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Neutron Spectrum from ^{5}He Breakup Process in the Continuous Emission Region

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【abstract】 *An unstable particle emission of ^{5}He has being included in the new version of LUNF code, as the evaluation tool and for interpreting experimental data. In order to describe the ^{5}He emission process, the formula on the double-differential cross sections of the neutron and the alpha-particle from $^5He \to n+\alpha$ was studied. Because of stronger recoil effect, the energy balance is strictly taken into account to meet the needs in nuclear engineering.*

Introduction

The possibility and the importance of ^{5}He emission in the neutron-induced reactions are elaborated [1,2]. Now ^{5}He emission has being included in the new version of LUNF code[3]. There is some new reaction mechanisms on ^{5}He emission process, one of which is the formulation on the double -differential cross sections of the neutron and the α-particle from the emitted ^{5}He breakup process. The formula of the double-differential cross sections of the neutron and the alpha-particle from the emitted ^{5}He breakup process to the discrete levels was obtained in Ref[1], while the formula of the double-differential cross sections of the neutron and the α-particle from the emitted ^{5}He breakup process to the continuous region is obtained in Section 1.

Meanwhile, it is proved that the energy balance is held exactly and given in Section 2. The conclusion remarks are given in the last section.

1 The Double-differential Cross Sections of the Neutron and α-particle from ^{5}He Separation in Continuous Region

In pre-equilibrium emission process, the emitted ^{5}He has forward angular distribution, so the neutron and the α-particle from ^{5}He two-body breakup process also has the same situation.

In this section the representation of the double-differential cross sections of the neutron and α-particle from ^{5}He breakup process in continuous region is given. The emitted ^{5}He in its ground state is assumed in the study, although ^{5}He has excited states.

Two motion systems are employed to set up the formulation, the physical quantities indicated by the superscript c and p are for center of mass system (CMS) and for emitted particle system (EPS), which is moving along with the emitted particle ^{5}He, respectively. Meanwhile, the physical quantities indicated by subscript 5 are for the emitted ^{5}He. The physical quantities indicated by subscripts n or α are for the neutron or the α-particle, respectively, from ^{5}He separation.

The double-differential cross section of emitted ^{5}He with the mass m_5 in CMS is represented in the standard form as

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}E_5^c\mathrm{d}\Omega_5^c}=\sum_l\frac{2l+1}{4\pi}f_l^c(E_5^c)P_l(\cos\theta_5^c) \qquad (1)$$

with normalization condition $\int_{E_{5,\min}^c}^{E_{5,\max}^c} f_0^c(E_5^c)dE_5^c=1$ within energy region $E_{5,\min}^c\le E_5^c\le E_{5,\max}^c$. Then the emitted ^{5}He is separated into one neutron and one α-particle, spontaneously, with the Q-value of 0.894 MeV. The energy ε_n^p and ε_α^p carried by the outgoing neutron and the α-particle in EPS are given, respectively, by

$$\varepsilon_n^p=\frac{m_\alpha}{m_5}Q,\qquad \varepsilon_\alpha^p=\frac{m_n}{m_5}Q \qquad (2)$$

Based on the velocity composition relation, the velocities of the neutron and the α-particle in CMS can be obtained by $\vec{v}_n^c=\vec{V}_5+\vec{v}_n^p$ and $\vec{v}_\alpha^c=\vec{V}_5+\vec{v}_\alpha^p$, respectively. Therefore, the outgoing neutron energy region $\varepsilon_{n,\min}^c\le\varepsilon_n^c\le\varepsilon_{n,\max}^c$ is obtained by

$$\varepsilon^c_{n,\substack{\min\\\max}} = \frac{1}{2}m_n(v^p_n \mp V_{5,\substack{\min\\\max}})^2 = \varepsilon^p_n[1 \mp \gamma_n(E^c_{n,\substack{\min\\\max}})]^2 \quad (3)$$

With the same procedure, the energy region $\varepsilon^c_{\alpha,\min} \le \varepsilon^c_\alpha \le \varepsilon^c_{\alpha,\max}$ of the α-particle is obtained by

$$\varepsilon^c_{\alpha,\substack{\min\\\max}} = \frac{1}{2}m_\alpha(v^p_\alpha \mp V_{5,\substack{\min\\\max}})^2 = \varepsilon^p_\alpha[1 \mp \gamma_\alpha(E^c_{5,\substack{\min\\\max}})]^2 \quad (4)$$

The functions used in eqs.(3) and (4) are defined by

$$\gamma_n(E_5) = \sqrt{\frac{m_n E_5}{m_5 \varepsilon^p_n}}, \quad \gamma_\alpha(E_5) = \sqrt{\frac{m_\alpha E_5}{m_5 \varepsilon^p_\alpha}} \quad (5)$$

Based on the Jacobian relation, the solid angle in different motion system has the equation

$$\mathrm{d}\varepsilon^p_n \mathrm{d}\Omega^p_n = \sqrt{\frac{\varepsilon^c_n}{\varepsilon^p_n}}\mathrm{d}\varepsilon^c_n \mathrm{d}\Omega^c_n \quad (6)$$

and the double-differential cross section of the neutron is isotropic in EPS with definite value of energy

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}\varepsilon^p_n \mathrm{d}\Omega^p_n} = \frac{1}{4\pi}\frac{\mathrm{d}\sigma}{\mathrm{d}\varepsilon^p_n} = \frac{1}{4\pi}\delta\left(\varepsilon^p_n - \frac{m_\alpha}{m_5}Q\right) \quad (7)$$

The double-differential cross section of the emitted neutron can be obtained by averaging the double-differential cross section of emitted neutron over that of emitted ^{5}He.

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}\varepsilon^c_n \mathrm{d}\Omega^c_n} = \int \frac{\mathrm{d}^2\sigma}{\mathrm{d}E^c_5 \mathrm{d}\Omega^c_5}\frac{\mathrm{d}^2\sigma}{\mathrm{d}\varepsilon^p_n \mathrm{d}\Omega^p_n}\sqrt{\frac{\varepsilon^c_n}{\varepsilon^p_n}}\mathrm{d}E^c_5 \mathrm{d}\Omega^c_5 \quad (8)$$

The normalized double-differential cross section of the neutron with the mass m_n in CMS is represented in the standard form as

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}\varepsilon^c_n \mathrm{d}\Omega^c_n} = \sum_l \frac{2l+1}{4\pi} f^c_l(\varepsilon^c_n) P_l(\cos\theta^c_n) \quad (9)$$

Substituting the representation of eq.(6) and eq.(7) into eq.(8), and using the orthogonal relation of Legendre polynomial, the Legendre coefficient of the outgoing neutron in eq.(9) can be obtained by

$$f^c_l(\varepsilon^c_n) = \int \frac{\mathrm{d}^2\sigma}{\mathrm{d}E^c_5 d\Omega^c_5}\frac{\mathrm{d}^2\sigma}{\mathrm{d}\varepsilon^p_n \mathrm{d}\Omega^p_n}\sqrt{\frac{\varepsilon^c_n}{\varepsilon^p_n}}\mathrm{d}E^c_5 \mathrm{d}\Omega^c_5 P_l(\cos\theta^c_n)\mathrm{d}\Omega^c_n \quad (10)$$

Denoting Θ as the angle between $\vec{V}_5$ and $\vec{v}^c_n$, and using the useful expansion equation

$$P_L(\cos\theta^c_5) = \frac{4\pi}{2L+1}\sum_m Y^*_{Lm}(\Theta,\Phi)Y_{Lm}(\Omega^c_n) \quad (11)$$

In eq.(10) the solid angles have the explicit expression $\mathrm{d}\Omega^c_5 \mathrm{d}\Omega^c_n = \mathrm{d}\cos\Theta \mathrm{d}\Phi \mathrm{d}\cos\theta^c_n \mathrm{d}\phi^c_n$, and carrying out the integration over $\mathrm{d}\cos\theta^c_n \mathrm{d}\Phi \mathrm{d}\phi^c_n$, the Legendre coefficient of the outgoing neutron is obtained by

$$f^c_l(\varepsilon^c_n) = \frac{1}{2}\int f^c_l(E^c_5)\frac{\mathrm{d}\sigma}{\mathrm{d}\varepsilon^p_n}\sqrt{\frac{\varepsilon^c_n}{\varepsilon^p_n}}P_l(\cos\Theta)\mathrm{d}E^c_5 \mathrm{d}\cos\Theta \quad (12)$$

From the velocity relation $\vec{v}^p_n = \vec{v}^c_n - \vec{V}^c_5$, in energy scale, the energy relation reads

$$\varepsilon^p_n = \varepsilon^c_n + \frac{m_n}{m_5}E^c_5 - 2\sqrt{\frac{m_n}{m_5}\varepsilon^c_n E^c_5}\cos\Theta \quad (13)$$

For a given ε^c_n we have

$$\mathrm{d}\cos\Theta = \frac{1}{2}\sqrt{\frac{m_5}{m_n \varepsilon^c_n E^c_5}}\mathrm{d}\varepsilon^p_n \quad (14)$$

Substituting eq.(7) and eq.(14) into eq.(12), and carrying out the integration over ε^p_n, the eq.(12) is reduced into the form as the following

$$f^c_l(\varepsilon^c_n) = \int \frac{m_5}{4\sqrt{m_n m_\alpha Q E^c_5}} f^c_l(E^c_5) P_l\left(\frac{\varepsilon^c_n + \frac{m_n}{m_5}E^c_5 - \frac{m_\alpha}{m_5}Q}{2\sqrt{\frac{m_n}{m_5}\varepsilon^c_n E^c_5}}\right)\mathrm{d}E^c_5 \quad (15)$$

Using $\gamma_n(E^c_5) = \sqrt{\frac{m_n E^c_5}{m_\alpha Q}} = \sqrt{\frac{m_n E^c_5}{m_5 \varepsilon^p_n}}$, the variable in Legendre polynomial of eq.(15) can be denoted by

$$\eta_n = \sqrt{\frac{\varepsilon^p_n}{\varepsilon^c_n}}\frac{\frac{\varepsilon^c_n}{\varepsilon^p_n} - 1 + \gamma^2_n(E^c_5)}{2\gamma_n(E^c_5)} \quad (16)$$

Hence eq.(15) becomes into the form as

$$f^c_l(\varepsilon^c_n) = \int_a^b \frac{f^c_l(E^c_5)}{4\varepsilon^p_n \gamma_n(E^c_5)}P_l(\eta_n)\mathrm{d}E^c_5 \quad (17)$$

For a given value of the outgoing neutron ε^c_n, the integration limits of E^c_5 can be given by

$$a = \max\left\{E_{5,\min}, \frac{m_5}{m_n}\left(\sqrt{\varepsilon^c_n} - \sqrt{\varepsilon^p_n}\right)^2\right\} \quad (18)$$

$$b = \min\left\{E_{5,\max}, \frac{m_5}{m_n}\left(\sqrt{\varepsilon^c_n} + \sqrt{\varepsilon^p_n}\right)^2\right\} \quad (19)$$

Obviously, from eq.(17) it can be seen that the forward tendency of the outgoing neutron is entirely determined by the forward tendency of ^{5}He emission.

With the same procedure, the Legendre coefficient of the outgoing α-particle can be obtained by exchanging m_n and m_α in the above formula.

2 Energy Balance

To meet the needs in engineering, the energy balance should be satisfied properly. In this section the energies carried by all kinds of particles, including the energies carried by the neutron and the α-particle from the ^{5}He breakup process, the energy carried by the residual nucleus as well as the gamma decay energy, are given analytically.

The energy carried by the neutron in CMS reads

$$E_n^c = \int \varepsilon_n^c f_0^c(\varepsilon_n^c) \mathrm{d}\varepsilon_n^c \tag{20}$$

Substituting the representation of eq.(16), and exchanging the integration order, then the integration region of ε_n^c for a given E_5^c is $\left(\sqrt{\varepsilon_n^p} - \sqrt{\frac{m_n}{m_5}E_5^c}\right)^2 \le \varepsilon_n^c \le \left(\sqrt{\varepsilon_n^p} + \sqrt{\frac{m_n}{m_5}E_5^c}\right)^2$. Therefore,

$$\begin{aligned} E_n^c &= \int_{E_{5,\min}^c}^{E_{5,\max}^c} \mathrm{d}E_5^c f_0^c(E_5^c)\left(\varepsilon_n^p + \frac{m_n}{m_5}E_5^c\right) \\ &= \varepsilon_n^p + \frac{m_n}{m_5}\int_{E_{5,\min}^c}^{E_{5,\max}^c} E_5^c f_0^c(E_5^c)\mathrm{d}E_5^c \end{aligned} \tag{21}$$

Analogously, The energy carried by the α-particle in CMS reads

$$E_\alpha^c = \varepsilon_\alpha^p + \frac{m_\alpha}{m_5}\int_{E_{5,\min}^c}^{E_{5,\max}^c} E_5^c f_0^c(E_5^c)\mathrm{d}E_5^c \tag{22}$$

The energy carried by residual nucleus after ^{5}He emission in CMS is given by

$$E_R^c = \frac{m_5}{M}\int_{E_{5,\min}^c}^{E_{5,\max}^c} E_5^c f_0^c(E_5^c)\mathrm{d}E_5^c \tag{23}$$

The residual excitation energy after ^{5}He emission is given by

$$E_R^* = E^* - B_5 - \left(1 + \frac{m_5}{M}\right)E_5^c \tag{24}$$

Averaged by the double-differential cross section of ^{5}He emission, if the reaction channel is ended by gamma decay, the gamma decay energy can be obtained by.

$$E_\gamma = E^* - B_5 - \left(1 + \frac{m_5}{M}\right)\int_{E_{5,\min}^c}^{E_{5,\max}^c} E_5^c f_0^c(E_5^c)\mathrm{d}E_5^c \tag{25}$$

where $E^* = \frac{M}{M_C}E_n + B_n$ is the excitation energy, E_n is the incident neutron energy, B_n , B_5 are the binding energy of neutron and emitted ^{5}He in the compound nucleus, respectively, M_C , M are the masses of compound nucleus and the residual nucleus, respectively,

Thus, the total released energy in CMS is given by

$$E_{total}^c = E_n^c + E_\alpha^c + E_R^c + E_\gamma = E^* - B_5 + Q \tag{26}$$

By means of the composition of velocities $\vec{v}' = \vec{v}^c + \vec{V}_C$, where $\vec{V}_C$ is the velocity of the center of mass, and $V_C = \frac{\sqrt{2m_nE_n}}{M_C}$. So the energy of a particle with mass m in LS can be obtained by

$$E_m^l = \frac{m}{2}\int (v_m^l)^2 \frac{\mathrm{d}^2\sigma}{\mathrm{d}\varepsilon_m^c \mathrm{d}\Omega_m^c}\mathrm{d}\varepsilon_m^c \mathrm{d}\Omega_m^c \tag{27}$$

In laboratory system, the energy carried by the neutron can be obtained by

$$\begin{aligned} E_n^l = {} & \frac{m_n^2}{M_C^2}E_n + \int \varepsilon_n^c f_0^c(\varepsilon_n^c)\mathrm{d}\varepsilon_n^c + \\ & \frac{2m_n}{M_C}\int \sqrt{E_n \varepsilon_n^c} f_1^c(\varepsilon_n^c)\mathrm{d}\varepsilon_n^c \end{aligned} \tag{28}$$

Substituting the representation of the Legendre coefficients of eq.(17) in eq.(28), and carrying the integration over ε_n^c by exchanging the integration order, the eq.(28) is reduced into the form as follows

$$\begin{aligned} E_n^l = {} & \frac{m_n^2}{M_C^2}E_n + E_n^c + \\ & \frac{2m_n}{M_C}\sqrt{\frac{m_n}{m_5}E_n}\int_{E_{5,\min}^c}^{E_{5,\max}^c} \sqrt{E_5^c} f_1^c(E_5^c)\mathrm{d}E_5^c \end{aligned} \tag{29}$$

and for α-particle we have

$$\begin{aligned} E_\alpha^l = {} & \frac{m_n m_\alpha}{M_C^2}E_n + E_\alpha^c + \\ & \frac{2m_\alpha}{M_C}\sqrt{\frac{m_n}{m_5}E_n}\int_{E_{5,\min}^c}^{E_{5,\max}^c} \sqrt{E_5^c} f_1^c(E_5^c)\mathrm{d}E_5^c \end{aligned} \tag{30}$$

The derivation procedure of eqs.(29) and (30) is straightforward but tedious. Similarly, the energy carried by the recoil residual nucleus is obtained by

$$E_R^l = \frac{m_n M}{M_C^2}E_n + E_R^c -$$

$$\frac{2m_5}{M_C}\sqrt{\frac{m_n}{m_5}E_n}\int_{E^c_{5,\min}}^{E^c_{5,\max}}\sqrt{E^c_5}f^c_1(E^c_5)\mathrm{d}E^c_5 \qquad (31)$$

Thus, the total released energy in laboratory system is obtained by

$$\begin{aligned}E^l_{total} &= E^l_n + E^l_\alpha + E^l_R + E_\gamma \\ &= \frac{m_n}{M_C}E_n + E^c_{total} = E_n + B_n - B_5 + Q\end{aligned} \qquad (32)$$

It is proved that the energy balance is held exactly in the analytical form. From the eqs.(29~31) it can be found that the energies carried by the neutron and the α-particle increase with the increasing of the forward tendency of emitted ^{5}He. The partial wave $l=1$ plays an important role in the energies carried by different kinds of emitted particles. The larger of $f^c_1(E^c_5)$, the more energies carried by the neutron and the α-particle. Meanwhile, the spectrum shape of $f^c_0(E^c_5)$ can also influence the energy distributions between the emitted particles, the residual nucleus and de-excitation γ energy. The harder of the spectrum, the more energies carried by the emitted particles, while the energies carried by the residual nucleus and de-excitation γ particle are reduced.

3 Conclusion Remarks

The formulation of the neutron and α-particle from ^{5}He separation in the continuous emission region has been obtained. Together with that in discrete level emissions [2], the ^{5}He emission process can be described with full energy balance. The energy balance is not only to meet the needs in nuclear engineering, but also could give the reasonable shape of the outgoing particles spectra. So far only the stable clusters, like d, t, ^{3}He, and α, have been included in the present widely used statistical model codes, now the ^{5}He emission, as an unstable cluster emission is added in the new code. It will be able to make the improvement to reproduce the experimental data.

^{5}He emission is a new reaction mechanism in both equilibrium and pre-equilibrium emission processes. The formula aforementioned will be employed in the new version of LUNF code.

References

[1] ZHANG Jingshang, CNDP No: 28 (2002) 3.

[2] ZHANG Jingshang, Science in China G 47 (2004) 137-145.

[3] ZHANG Jingshang, UNF code for Fast Neutron Reaction Data Calculations, Nucl. Sci.Eng., 133 (1999) 218.

Pre-Formation Probability of ^{5}He Cluster in Pre-Equilibrium Mechanism

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【abstract】 *The ^{5}He cluster is apt to be emitted than ^{3}He from point of view on the threshold energy. In terms of Iwamoto-Harada model, the theoretical formula of pre-formation probability of ^{5}He cluster including 1p shell nucleon in pre-equilibrium mechanism has been established and calculated. In the case of low incident energies, the configuration of [1,m] for ^{5}He cluster is the dominant part in the nuclear reaction. The calculated results indicate that pre-formation probability of configuration [1,m] for the unstable ^{5}He cluster is much smaller than that of d,t,^{3}He, and ^{4}He light stable composite particles, which is consisted of only 1s shell nucleons. However, it is propitious to the emission of ^{5}He from the point of view on threshold energies, since the binding energies of ^{5}He are generally lower than that of ^{3}He in compound system. The corresponding model formula has been given in this paper for describing pre-formation probability of ^{5}He cluster in pre-equilibrium mechanism.*

Introduction

In recent years, the total outgoing neutron double -differential cross sections for neutron induced light nuclear reaction have been calculated by using the unitive Haser-feshbach theory and exciton model, which can reproduce the experimental measured data nicely[1~4]. The total outgoing neutron double -differential cross sections consist of the neutrons emitted from various reaction channels to discrete level of residual nucleus, which strongly differ from each other. It is shown that ^{5}He emission should be taken into account for fitting the experimental data. Otherwise, it will appear obvious deficiency at low energy part of neutron emission angle-energy spectra, which can be reparative when ^{5}He emission channel has been considered[5].

In addition, it is shown that the composite particle emissions mainly come from the pre-equilibrium reaction processes in light nuclear reaction. The angular momentum dependent exciton model is able to describe the pre-equilibrium reaction successfully[6]. In the exciton model the pre -formation probabilities of all kinds of composite particles in compound system are the important factors to describe composite particle emissions, which should be given by theory. The Iwamoto -Harada model has been proposed to give the pre-formation probabilities of d, t, ^{3}He, and ^{4}He reasonably, which only include 1s shell nucleons for pre-equilibrium emission mechanism[7,8]. The basic physical idea of this model is that the pick-up mechanism in pre-equilibrium processes of composite particles emission is employed, namely, the emitted single nucleon picks up other nucleons to form composite particle before emitting. The nucleon inside nucleus can be described by wave function of shell model, and the associate condition of momentum and position of every degree of freedom can be given by the shell model. Then the pre -formation probability of each kind of configuration for a composite particle could be obtained by means of the volume of the phase space occupied by the composite particles.

Based on the idea of Iwamoto-Harada model, and extending to the composite particle of ^{5}He cluster including 1p shell nucleon, the formulation of the pre-formation probability in compound nucleus for ^{5}He has been established and calculated. Compared with the results of pre-formation probabilities for other composite particles, the calculated results for ^{5}He cluster is reasonable. The theoretical formula of pre-formation probability for ^{5}He cluster are given in section II. The calculated results and discussion are given in the last section.

1 The Formula of Pre-formation Probability for ^{5}He Cluster

The model has been used to calculate pre-formation probability of light composite particles, which only consist of 1s state nucleons described by using the harmonic oscillator model[9,10]. The coordinate $\vec{R}$ of the center of mass for a cluster including N nucleons is given by

$$\vec{R} = \frac{1}{N}\sum_{i=1}^{N}\vec{r}_i \tag{1}$$

where $\vec{r}_i$ stands for the coordinate of the i_{th} nucleon. The total wave function of composite particle is constructed by

$$\psi = \prod_{i=1}^{N}\phi(r_i) = \Phi(R)\varphi_{\text{int}} \tag{2}$$

It can be divided into two parts, $\Phi(R)$ is movement wave function of the motion of the center of mass, and φ_{int} is intrinsic wave function. The normalized $\Phi(R)$ and the single particle wave function for 1s shell nucleons are given by the harmonic oscillator model as follows

$$\Phi(R) = \left(\frac{N\beta}{\pi}\right)^{3/4} e^{-\frac{\beta}{2}NR^2} \tag{3}$$

$$\varphi(\vec{r}_i) = \left(\frac{\beta}{\pi}\right)^{3/4} e^{-\frac{\beta}{2}r_i^2} \tag{4}$$

The configuration of composite particle with N nucleons is denoted by $[\lambda,m]$, namely, λ and m are the particle numbers above and below the Fermi surface respectively, and $\lambda + m = N$. The parameter β in the wave functions can be determined by harmonic oscillator parameter $\hbar\omega$.

Thus the conditions of the momentum for each nucleon is given by

$$\left.\begin{array}{ll} |\vec{p}_i| > p_f & i = 1,2,\cdots,\lambda \\ |\vec{p}_j| < p_f & j = 1,2,\cdots,m \end{array}\right\} \tag{5}$$

The intrinsic relative coordinates of ^{5}He cluster can be defined as follows

$$\left.\begin{array}{l} \vec{r} = \vec{r}_1 - \vec{r}_2 \\ \vec{r}\,' = \vec{r}_3 - \vec{r}_4 \\ \vec{r}\,'' = \frac{1}{2}(\vec{r}_1 + \vec{r}_2) - \frac{1}{2}(\vec{r}_3 + \vec{r}_4) \\ \vec{r}\,''' = \frac{1}{4}(\vec{r}_1 + \vec{r}_2 + \vec{r}_3 + \vec{r}_4) - \vec{r}_5 \\ d\vec{r}_1 d\vec{r}_2 d\vec{r}_3 d\vec{r}_4 d\vec{r}_5 = d\vec{R}d\vec{r}d\vec{r}\,'d\vec{r}\,''d\vec{r}\,''' \end{array}\right\} \tag{6}$$

where the $\vec{r},\vec{r}\,',\vec{r}\,''$ are the intrinsic relative coordinates of an α cluster, while the fourth relative coordinates $\vec{r}\,'''$ stand for the intrinsic coordinate between the fifth nucleon and the α cluster in ^{5}He nucleus. Hence the corresponding intrinsic momentum coordinates can be obtained as the following

$$\left.\begin{array}{l} \vec{p}_r = \frac{1}{2}(\vec{p}_1 - \vec{p}_2) \\ \vec{p}_{r'} = \frac{1}{2}(\vec{p}_3 - \vec{p}_4) \\ \vec{p}_{r''} = \frac{1}{2}(\vec{p}_1 + \vec{p}_2) - \frac{1}{2}(\vec{p}_3 + \vec{p}_4) \\ \vec{p}_{r'''} = \frac{1}{5}(\vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \vec{p}_4) - \frac{4}{5}\vec{p}_5 \\ d\vec{p}_1 d\vec{p}_2 d\vec{p}_3 d\vec{p}_4 d\vec{p}_5 = d\vec{p}d\vec{p}_r d\vec{p}_{r'} d\vec{p}_{r''} d\vec{p}_{r'''} \end{array}\right\} \tag{7}$$

The momentum of ^{5}He cluster is given by

$$\vec{p} = \sum_{i=1}^{5}\vec{p}_i \tag{8}$$

The construction of ^{5}He cluster is different from d,t,^{3}He and α. The latter only include 1s shell nucleons. While for ^{5}He, besides the 1s shell nucleons the 1p shell nucleon is involved, in which the former forth nucleons are described by 1s shell wave function and the fifth nucleon is described by 1p shell wave function as

$$\varphi_{\text{int}} = R_{01}(r)Y_{1m}(\Omega) \tag{9}$$

where $R_{01}(r)$ stands for normalized radial wave function of 1p shell nucleon.

$$R_{01} = \beta\left(\frac{64\beta}{9\pi}\right)^{\frac{1}{4}} e^{-\frac{1}{2}\beta r^2} \tag{10}$$

here $Y_{1m}(\Omega)$ is spherical harmonic function.

According to the knowledge of the shell model, for each intrinsic degree of freedom the occupied energy is $\frac{3}{2}\hbar\omega$ for 1s shell nucleons, but it is $\frac{5}{2}\hbar\omega$ for 1p shell nucleons inside ^{5}He cluster. The kinetic energy and potential energy for each degree of freedom satisfy the equation obtained by the shell model as

$$\begin{aligned} &\frac{p_r^2}{m}+\frac{1}{4}m\omega_5^2 r^2=\frac{3}{2}\hbar\omega_5 \\ &\frac{p_{r'}^2}{m}+\frac{1}{4}m\omega_5^2 r'^2=\frac{3}{2}\hbar\omega_5 \\ &\frac{p_{r''}^2}{2m}+\frac{1}{2}m\omega_5^2 r''^2=\frac{3}{2}\hbar\omega_5 \\ &\frac{5p_{r'''}^2}{8m}+\frac{2}{5}m\omega_5^2 r'''^2=\frac{5}{2}\hbar\omega_5 \end{aligned} \tag{11}$$

Based on the physical idea of Iwamoto-Harada model, the volume in phase space occupied by the ^{5}He cluster with the configuration $[\lambda,m]$ constrained by conditions of eq.(11) is given in the following form

$$F_{[\lambda,m]}(\varepsilon_5) = \frac{C}{(2\pi\hbar)^{12}} \int_{P-fixed[\lambda,m]} d\vec{r}d\vec{r}'d\vec{r}''d\vec{r}'''d\vec{p}_r d\vec{p}_{r'} d\vec{p}_{r''} d\vec{p}_{r'''} \tag{12}$$

The normalized coefficient C in eq.(12) is determined with the normalized condition

$$\sum_{lm} F_{lm}(\varepsilon_5)=1 \tag{13}$$

Since the former three intrinsic relative coordinates and relative momentum are designed as the coordinates of the α cluster. The integrated analytical results were obtained in reference [8], which can be employed in eq.(12). By using the last condition in eq.(11), and carrying out the integration over the relative coordinate $\vec{r}'''$ and the relative coordinate $\vec{p}_{r'''}$ with the constrain condition of the configuration $[\lambda,m]$, the integrated analytical expression of configuration [5,0], [4,1], [3,2], [2,3], [1,4], [0,5] can be obtained.

Denoting

$$B=\left[\frac{1}{25}p^2+p_{r'''}^2-p_f^2\right]\Big/\frac{2}{5}pp_{r'''} \tag{14}$$

here p_f is the Fermi momentum.

The expression of relative momentum coordinates between the fifth nucleon and the α cluster by using eq.(11) for a given total momentum $\vec{p}$ of ^{5}He is obtained by

$$\vec{p}_5=\frac{1}{5}\vec{p}+\vec{p}_{r'''} \tag{15}$$

Based on the conditions mentioned above, the representation of the pre-formation probability of ^{5}He cluster can be obtained with two mathematical cases.

(1) If $p<5p_f$ we have

$$\begin{aligned}
F_{50}(p) &= \int_{p_f-\frac{1}{5}p}^{\sqrt{5\mu\hbar\omega_5}} dp'T(p')\int_{-1}^{B} d\cos\beta F_{40}(\vec{p}) \\
F_{41}(p) &= \int_0^{p_f-\frac{1}{5}p} dp'T(p')\int_{-1}^{1} d\cos\beta F_{40}(\vec{p}) + \int_{p_f-\frac{1}{5}p}^{\sqrt{5\mu\hbar\omega_5}} dp'T(p')\left[\int_{-1}^{B} d\cos\beta F_{31}(\vec{p}) + \int_{B}^{1} d\cos\beta F_{40}(\vec{p})\right] \\
F_{32}(p) &= \int_0^{p_f-\frac{1}{5}p} dp'T(p')\int_{-1}^{1} d\cos\beta F_{31}(\vec{p}) + \int_{p_f-\frac{1}{5}p}^{\sqrt{5\mu\hbar\omega_5}} dp'T(p')\left[\int_{-1}^{B} d\cos\beta F_{22}(\vec{p}) + \int_{B}^{1} d\cos\beta F_{31}(\vec{p})\right] \\
F_{23}(p) &= \int_0^{p_f-\frac{1}{5}p} dp'T(p')\int_{-1}^{1} d\cos\beta F_{22}(\vec{p}) + \int_{p_f-\frac{1}{5}p}^{\sqrt{5\mu\hbar\omega_5}} dp'T(p')\left[\int_{-1}^{B} d\cos\beta F_{13}(\vec{p}) + \int_{B}^{1} d\cos\beta F_{22}(\vec{p})\right] \\
F_{14}(p) &= \int_0^{p_f-\frac{1}{5}p} dp'T(p')\int_{-1}^{1} d\cos\beta F_{13}(\vec{p}) + \int_{p_f-\frac{1}{5}p}^{\sqrt{5\mu\hbar\omega_5}} dp'T(p')\left[\int_{-1}^{B} d\cos\beta F_{04}(\vec{p}) + \int_{B}^{1} d\cos\beta F_{13}(\vec{p})\right] \\
F_{05}(p) &= \int_0^{p_f-\frac{1}{5}p} dp'T(p')\int_{-1}^{1} d\cos\beta F_{04}(\vec{p}) + \int_{p_f-\frac{1}{5}p}^{\sqrt{5\mu\hbar\omega_5}} dp'T(p')\left[\int_{B}^{1} d\cos\beta F_{04}(\vec{p})\right]
\end{aligned} \tag{16}$$

(2) If $p > 5p_f$ we have

$$F_{50}(p) = \int_0^{\frac{1}{5}p-p_f} \mathrm{d}p' T(p') \int_{-1}^{1} \mathrm{d}\cos\beta F_{40}(\bar{p}) + \int_{\frac{1}{5}p-p_f}^{\sqrt{5\mu\hbar\omega_5}} \mathrm{d}p' T(p') \int_{-1}^{B} \mathrm{d}\cos\beta F_{40}(\bar{p})$$

$$F_{41}(p) = \int_0^{\frac{1}{5}p-p_f} \mathrm{d}p' T(p') \int_{-1}^{1} \mathrm{d}\cos\beta F_{31}(\bar{p}) + \int_{\frac{1}{5}p-p_f}^{\sqrt{5\mu\hbar\omega_5}} \mathrm{d}p' T(p') \left[\int_{-1}^{B} \mathrm{d}\cos\beta F_{31}(\bar{p}) + \int_{B}^{1} \mathrm{d}\cos\beta F_{40}(\bar{p}) \right]$$

$$F_{23}(p) = \int_0^{\frac{1}{5}p-p_f} \mathrm{d}p' T(p') \int_{-1}^{1} \mathrm{d}\cos\beta F_{13}(\bar{p}) + \int_{\frac{1}{5}p-p_f}^{\sqrt{5\mu\hbar\omega_5}} \mathrm{d}p' T(p') \left[\int_{-1}^{B} \mathrm{d}\cos\beta F_{13}(\bar{p}) + \int_{B}^{1} \mathrm{d}\cos\beta F_{22}(\bar{p}) \right]$$

$$F_{14}(p) = \int_0^{\frac{1}{5}p-p_f} \mathrm{d}p' T(p') \int_{-1}^{1} \mathrm{d}\cos\beta F_{04}(\bar{p}) + \int_{\frac{1}{5}p-p_f}^{\sqrt{5\mu\hbar\omega_5}} \mathrm{d}p' T(p') \left[\int_{-1}^{B} \mathrm{d}\cos\beta F_{04}(\bar{p}) + \int_{B}^{1} \mathrm{d}\cos\beta F_{13}(\bar{p}) \right]$$

$$F_{05}(p) = \int_{\frac{1}{5}p-p_f}^{\sqrt{5\mu\hbar\omega_5}} \mathrm{d}p' T(p') \int_{B}^{1} \mathrm{d}\cos\beta F_{04}(\bar{p}) \tag{17}$$

where $\mu = \frac{4}{5}m$ is the reduced mass between the fifth nucleon and the α cluster, and

$$T(p) = \frac{16}{27\pi(\mu\hbar\omega_5)^3}(3\mu\hbar\omega_5 - p^2)^{\frac{3}{2}} p^2 \tag{18}$$

Summing over all of the configurations, one can prove that the normalization condition of ^{5}He cluster is held analytically.

2 The Calculated Results and Discussion

Based on the idea of Iwamoto-Harada model, the formulation of the pre-formation probabilities of ^{5}He cluster with various configurations are established and calculated. The results are shown in Fig.1.

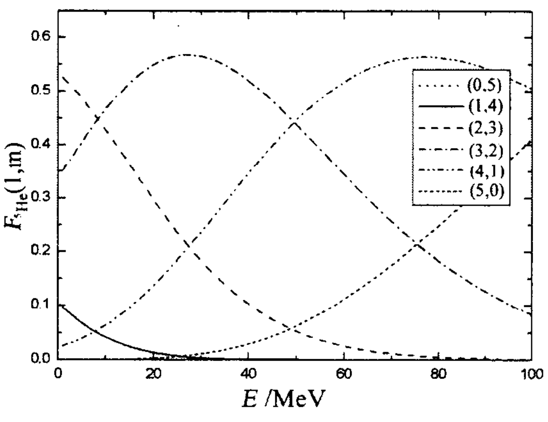

Fig.1 Pre-formation probability of ^{5}He cluster

where ε_b is the observational energy of outgoing composite particle and B_b is the binding energy.

From Fig.1, the pre-formation probability of [2,3], [3,2], [4,1], [5,0] are larger than that of [1,4] with the observational energy of outgoing composite particle increasing. But when incident neutron energies are below 20 MeV, the configuration of [λ=1,m] is the dominant part, and other configurations can be ignored. On the other hand, in the emission rate the two terms are always occurred simultaneous, then the following inequality

$$\begin{aligned} &F_{b[\lambda=1,m]}(\varepsilon)Q_{b[\lambda=1,m]}\omega(n-1,E') \\ &> F_{b[\lambda=2,m]}(\varepsilon)Q_{b[\lambda=2,m]}\omega(n-2,E') \end{aligned} \tag{19}$$

is always held, because the exciton state densities reduce rapidly with the decreasing of the exciton number. This is the reason that the configuration of [λ=1,m] is only considered in low energy nuclear reaction.

The comparison of pre-formation probabilities for configuration of [λ=1,m] in composite particle emissions, such as d, t, ^{3}He, α, as well as ^{5}He cluster is shown in Fig.2.

From Fig.2 one can see that the formation probability of configuration of [λ=1,m] for ^{5}He cluster is obviously less than that of d, t, ^{3}He and α.

The calculated results indicate that the pre-formation probability of unstable ^{5}He cluster is much less than that of the stable composite particle clusters. In general, the binding energies of ^{5}He are smaller than that of ^{3}He clearly[5], and the ^{5}He cluster is apt to be emitted than ^{3}He. However, combining with optical model potential, the values of cross section for ^{5}He cluster emission should be taken into account synthetically. The theoretical model formulation for describing the pre-equilibrium emission of ^{5}He has been given in this paper, which can be used in the statistical model calculations for neutron induced reactions.

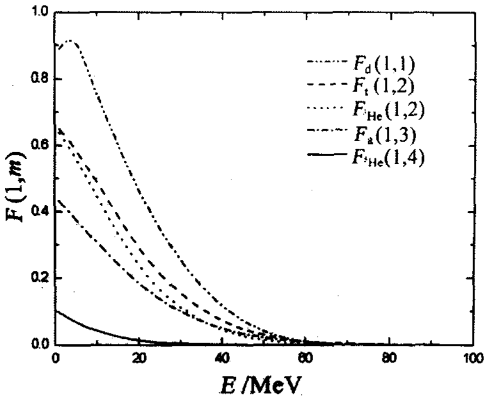

Fig.2 Comparison of pre-formation probabilities with configuration [1,m] for d,t,^{3}He,α,^{5}He

References

[1] ZHANG J. S., HAN Y. L., Cao L. G. "Model Calculation of n+^{12}C Reactions from 4.8 to 20 MeV". Nucl. Sci. Eng., 133 (1999) 218-234.

[2] ZHANG J. S., HAN Y. L., Fan X. L., "Theoretical analysis of the neutron double differential cross section of n+^{16}O at E_n=14.1 MeV". Commun. Theor. Phys. (Beijing, China), 35 (2001) 579-584.

[3] ZHANG J. S., "Theoretical analysis of the neutron double differential cross section of n+^{11}B at E_n=14.2 MeV". Commun. Theor. Phys. (Beijing, China), 39 (2003) 83-88.

[4] ZHANG J. S., "Theoretical analysis of the neutron double differential cross section of n+^{10}B at E_n=14.2 MeV". Commun. Theor. Phys. (Beijing, China), 39 (2003) 433-438.

[5] ZHANG J. S., "Possibility of ^{5}He emission in Neutron induced Reactions" Science in China 47-2 (2004) (to be published)

[6] ZHANG J. S., "UNF Code for Fast Neutron Reaction Data Calculations", Nucl Sci Eng, 142 (2002) 207-219

[7] Iwamoto A. and Harada K.,"Mechanism of cluster emission in ucleon-induced pre-equilibrium reactions". Phys. Rev., C26, (1982) 1821.

[8] Sato. K., Iwamoto A. and Harada K., "Pre-equilibrium emission of light omposite particles in the framework of the exciton model". Phys. Rev., C28, (1983) 1527.

[9] ZHANG J. S., YAN S. W., Wang C. L., "The Pick-up Mechanism in Composite Particle Emission Processes". Z. Phys. A 344 (1993) 251-258.

[10] ZHANG J. S., WEN Y. Q., Wang S. N., Shi X. J., "Formation and emission of light particles in fast neutron induced reaction-a united compound pre-equilibrium model". Commun. Theor. Phys (Beijing, China). 10 (1988) 33-44.

Double-Differential Cross Section of ^{5}He Emission

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【abstract】 *The probability of ^{5}He particle emission has been affirmed theoretically. In order to describe the ^{5}He emission, the theoretical formula of the double-differential cross section of emitted ^{5}He is to be established. Based on the pick-up mechanism, used for calculating the formula of d,t,^{3}He, α emissions, the theoretical formula of double-differential cross section of ^{5}He is obtained, which is expressed in the form of Legendre coefficients. The calculated result indicates that the forward peaked angular distribution of the composite particle emission is weaker than that of the emitted single nucleon due to pick-up nucleon from the Fermi sea. As an example, the reactions of n+^{14}N weren calculated, and the Legendre coefficients of d,t,^{3}He, α,^{5}He emissions were obtained respectively. The results show that the forward tendency is decided by the average momentum per nucleon in the emitted composite particles. The larger of the average momentum, the stronger of the forward tendency.*

Introduction

In recent years, the total outgoing neutron double-differential cross section of neutron induced light nuclear reaction has been calculated by using a new nuclear reaction model. In this model the ^{5}He emission is involved[1]. When the ^{5}He emission is taken into account, the calculated result can reproduce the experimental data successfully[2~5].

There are very few measurements on the double-differential cross section of composite particle emissions at present. Internationality, the Kalbach systematic formula[6] has been employed to set up the neutron data libraries. This formula is very useful for the middle and heavy nuclei, while it's imperfect to the 1p shell light nuclei. The model for describing the double-differential cross section of composite particle emissions of d, t, ^{3}He, α emissions has been proposed, which can reproduce the double-differential measurements of deuterium and alpha particle emissions nicely[7,8], and has been employed in the UNF code[9]. Now, extending this method for ^{5}He emission, the theoretical formula of the double-differential cross section of ^{5}He emission is established.

The composite particle emissions are mainly from pre-equilibrium reaction processes, which could be described successfully by the angular momentum dependent exciton model[9]. The pre-formation probability of ^{5}He in pre-equilibrium mechanism has been obtained[10]. The method for calculating the angular factor of ^{5}He emission in pre-equilibrium reaction processes is introduced in Sec.2, and the theoretical formulae of double-differential cross section of composite particle emission are given. As an example, the angular factor of d, t, ^{3}He, α, as well as ^{5}He of n+^{14}N reaction are calculated at incident neutron energies of 15, 20 MeV, respectively, meanwhile the angular distribution of single nucleon, α and ^{5}He particle are also calculated, the results and discussion are given in Sec.2. The summary is given in Sec.3.

1 The Angular Factor of ^{5}He in Pre-equilibrium Emission Process

The basic idea of the model for describing the double-differential cross section of composite particle emission is as follows:

(1) If the outgoing single nucleon picks up other nucleons in compound nucleus formed a composite particle during the emission process, then the composite particle emission would be occurred, otherwise only single particle emission would be happen.

(2) The double-differential cross section of single particle emission is obtained by the linear momentum dependent exciton model[11,12], in which the Fermi motion and the Pauli exclusion effects are taken into account.

(3) The Fermi gas model is employed for the compound nucleus system.

(4) The momentum of the composite particle is the summation over the outgoing single nucleon and the picked-up nucleons with the momentum conserving.

(5) The pre-formation probability is determined by the pick-up mechanism.

Based on the idea mentioned above, the normalized angular factor of composite particle b at the exciton state n with the emitted energy ε and the outgoing direction Ω is proposed by [9,13]

$$A(n,\varepsilon,\Omega)=\frac{1}{N}\int_{[lm]}\mathrm{d}\bar{p}_1\cdots\mathrm{d}\bar{p}_{A_b}\delta(\bar{P}-\sum_{i=1}^{A_b}\bar{p}_i)\tau(n,\Omega_1) \quad (1)$$

where N is the normalization factor, A_b is mass number of the composite particle.

For ^{5}He, the angular factor reads

$$A(n,\varepsilon,\Omega)=\frac{1}{N}\int_{[lm]}\mathrm{d}\bar{p}_1\mathrm{d}\bar{p}_2\mathrm{d}\bar{p}_3\mathrm{d}\bar{p}_4\mathrm{d}\bar{p}_5\delta(\bar{P}-\bar{p}_1-\bar{p}_2-\bar{p}_3-\bar{p}_4-\bar{p}_5)\tau(n,\Omega_1) \quad (2)$$

where $\bar{P}$ is the momentum of ^{5}He, $\bar{p}_i$, i=1, 2, 3, 4, 5 are the momenta of the nucleons formed ^{5}He, and the outgoing single nucleon is denoted by i=1. $\tau(n,\Omega_1)$ stands for the lifetime of the n exciton state with the direction Ω_1. The δ function in eq.(2) implies the momentum conservation.

In the case of low incident energies, the configuration [1,m] is the dominant part in the pre-equilibrium emission mechanism[9,10], which means that the outgoing single nucleon picked up four nucleons below the Fermi surface for ^{5}He emission. When the value of $\bar{P}$ is given, and the nucleon 1 is forward-peaked angular distribution in the pre-equilibrium emission process, then the probability of ^{5}He emission in direction Ω is determined by the integrated momentum space volume occupied by the other four picked up nucleons restricted by the δ function for momentum conserving.

Except nucleon 1, the other four picked-up nucleons corresponds to a α-particle, their relative intrinsic momenta are defined by

$$\bar{p}_{23}=\bar{p}_2+\bar{p}_3 \quad \bar{p}_r=\frac{1}{2}(\bar{p}_2-\bar{p}_3)$$

$$\bar{p}_{45}=\bar{p}_4+\bar{p}_5 \quad \bar{p}_{r'}=\frac{1}{2}(\bar{p}_4-\bar{p}_5) \quad (3)$$

then $\mathrm{d}\bar{p}_2\mathrm{d}\bar{p}_3\mathrm{d}\bar{p}_4\mathrm{d}\bar{p}_5=\mathrm{d}\bar{p}_{23}\mathrm{d}\bar{p}_r\mathrm{d}\bar{p}_{45}\mathrm{d}\bar{p}_{r'}$ is held. Thus, the angular factor of ^{5}He becomes

$$A(n,\varepsilon,\Omega)=\frac{1}{N}\int_{[lm]}\mathrm{d}\bar{p}_1\mathrm{d}\bar{p}_{23}\mathrm{d}\bar{p}_r\mathrm{d}\bar{p}_{45}\mathrm{d}\bar{p}_{r'}\delta(\bar{P}-\bar{p}_1-\bar{p}_{23}-\bar{p}_{45})\tau(n,\Omega_1) \quad (4)$$

When $\bar{P}$ and $\bar{p}_1$ are given, then the value of $\bar{p}_\alpha$ is definite due to $\bar{p}_\alpha=\bar{p}_{23}+\bar{p}_{45}$, so the relationship of $\bar{p}_{23}$ and $\bar{p}_{45}$ can be obtained. The observable emitted energy ε of ^{5}He is obtained by

$$\varepsilon=\frac{P^2}{10m}+\frac{7}{2}\hbar\omega_5-5\varepsilon_f-B_5$$

where $\varepsilon_f=p_f^2/2m$ is the Fermi energy, p_f refers to the Fermi momentum and B_5 stands for the binding energy of ^{5}He in compound nucleus. So the inequality $P^2>16P_f^2$ could be reduced into

$$9\varepsilon_f>5\left(\frac{7}{2}\hbar\omega_5-\varepsilon-B_5\right)$$

which is always holds since B_5, ε are always less than 20 MeV, and $\varepsilon_f\approx$30 MeV, $\hbar\omega_5\approx 7.63$ MeV. Therefore, the inequality $P>4p_f$ is always holds. Meanwhile, p_1 satisfies $p_1^2/2m\le E^*+\varepsilon_f$, where E^* is the excitation energy. In the case of low energies, if $E^*<\varepsilon_f$, then $p_1<2p_f$, and by using the momentum relation $\bar{P}=\bar{p}_1+\bar{p}_\alpha$, one has $|\bar{P}-\bar{p}_1|\equiv\bar{p}_\alpha>2p_f$.

The configuration [1,4] hints that $p_2\le p_f$, $p_3\le p_f$. Since $\bar{p}_r$ and $\bar{p}_{r'}$ do not appear in the δ function of eq.(4), so we ought to carry out the integration over them in advance. At first, for the integration $\int\mathrm{d}\bar{p}_{23}\mathrm{d}\bar{p}_r$, once $\bar{p}_{23}$ is given, the integration limits of $\bar{p}_r$ can be confirmed. The relationship of $\bar{p}_2$, $\bar{p}_3$ and their relative momenta $\bar{p}_{23}$, $\bar{p}_r$ is shown in Fig.1, where θ_r is the angle between $\bar{p}_{23}$ and $\bar{p}_r$, then we have

$$p_2^2=\frac{1}{4}p_{23}^2+p_r^2-p_{23}p_r\cos\theta_r\le p_f^2$$

$$p_3^2=\frac{1}{4}p_{23}^2+p_r^2+p_{23}p_r\cos\theta_r\le p_f^2 \quad (5)$$

Hence, the integration limits over θ_r satisfy the inequality $-\beta\le\cos\theta_r\le\beta$.

Where

$$\beta=\frac{p_f^2-\frac{1}{4}p_{23}^2-p_r^2}{p_{23}p_r} \quad (6)$$

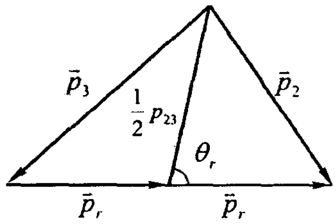

Fig.1 The relations of $\bar{p}_2, \bar{p}_3$ and $\bar{p}_{23}, \bar{p}_r$

The condition for the integration existence needs $\beta \geq 0$, which gives

$$p_r \leq \sqrt{p_f^2 - \frac{1}{4} p_{23}^2}$$

In addition, from the condition of $\beta \leq 1$, one gets

$$p_r \leq p_f - \frac{1}{2} p_{23}$$

Then the integration $\int d\bar{p}_{23} d\bar{p}_r$ can be written explicitly as

$$\int d\bar{p}_{23} d\bar{p}_r = \int d\bar{p}_{23} \left\{ \int_0^{p_r - \frac{1}{2}p_{23}} p_r^2 dp_r \int_{-1}^{1} d\cos\theta_r \int_0^{2\pi} d\varphi_r + \int_{p_r - \frac{1}{2}p_{23}}^{\sqrt{p_r^2 - \frac{1}{4}p_{23}^2}} p_r^2 dp_r \int_{-\beta}^{\beta} d\cos\theta_r \int_0^{2\pi} d\varphi_r \right\} \quad (7)$$

Carrying out the integration over $\bar{p}_r$, the result is

$$\int d\bar{P}_r = \frac{\pi}{12}\left(16 p_f^3 - 12 p_f^2 p_{23} + p_{23}^3\right) \quad (8)$$

With the same procedure for the integration over $\bar{p}_{r'}$, one can get

$$\int d\bar{P}_r = \frac{\pi}{12}\left(16 p_f^3 - 12 p_f^2 p_{45} + p_{45}^3\right) \quad (9)$$

Thus the eq.(4) is reduced into the form as follows

$$A(n,\varepsilon) = \frac{\pi^2}{144N} \iiint_{[1,4]} d\bar{p}_1 d\bar{p}_{23} d\bar{p}_{45} \delta (\bar{P} - \bar{p}_1 - \bar{p}_{23} - \bar{p}_{45}) (16 p_f^3 - 12 p_f^2 p_{23} + p_{23}^3) (16 p_f^3 - 12 p_f^2 p_{45} + p_{45}^3) \tau(n, \Omega_1) \quad (10)$$

Since $\bar{p}_\alpha = \bar{p}_{23} + \bar{p}_{45}$, and θ is the angular between $\bar{p}_\alpha$ and $\bar{p}_{23}$.

$$p_{45} = \sqrt{p_\alpha^2 + p_{23}^2 - 2 p_\alpha p_{23} \cos\theta} \leq 2 p_f \quad (11)$$

The integration limits of θ satisfy the inequality as

$$\cos\theta \geq \frac{p_\alpha^2 + p_{23}^2 - 4 p_f^2}{2 p_\alpha p_{23}} \equiv \gamma \quad (12)$$

If $\gamma \leq -1$ were held, one could get $p_\alpha + p_{23} \leq 2 p_f$, which is contradictory with the condition $p_\alpha > 2 p_f$, so $\gamma \leq -1$ is impossible. Thus the integration limits of θ are given by $\gamma \leq \cos\theta \leq 1$. The condition $\gamma < 1$ needs $p_{23} \geq p_\alpha - 2 p_f$. Therefore, the integration over $\bar{p}_{23}$ is obtained explicitly by

$$\int p_{23} = \int_{p_\alpha - 2p_f}^{2p_f} p_{23}^2 dp_{23} \int_\gamma^1 d\cos\theta \quad (13)$$

For convenience the dimensionless quantities are introduced as follows

$$\bar{x}_b \equiv \frac{\bar{P}}{p_f}, \quad \bar{x}_1 \equiv \frac{\bar{P}_1}{p_f}, \quad \bar{x}_{23} \equiv \frac{\bar{P}_{23}}{p_f}$$

$$\bar{x}_{45} \equiv \frac{\bar{P}_{45}}{p_f}, \quad \bar{y} \equiv \frac{\bar{P}_\alpha}{p_f}, \quad t \equiv \cos\theta$$

In spite of the constant factor, and inserting the factor $\int d\bar{y} \delta(\bar{y} - \bar{x}_{23} - \bar{x}_{45})$ in eq.(10), then it is rewritten by

$$A(n,\varepsilon,\Omega) = \iiint_{[1,4]} d\bar{x}_1 d\bar{y} d\bar{x}_{23} d\bar{x}_{45} \delta(\bar{x}_b - \bar{x}_1 - \bar{y}) \delta(\bar{y} - \bar{x}_{23} - \bar{x}_{45}) (16 - 12 x_{23} + x_{23}^3)(16 - 12 x_{45} + x_{45}^3) \tau(n, \Omega) \quad (14)$$

Using the second δ function in eq.(14), $\bar{x}_{45}$ can be replaced by $\bar{x}_{23}$ and $\bar{y}$.

Carrying out the integrating over *t*, and x_{23}, the angular factor is reduced into the form as

$$A(n,\varepsilon,\Omega) = \iiint_{[1,4]} d\bar{x}_1 d\bar{y} \delta(\bar{x}_b - \bar{x}_1 - \bar{y}) Z_b(y) \tau(n, \Omega_1) \quad (15)$$

The function $Z_b(y)$ in eq.(15), as used for the other composite particles [7,8,10], reads

$$Z_b(y) = \frac{(y-4)^6}{y} (-144 + 224 y + 156 y^2 + 24 y^3 + y^4) \quad (16)$$

Using the δ function in eq.(15), the integration is reduced only over $d\bar{y}$, where *y* can be expressed by $\bar{x}_b$ and $\bar{x}_1$ as

$$y = \sqrt{x_b^2 + x_1^2 - 2 x_b x_1 \cos\Theta} \leq 4$$

or