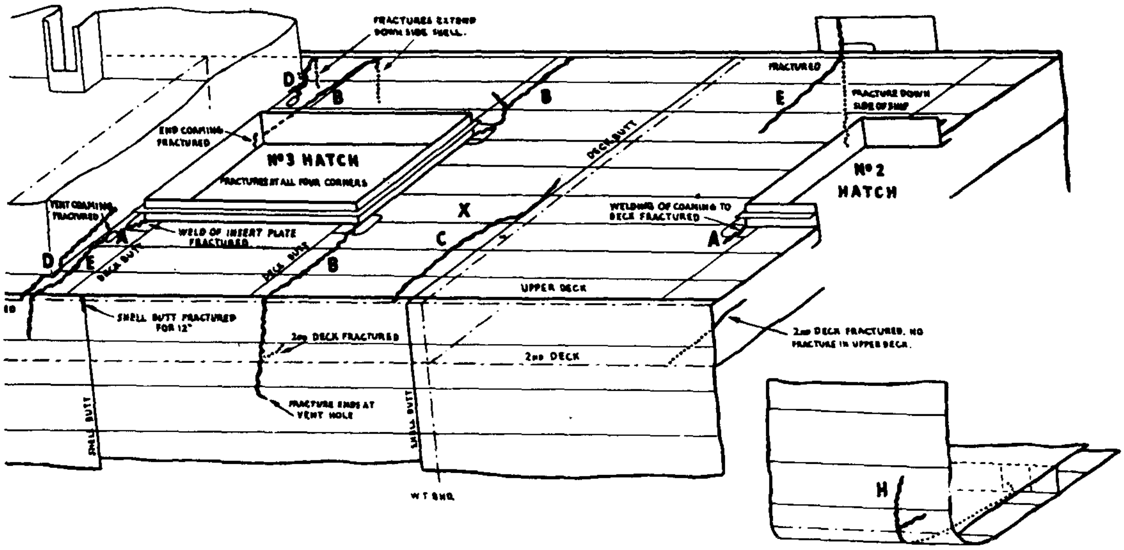


Fig. 6: Locations of cracks at various features of the ships construction (Brittle Fracture in Mild Steel Plates, Anon. 1945)

were made and many discussion meetings were held. Typical of these was a meeting held in Cambridge just after the War (Anon, 1945), to discuss the British and American experiences. The overemphasis of the metallurgy of the problem is evident, as is the bafflement of the stars of the scientific community (Baker, Bragg, Orowan, Mott, G I Taylor) concerning the mechanics of the situation. Later understanding of the roots of the problem can be summarised as bad geometry of design (sharp corners), no crack stopping devices in the all welded constructions coupled with occurrences of low temperatures, poor material and slack quality control of the welding. As far as fracture theory was concerned, the imposition of near useless impact tests onto the engineering community, was outweighed by the advantages which stemmed from the concentration over a period of years of the best brains available onto the problems of cracks in metals.

The theory of stress concentration round elliptical due to Inglis (1913), produced the classic simple formula for the maximum stress at a notch root:

$$\sigma_{max} = \sigma(1 + 2\sqrt{\frac{D}{\rho}})$$

where σ is the remotely applied nominal stress, D is the notch depth and ρ its root radius. Whilst this formula proved very useful for predicting stress levels at relatively blunt notches, a difficulty arose when cracks were considered, since as $\rho \rightarrow 0$ the maximum local stress $\rightarrow \infty$. Now in ductile materials it was clear that these high stresses would be limited by yielding of the material, but in both brittle and ductile materials it was a commonsense practical observation that larger cracks were more dangerous than smaller ones, a fact which could not be accounted for by the theory of stress concentrations.

Griffith (1921, 1924) made the important step of diverting attention from local crack tip conditions by deriving a global instability criteria. His first paper (1921), "The Phenomena of Rupture of Flow in Solids", starts by stating the origin and object of the work, which may come as a surprise to many,

> "the discovery of the effect of surface treatment - such as filing, grinding or polishing, on the strength of metallic machine parts subjected to alternating or repeated loads"

In the event, this paper dealt with the unstable fracture of cracks subjected to monotonic loading, by formulating a necessary thermodynamic criterion for fracture, that the elastic energy available to propagate the crack must be greater than the energy absorbed in creating new crack surfaces. The first paper, although containing interesting detail on experiments with glass, was rather long and discursive, contained an error in the application of Inglis' earlier stress analysis, but nevertheless was an important milestone. The second paper (Griffith, 1924), "The Theory of Rupture", was much more concise. A formula for the remote stress to propagate a central crack in a uniform biaxially loaded sheet was derived:

$$\sigma = \sqrt{\frac{2ET}{\pi a}}$$

with E, the Young's Modulus, a as the semi-crack length and T the surface energy. The theory was applied to inclined cracks in both tensile and compressive stress fields, in the development of which Griffith clearly recognised that a sufficient local condition for fracture must be added to his global necessary condition - that of tensile stresses acting across the prospective crack path (present adherents of "strain-energy density criteria" please note!).

It was the use of surface energy as the only crack tip energy sink that caused the Griffith predictions to be in large error for ductile materials. Griffith's own conclusion was that

> "In the case of plastic crystals, we are further hindered by the fact that rupture is almost invariably preceded by plastic flow, whose nature is still the subject of hot controversy."

It was largely for this reason that Griffith's theory lay unused (and unsung) for nearly twenty years, before the pressing problem of brittle fracture caused it to be re-examined.

THE ORIGINS OF THE STRESS INTENSITY FACTOR

Sneddon (1946) could well claim to have made the first observation that the local stress fields near crack tips were always spatially similar even if the bulk geometries in which the cracks occurred are different. Based on crack tip polar coordinates (r, θ), he observed that for a Griffith type crack, the local stresses could be written as

$$\sigma_{xx} = \sigma\left(\frac{a}{2r}\right)^{\frac{1}{2}}\left(\frac{3}{4}\cos\frac{\theta}{2} + \frac{1}{4}\cos\frac{5\theta}{2}\right)$$

$$\sigma_{yy} = \sigma\left(\frac{a}{2r}\right)^{\frac{1}{2}}\left(\frac{5}{4}\cos\frac{\theta}{2} - \frac{1}{4}\cos\frac{5\theta}{2}\right)$$

$$\tau_{xy} = \sigma\left(\frac{a}{8r}\right)^{\frac{1}{2}}\sin\theta\cos\frac{3\theta}{2}$$

and that for a uniformly stressed penny shaped crack;

> "The most striking feature of the analysis ... is that the expressions for the components of stress in the neighbourhood of the crack differ from those of the two-dimensional case by a numerical factor only." [That factor being 2/π.]

He went further by recognising that these local stress fields would still apply if the extent of plastic yielding was small:

> "Thus even for small stresses plastic flow occurs at the corners [tips] of the crack to remove this infinite stress. There is, in fact, no purely <u>elastic</u> solution of the problem; if, however, the internal pressure [stress] is not too large the region of plastic flow will be small and not appreciably affect the distribution of stress at points in the solid at a distance from the corners [tips] of the crack."

The paper goes on to calculate a Griffith criterion for the internal penny shaped crack.

Here the matter lay, until a decade or so later when contributions were made by Irwin (1957a, b) and Williams (1957). Irwin (1957a) presented the local crack tip stresses for the Griffith crack, in the typical form:

$$\sigma_{yy} = \left(\frac{EG}{\pi}\right)^{\frac{1}{2}}\frac{1}{\sqrt{2r}}\cos\frac{\theta}{2}\left(1 + \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$$