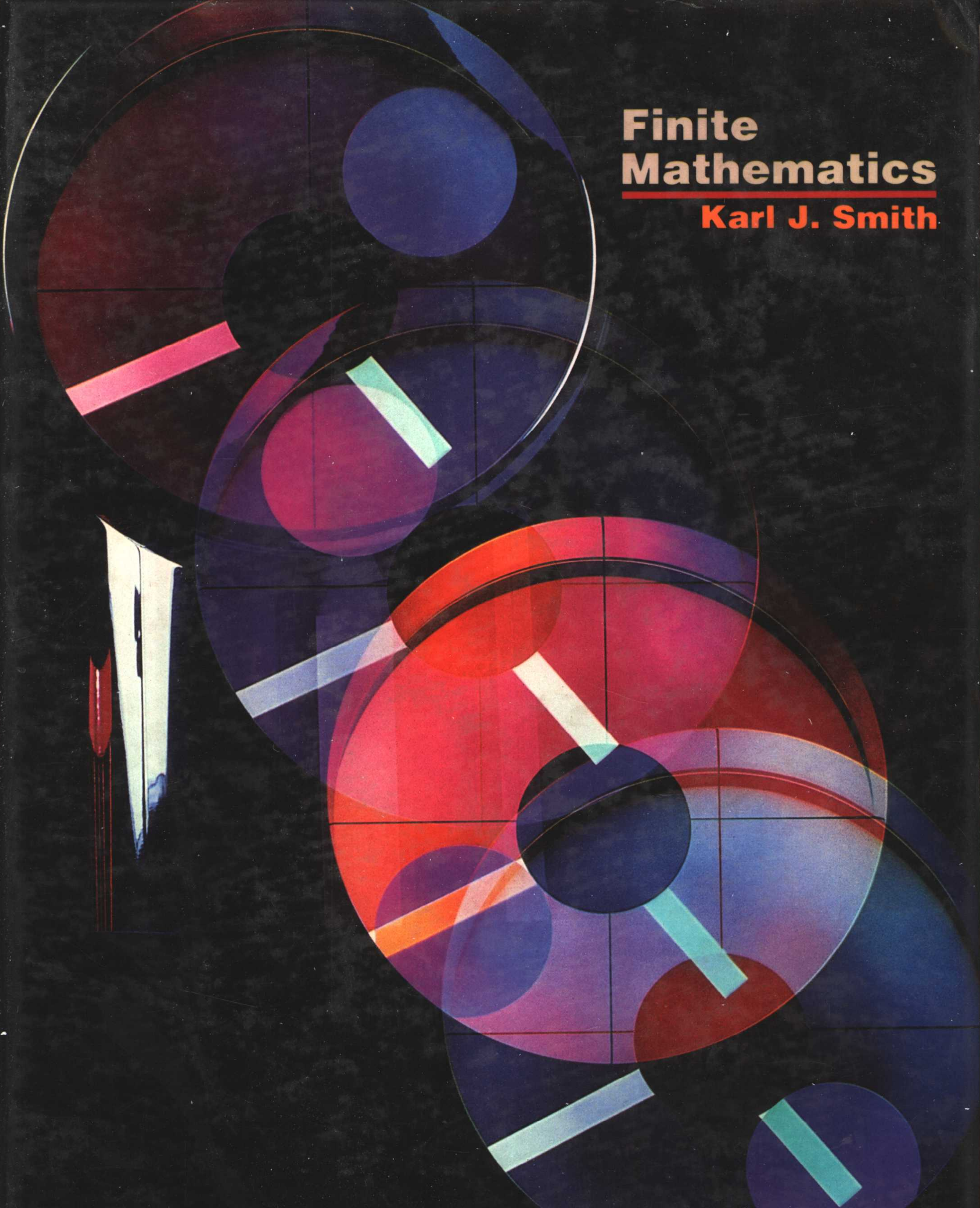
Finite Mathematics

Karl J. Smith



Finite Mathematics

Karl J. Smith
Santa Rosa Junior College

Brooks/Cole Publishing Company
Monterey, California

This book is dedicated, with love,
to my son, Shannon J. Smith

The cover photograph, by Al Satterwhite, is a quadruple exposure of an Aurora Clock. The face of this clock changes color as time progresses.

Brooks/Cole Publishing Company A Division of Wadsworth, Inc.

Printed in the United States of America

10 9 8 7 6

Library of Congress Cataloging in Publication Data

Smith, Karl J.
Finite mathematics.

Includes index.
1. Mathematics—1961- . I. Title.
QA39.2.S588 1985 510 84-21337
ISBN 0-534-03975-8

Sponsoring Editor: Craig Barth
Production Service: Phyllis Niklas
Production Coordinator: Joan Marsh
Design: Albert Burkhardt
Illustrations: Carl Brown

Cover Photo: © Al Satterwhite, The Image Bank West
Typesetting: Bi-Comp, Inc.
Printing and Binding: Vail-Ballou Press

Preface

Finite Mathematics provides the noncalculus mathematics background necessary for students in business, social science, and biological science. The only prerequisite for the course is intermediate algebra.

I have written the material to be mathematically comprehensive and correct, but at the same time my goal has been to make the material clear and interesting for the reader. For example, one of the most difficult tasks for students in solving linear programming problems is not finding a solution, but rather building the model to be solved. This skill is rarely *developed* in a textbook, but this book devotes an entire section to the nature of the linear programming problem and how to go about formulating an appropriate model (see Section 3.2). Throughout the book, I have made every effort to point out the important ideas and pitfalls. Important terms are set off for easy reference, and problems are keyed to examples in the text. There are frequent summaries and reviews to help the student decide what to do, as well as how to do it. The text is divided into sections of about equal size which each take about one class day to develop. Since there are 51 numbered sections plus 10 review sections, there is ample opportunity for the instructor to select material to tailor the book to individual classes. Sample course outlines are provided in the *Instructor's Manual,* but the interdependence of the chapters is shown in the table:

Chapter	Prerequisite Chapters	
1	None	Chapter 1 may be assumed if you have had a recent intermediate algebra course.
2	1	
3	1	
4	1, 3	
5	1	
6	1, 5	
7	1, 5, 6	
8	1, 2, 6	
9	1, 2, 6	
10	1	

The features of the text include the following:

Exercises

The greatest strength or weakness of a mathematics textbook is the problem sets. An unusual amount of effort has gone into the construction of these problem sets. There are more than 2,500 problems in this text. The problems are arranged in matching pairs with the answers to the odd-numbered problems in the back of the book (Appendix E). All problems are graded in difficulty from easy to hard.

Drill. Problems are keyed to examples so that the reader has a clear idea of what is expected.

Applications. Self-contained applied-type problems designated by discipline are provided to give relevance and practicality to the topic at hand.

Extended Applications. Each chapter concludes with an optional real-life extended application to allow the material learned to be applied to a *real* (rather than a textbook) problem, but at the same time at a level of difficulty that is manageable.

CPA, CMA, and Actuary Exam Questions. Actual questions from Actuary, CPA, and CMA exams are scattered throughout the textbook. These test questions provide a link between textbook and profession.

Historical Questions. Historical notes provide insight into the people and humanness of mathematics, but instead of being superfluous commentaries, they are integrated into the problem sets to give students a taste of some of the ways the topics were originally developed as solutions to mathematical problems.

Chapter Review Problems. I have provided extensive lists of review problems that are keyed to specific examples in the chapter. Each chapter review is designed to make it clear to the reader what specific skills from the chapter need to be mastered.

Examples

After the problems, the next most important part of any textbook is the number and nature of the examples. A student learns by seeing examples both in and out of class. There are over 330 examples—many more than is found in most finite mathematics textbooks. I have provided additional hints and comments to help the reader through the examples, and have also included the "intermediate steps" showing the algebraic manipulations in a way that can be easily followed. A second color is used to focus on the important steps or particular parts of an equation or formula.

Chapter Reviews

Each chapter has an extensive review, listing important terms and problems keyed to stated objectives. The problems include references to specific examples and pages within the chapter.

Calculators

Almost all students have access to an inexpensive calculator. Many of the examples and applications in this text require the use of a calculator. See Appendix B for a discussion of calculator use.

Computers

Many of the problems in finite mathematics can be programmed and solved on a microcomputer. Many students have access to an Apple, IBM PC, or other microcomputer. A computer supplement (with accompanying disk), *Computer Aided Finite Mathematics*, prepared by Chris Avery and Charles Barker, is available for use with this text. Appendix C lists the programs available in this supplement.

Instructional Aids

The answers to the odd-numbered problems appear in the back of this book. In addition, the following instructional aids are available:

Instructor's Manual, including sample tests
Student's Solution Manual
Computer Manual with BASIC Programs (see above)

Acknowledgments

The production of a textbook is a team effort. I would like to thank Craig Barth, Joan Marsh, Carl Brown, and especially Phyllis Niklas for their help in producing this book. Their many suggestions and commitment to this book is very much appreciated.

The reviewers of the manuscript made many helpful suggestions, and I thank them all: Joseph S. Evans, Middle Tennessee State University; James L. Forde, Concordia College; Marjorie S. Freeman, University of Houston; Matthew Gould, Vanderbilt University; Norman Mittman, Northeastern Illinois University; and Wendell L. Motter, Florida A&M University. The problem solvers were John Spellman, Ernest Ratliff, Ricardo Torrejon, and Gregory Passty of Southwest Texas State University, and I thank them not only for reviewing the manuscript, but for the long hours spent working all the examples and problems. In addition, Terry Shell of Santa Rosa Junior College worked all the problems (at least twice), and offered many helpful suggestions, and I give him my heartfelt thanks. Chris Avery and Charles Barker from De Anza College were most cooperative and helpful in modifying their computer material to match the development in this book.

And last, but not least, my thanks go to my family, Linda, Missy, and Shannon, for their love, support, and continued understanding of my involvement in writing this book.

Contents

*Optional sections

*Optional sections

Finite Mathematics

CHAPTER 1

Linear Models

Chapter Contents

Chapter Overview

In this chapter you will learn what we mean by a mathematical model, which will form the foundation for much of the remaining material in the text. Mathematical modeling, in short, is the application of mathematics to real-life situations. Many of the preliminary ideas you will need for this course are also introduced in this chapter and include working with formulas, graphs, lines, and systems of equations. You will then be able to apply these ideas to answer a variety of applied problems as you progress through this book.

Extended Application*

Solartex manufactures solar collector panels. During the first year of operation, rent, insurance, utilities, and other fixed costs averaged $8,500 per month. The panels sold for $320 each and cost the company $95 each in material and $55 each in labor. Since company resources are limited, Solartex cannot spend more than $20,000 in any one month.

Sunenergy, another company selling similar panels, competes directly with Solartex. Last month, Sunenergy manufactured 85 panels at a total cost of $20,475, but the previous month produced only 60 panels at a total cost of $17,100.

Should Solartex employ a consulting firm to help increase efficiency and enable them to compete against Sunenergy more effectively?

1.1 Mathematical Modeling

Mathematics is sometimes classified as an art and sometimes as a science. This dual nature of mathematics is apparent when you look at a college catalogue of courses offered. There are courses in applied mathematics and courses in pure mathematics. At the elementary level, both these types of courses translate into very simplified application problems with most emphasis on learning the mechanics of mathematics (such as solving equations, drawing graphs, and manipulating symbols). In this course we will not only learn the mechanics of mathematics, but also apply it to particular

* The extended application is an optional section beginning on page 41. It incorporates and reviews the main ideas of this chapter.

real-life situations in the business, social science, and biological science areas.

When a real-life situation is analyzed, it usually does not easily lend itself to mathematical analysis because the real world is far too complicated to be precisely and mathematically defined. It is therefore necessary to develop what is known as a **mathematical model.** This model is based on certain assumptions about the real world and is modified by experimentation and accumulation of data. It is then used to predict some future occurrence in the real world. A mathematical model is not static or unchanging but is continually being revised and modified as additional relevant information becomes known.

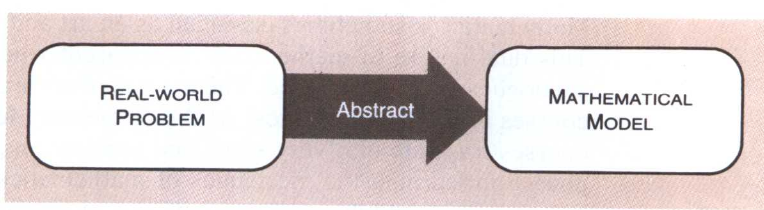
Some mathematical models are quite accurate, particularly in the physical sciences. For example, the path of a projectile, the distance that an object falls in a vacuum, or the time of sunrise tomorrow all have mathematical models that provide very accurate predictions about future occurrences. On the other hand, in the fields of social science, psychology, and management, models provide much less accurate predictions because they must deal with situations that are often random in character. It is therefore necessary to consider two types of models:

Types of Models

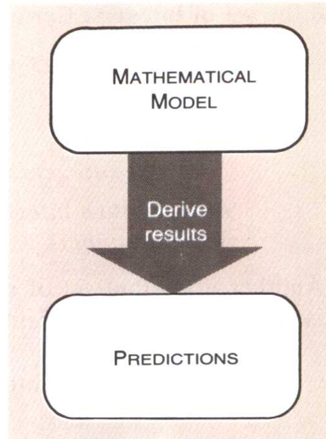
A **deterministic model** predicts the exact outcome of a situation because it is based on certain known laws.

A **probabilistic model** deals with situations that are random in character and can predict the outcome within a certain stated or known degree of accuracy.

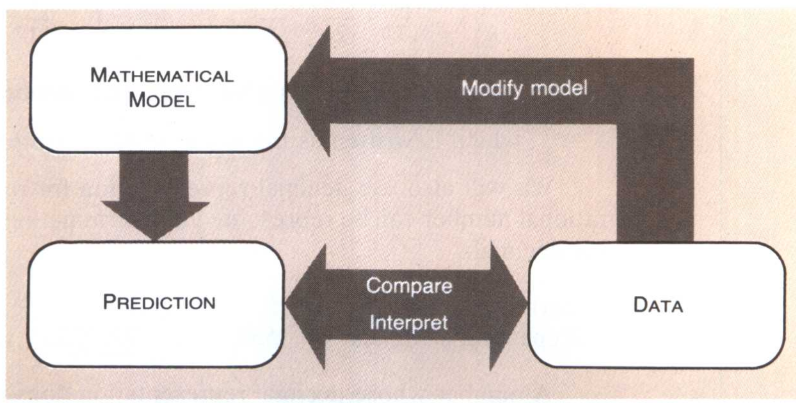
How can you go about constructing a model? This is what this book is all about. You need to observe a real-world problem and make assumptions about influencing factors. This is called *abstraction*.



You must know enough about the mechanics of mathematics to *derive results* from the model.



The next step is to gather data. Does the prediction given by the model fit all the known data? If not, you will use the data to *modify* the assumptions used to create the model. This is an ongoing process.



Thus, we must begin this course by learning some mathematics. It is assumed that you have had a previous course in algebra, but we will review many of the basic ideas here in order to refresh your memory.

Sets and Numbers

One of the most important fundamental ideas in mathematics is the idea of a **set.** The objects of a set S are called the *elements,* or *members,* of S. The set with no elements is called the *empty,* or *null,* set and is denoted by the symbol $\varnothing$. It is assumed that you are familiar with the set

$$\{. . ., -3, -2, -1, 0, 1, 2, 3, . . .\}$$

called the **set of integers.** Certain *subsets* of this set are

Natural numbers *Counting numbers* *Positive integers*	$\{1, 2, 3, \ldots\}$
Negative integers	$\{-1, -2, -3, \ldots\}$
Nonnegative integers *Whole numbers*	$\{0, 1, 2, 3, \ldots\}$

Another important set of numbers is the set of **rational numbers,** Q, consisting of all quotients of integers:

$$Q = \left\{\frac{p}{q} \,\middle|\, p \text{ is an integer and } q \text{ is a nonzero integer}\right\}$$

The above line is read "Q is the set of all $\frac{p}{q}$ such that p is an integer and q is a nonzero integer." The notation used is sometimes called **set-builder** notation. Examples of rational numbers are

$$\frac{2}{3},\quad \frac{-1}{2},\quad \frac{15}{4},\quad 5,\quad -2,\quad 0,\quad \frac{36}{11},\quad \ldots$$

Notice that each integer is also a rational number, since, for example, $5 = \frac{5}{1}$, which is written as the quotient of two integers.

We will also use decimal representation for rational numbers. Every rational number can be represented as a terminating decimal or as a repeating decimal:

Terminating decimals:	.5,	.75,	.006333,	. . .
Repeating decimals:	.555. . .,	.757575. . .,	.006333. . .,	. . .

A number whose decimal representation does not terminate or repeat is called an **irrational number.** Examples of irrational numbers you have probably seen are π, $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, $\sqrt{6}$, $\sqrt{7}$, $\sqrt{8}$, $\sqrt{10}$, ...

If we put together all the rational numbers and all the irrational numbers, the resulting set is called the set of **real numbers.** The real numbers can most easily be visualized by using a *one-dimensional coordinate system called the* **real number line** (Figure 1.1).

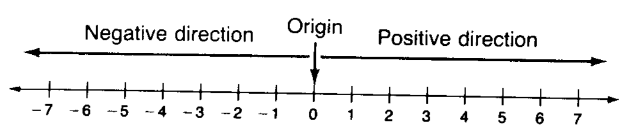


Figure 1.1 A real number line

A *one-to-one correspondence* is established between all real numbers and all points on a real number line:

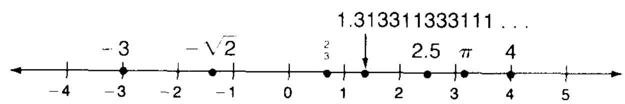
1. Every point on the line corresponds to precisely one real number.
2. Every real number corresponds to precisely one point.

A point associated with a particular number is called the **graph** of that number.

EXAMPLE 1 Graph the following numbers on a real number line:

$$4, \quad -3, \quad 2.5, \quad 1.313311333111\ldots, \quad \frac{2}{3}, \quad \pi, \quad -\sqrt{2}$$

Solution When graphing, the exact positions of the points are approximated.



Linear Relationships

There are certain relationships between real numbers with which you should be familiar.

Less Than $a < b$, read "a is less than b," means the graph of a is to the left of the graph of b.

Greater Than $a > b$, read "a is greater than b," means the graph of a is to the right of the graph of b.

Equal To $a = b$, read "a is equal to b," means that a and b represent the same point on the number line.

Less Than or Equal To $a \leq b$, read "a is less than or equal to b," means that either $a < b$ or $a = b$ (but not both).

Greater Than or Equal To $a \geq b$, read "a is greater than or equal to b," means that either $a > b$ or $a = b$ (but not both)

Between $b < a < c$, read "a is between b and c," means *both* $b < a$ and $a < c$. Additional "between" relationships are also used:

$$b \leq a \leq c \quad \text{means} \quad b \leq a \quad \text{and} \quad a \leq c$$
$$b \leq a < c \quad \text{means} \quad b \leq a \quad \text{and} \quad a < c$$
$$b < a \leq c \quad \text{means} \quad b < a \quad \text{and} \quad a \leq c$$

Graphs of inequality statements are drawn on a one-dimensional coordinate system. For example, $x < 7$ denotes the interval shown in Figure 1.2a. Notice in Figure 1.2a that $x \neq 7$, and this fact is shown by an open

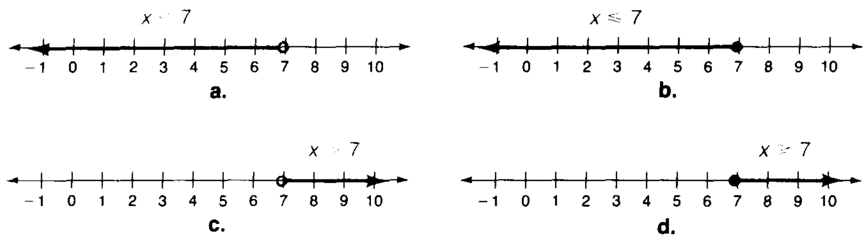


Figure 1.2 Graphs of inequality statements

circle as the end point of the graph. Compare this with $x \leq 7$ (Figure 1.2b) in which the end point $x = 7$ is included, as indicated by the solid dot. Notice that to sketch the graph, you darken (or color) the appropriate portion. Graphs for $x > 7$ and $x \geq 7$ are drawn in a similar way (Figures 1.2c and 1.2d).

Some Models

In the first part of this text we will focus on deterministic models for which the abstraction step has already been completed. Models are often stated in terms of formulas. A **formula** is an equation or other relationship given in mathematical symbols that can be applied to some specific situation. For example, the formula

$$d = 16t^2$$

is used to predict the distance an object will fall in a vacuum as a function of time. A **function** is a rule that assigns to each number of one set, called the **domain,** exactly one number from another set, called the **range.** In this example, for each nonnegative value of time (t) there is exactly one value for the distance (d) that the object falls. The variable t is called the **independent variable,** and d is called the **dependent variable.**

You will also notice that the formula specifying the model of a free-falling object uses an **exponent.** Recall the definition of exponent from algebra:

Exponential Notation

If b is any real number and n is a positive integer, then

$$b^n = \underbrace{b \cdot b \cdot b \cdot \cdots \cdot b}_{n \text{ factors}}$$

$$b^0 = 1 \qquad b \neq 0$$

$$b^{-n} = \frac{1}{b^n} \qquad b \neq 0$$