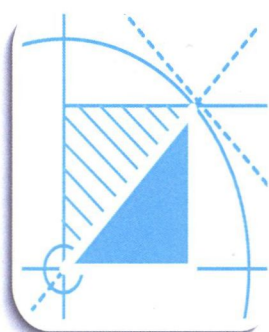
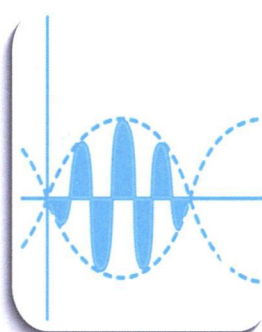


Modern Nonlinear Theory and Application

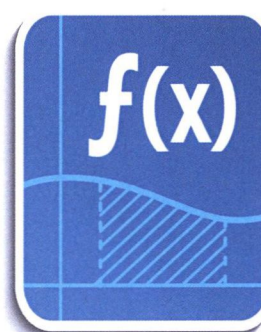
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Trigonometry



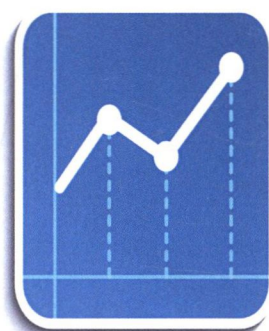
Linear algebra



Calculus 1



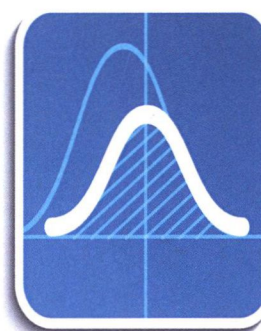
Calculus 2



Statistics 1



Statistics 2



Probability theory

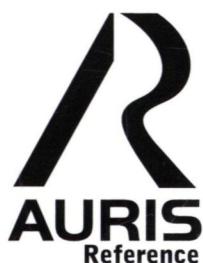


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About the Book

Nonlinear problems are of interest to engineers, physicists and mathematicians and many other scientists because most systems are inherently nonlinear in nature. Nonlinear systems may appear chaotic, unpredictable or counterintuitive, contrasting with the much simpler linear systems. Typically, the behavior of a nonlinear system is described in mathematics by a nonlinear system of equations, which is a set of simultaneous equations in which the unknowns (or the unknown functions in the case of differential equations) appear as variables of a polynomial of degree higher than one or in the argument of a function which is not a polynomial of degree one. In other words, in a nonlinear system of equations, the equation(s) to be solved cannot be written as a linear combination of the unknown variables or functions that appear in them. It does not matter if nonlinear known functions appear in the equations. In particular, a differential equation is linear if it is linear in terms of the unknown function and its derivatives, even if nonlinear in terms of the other variables appearing in it. Nonlinearity is ubiquitous in physical phenomena. Fluid and plasma mechanics, gas dynamics, elasticity, relativity, chemical reactions, combustion, ecology, biomechanics, and many, many other phenomena are all governed by inherently nonlinear equations. (The one notable exception is quantum mechanics, which is a fundamentally linear theory, although recent attempts at grand unification of all fundamental physical theories, such as string theory and conformal field theory, are nonlinear.) For this reason, an ever increasing proportion of modern mathematical research is devoted to the analysis of nonlinear systems and nonlinear phenomena.

The goal of *Modern Nonlinear Theory and Application* is to provide some of the most important ideas, mathematical techniques, and new physical phenomena in the nonlinear realm.



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Preface

Nonlinear problems are of interest to engineers, physicists and mathematicians and many other scientists because most systems are inherently nonlinear in nature. Nonlinear systems may appear chaotic, unpredictable or counterintuitive, contrasting with the much simpler linear systems. Typically, the behavior of a nonlinear system is described in mathematics by a nonlinear system of equations, which is a set of simultaneous equations in which the unknowns (or the unknown functions in the case of differential equations) appear as variables of a polynomial of degree higher than one or in the argument of a function which is not a polynomial of degree one. In other words, in a nonlinear system of equations, the equation(s) to be solved cannot be written as a linear combination of the unknown variables or functions that appear in them. It does not matter if nonlinear known functions appear in the equations. In particular, a differential equation is linear if it is linear in terms of the unknown function and its derivatives, even if nonlinear in terms of the other variables appearing in it. Nonlinearity is ubiquitous in physical phenomena. Fluid and plasma mechanics, gas dynamics, elasticity, relativity, chemical reactions, combustion, ecology, biomechanics, and many, many other phenomena are all governed by inherently nonlinear equations. (The one notable exception is quantum mechanics, which is a fundamentally linear theory, although recent attempts at grand unification of all fundamental physical theories, such as string theory and conformal field theory, are nonlinear.) For this reason, an ever increasing proportion of modern mathematical research is devoted to the analysis of nonlinear systems and nonlinear phenomena.

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STABILITY OF THE SHALLOW AXISYMMETRIC PARABOLIC-CONIC BIMETALLIC SHELL BY NONLINEAR THEORY

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ABSTRACT

In this contribution, we discuss the stress, deformation, and snap-through conditions of thin, axi-symmetric, shallow bimetallic shells of so-called parabolic-conic and plate-parabolic type shells loaded by thermal loading. According to the theory of the third order that takes into account the balance of forces on a deformed body, we present a model with a mathematical description of the system geometry, displacements, stress, and thermoelastic deformations. The equations are based on the large displacements theory. We numerically calculate the deformation curve and the snap-through temperature using the fourth-order Runge-Kutta method and a nonlinear shooting method. We show how the temperature of both snap-through depends on the point where one type of the rotational curve transforms into another.

INTRODUCTION

The development of machine sciences in recent centuries has led to the manufacture of various devices from relatively simple mechanisms to the very complex mechanical devices used by mankind in the process of transforming material goods. Although modern equipment comes in very different forms, functions, and structure, owing to the importance of smooth, reliable operation and their value, a demand for protection against a number of overloads is expressed. It is especially necessary to provide reliable protection against thermal overload for machines that convert one form of energy into another and heat up in the process. For this purpose, elements are built into devices to serve as a “thermal fuse” turning the machine off as soon as an individual part reaches the maximum permissible temperature. Due to their operational reliability, both line and plane bimetallic structural elements are used in protection against thermal overload, whose operation is based on the known physical fact that bodies expand with the increase of temperature. With a suitable technical act, we can connect a bimetallic construction element, Figure 1, with electrical contacts

and make a so-called “thermal switch”. Displacements created on the bimetallic element due to a combination of temperature and mechanical loads turn the device on and off dependent on the temperature T . For this, it is of course necessary to know the connection between the displacements and loading of the bimetallic construction element. Apart from material properties, this connection is also dependent on the geometric properties of the bimetal, as the line bimetallic construction elements respond with different stresses and deformational states to temperature loads as compared to plane construction elements. In practice, the difference in the stability conditions is the most important. Thin and shallow bimetallic shells with suitable material and geometric properties have the characteristic of snapping-through into a new equilibrium position at a certain temperature. The result of such a fast snap-through of a bimetallic shell acting as a switching element in a thermal switch is the instantaneous shutdown of electric power and the machine. The snap-through of the bimetallic shell is a dynamic occurrence that lasts a very short time and as such prevents the damaging sparking and melting of electric contacts and extends the life expectancy of the thermal switch.

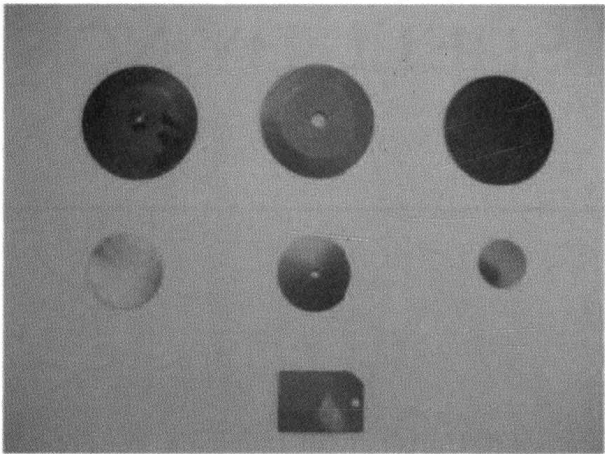


Figure 1: Some of the different types of bimetallic shells made by the Austrian manufacturer “Elektronik Werkstatte”.

Panov, Timoshenko, Videgren, Witrick, Aggarwala, Saibel, Huai, Vasudevan, Johnson, Keller, Reiss, Brodland, Cohen, Kosel, Batista, Drole, Jakomin, and some other authors have engaged in the research of bimetallic shell elements in a homogenous temperature field. S. Timoshenko [1] researched the problem of the stability of bimetallic lines and plates. The authors Panov [2] and Witrick et al. [3] researched the problem of the stability for shallow axi-rotational symmetric spherical shells during heating. The related problem of stability of a spherical shell under normal loads has been treated by Keller and Reiss [4]. The problem of thermal stability of a multimetallic strip has been treated by Vasudevan and Johnson in [5]. Aggarwala and Saibel researched the thermal stability of thin spherical shells [6]. The occurrence of the snap-through of an open bimetallic shell was treated by Ren Huai [7] using approximative methods. The problem of finite axisymmetric deflection and snapping of spherical shells which are point loaded at the apex and simply supported at the boundary is analysed by Brodland and Cohen, in [8]. Kosel et al. researched the stability conditions during the temperature and mechanical loading of rotational axisymmetric [9–12] and translation shells [13, 14] with and without an opening at the apex of the shell.

Apart from the spherical and parabolic bimetallic shells, the market also offers shells with more complex initial geometries. “Elektronik Werkstatte” the manufacturer of bimetallic shells from Eichgraben in Austria offers several different types of combined axisymmetric shells (Figure 1, the first and the second shell from the left in the top row) that have an advantage over spherical shells in that the temperature range of the snap-through (the difference in temperature of the upper and lower snap-through) can be changed at the constant of the initial bimetallic shell height. Even with a relatively small initial shell height, using a suitable combination of a parabola and cone rotational curve, it is possible to achieve snap-through of the shell only at very high temperature loads. Due to the smaller initial height, it is important that the stresses in these shells are smaller relative to the stresses in parabolic or spherical shells.

So, this time, we are discussing the stability and deformation conditions for a thin axisymmetric shallow bimetallic shell, composed of a parabola and cone and a plane and parabola. Apart from the temperature, the shell is additionally burdened with a force at the apex and with pressure. When executing a nonlinear mathematical model for the snap-through of a bimetallic translation shell, we will assume a small strain and the moderate rotation of the shell element. In the strain tensor, we will also consider nonlinear terms, while placing equilibrium equations on the deformed part of the shell. For the first time, we will show in both table and graphic form how the temperature of both snap-through depends on the point where the parabolic shape of the rotational curve changes into a cone shape. We will explain the effect of the concentrated force at the apex for the example of a bimetallic shell of the plate-parabola type!

The defined thermoelastic problem will be solved with the following steps:

- (1) Defining the geometry of the undeformed shell,
- (2) Deriving the displacement vector as relation between the geometry of the undeformed and the deformed shell,
- (3) Defining the geometry of the deformed shell,
- (4) Deriving the elements of the strain and stress tensor,
- (5) Introducing the forces and moments per unit of length,
- (6) Deriving the equilibrium equations of unit forces and moments acting in the deformed shell element, and
- (7) Calculating the deformation curve and the snap-through temperature using the fourth-order Runge-kutta method and a nonlinear shooting method.

GEOMETRY OF THE UNDEFORMED SHELL

The axisymmetric shape of the undeformed shell is formed by rotation of the curve $y=(x)$, about the y axis [15], Figure 2. The middle surface of the undeformed axisymmetric shell in the Lagrange Coordinate System is, therefore, defined by the function $y=y(x)$. Figure 3 shows the middle surface of a thin rotationally symmetric bimetallic shell.

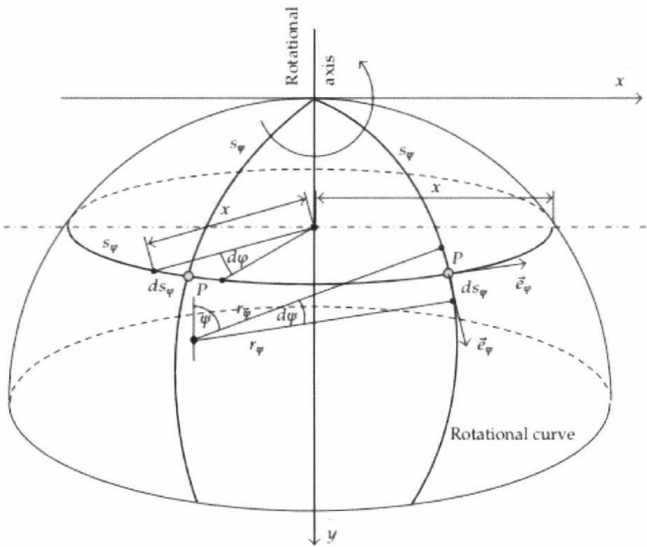


Figure 2: Geometry of the undeformed shell and basis unit vectors.

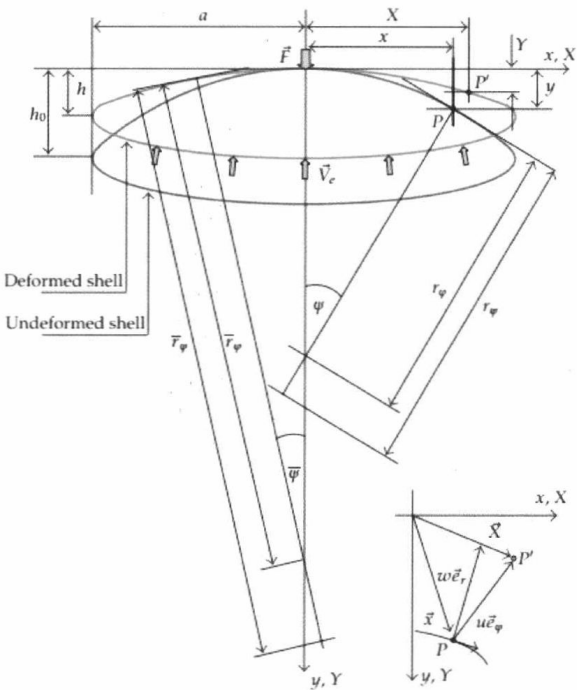


Figure 3: The axisymmetric shell in a homogeneous temperature field and the connection between the Euler and Lagrange coordinate system.

From Figures 2 and 3, we can obtain the geometric properties of the unreformed shell.

Differential of the length in the meridian direction ds_ψ :

$$ds_\psi = r_\psi d\psi. \quad (2.1)$$

Differential of the length in the circular direction ds_φ is

$$ds_\varphi = r_\varphi \sin \psi \quad d\varphi = x d\varphi. \quad (2.2)$$

Meridian angle ψ

$$\psi = \arctan\left(\frac{dy}{dx}\right) = \arctan(y') \approx y'. \quad (2.3)$$

The flexion curvature $1/r_\psi$ of the rotational curve (x) in the meridian direction is, Figures 2 and 3

$$\frac{1}{r_\psi} = \frac{y''}{\sqrt{(1 + y'^2)^3}} \approx y''. \quad (2.4)$$

The flexion curvature $1/r_\varphi$ of the circle formed by rotation of the curve $y=(x)$, in the circular direction is, Figures 2 and 3,

$$\frac{1}{r_\varphi} = \frac{\sin \psi}{x} = \frac{\sin \arctan(y')}{x} = \frac{1}{x} \frac{y'}{\sqrt{1 + (y')^2}} \approx \frac{y'}{x}. \quad (2.5)$$

The simplifications in (2.3), (2.4), and (2.5) are justified since we are discussing a shallow shell, where,

$$y'(x)^2 \ll 1, \quad (2.6)$$

and consequently,

$$\sin \psi \approx \psi, \quad \sin \varphi \approx \varphi, \quad \cos \psi \approx 1. \quad (2.7)$$

DISPLACEMENT VECTOR AND GEOMETRY OF DEFORMED SHELL

Due to the temperature change, the shell deforms into a new shape defined by the function (X) in the Euler Coordinate System. Since we are discussing a thin double-layered shell, the displacement field is selected to satisfy the Kirchhoff hypothesis [16]:

- (1) Straight lines perpendicular to the shell's middle surface before deformation, remain straight after deformation,
- (2) The transverse normals do not experience elongation,
- (3) The transverse normals rotate, so that they remain perpendicular to the shell's middle surface after deformation.

The reader should note that the Kirchhoff hypothesis stating that the thickness of the shell before and after deformation remains the same when the shell is subjected to kinematic constraint is not realistic when large strains are admitted in the deformation process [17]. The so-called Zig-Zag theory is known in literature and describes a piecewise continuous displacement field in the direction of the multilayered shell thickness and accomplishes the continuity of transverse stresses at each layer [18–20]. However, given the fact that the ratio between the thickness and length of bimetallic shells, used as safety constructional elements against temperature overheating, is about 1/100, we found the Kirchhoff hypothesis is fully acceptable for our mathematical model.

The new shape of the shell in a deformed state is axisymmetric due to the homogeneous temperature field and axisymmetric mechanical load. Therefore, the displacement v in the circular direction as well as other shell properties relative to the angle φ do not change: