

ELECTRONS AND WAVES

AN INTRODUCTION TO ATOMIC PHYSICS

BY

H. STANLEY ALLEN, F.R.S.

PROFESSOR OF NATURAL PHILOSOPHY AND
DIRECTOR OF THE PHYSICS RESEARCH LABORATORY IN THE
UNITED COLLEGE OF THE UNIVERSITY OF ST. ANDREWS

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PREFACE

DISCOVERY in the realm of physical science has made rapid progress since the closing years of last century. The most important advances have been connected with the electrical constitution of matter and the nature of radiation. The title of this book—*Electrons and Waves*—is meant to include the two concepts of particle and wave, which used to be regarded as forming an antithesis fundamental in all physical theory. But the title is meant also to suggest the possibility of a relationship between these “complementary” ideas, such as is envisaged in the “wavicles” of Eddington.

Most books dealing with modern physics fall into one of two classes. They are either of a popular character and avoid all technical difficulties, or they are serious works demanding previous scientific knowledge and considerable mathematical equipment. In writing this book I have attempted to follow the middle course, and though the task has been difficult, I am not without hope that the result may prove satisfactory to the layman and perhaps afford new light even to the expert. The work is based on lectures delivered on various occasions in the University of St. Andrews, including a course given to teachers attending a summer school. The reception accorded to those lectures encourages me in the belief

that readers who have not specialised in physics will welcome this account of recent progress. Mathematical symbols have not been entirely eliminated, but their use has been reduced to a minimum, and only the simplest algebraic equations have been employed. The non-mathematical reader will probably be able to understand the gist of the argument while taking the mathematical results for granted. The origin of the book accounts for the mode of presentation, and for certain repetitions which have been deliberately retained. The last chapter, which is a summary of the whole volume, contains the substance of a lecture published as a Supplement to *Nature* for December 8, 1928, and I am indebted to the editor for permission to quote freely from the report.

In compiling the lectures, extensive use was made of quotations from original investigators, and this feature has been retained in the book. I desire to thank the authors concerned and also Messrs. J. M. Dent, who have given permission to quote passages from W. E. Leonard's metrical translation of *Lucretius* in Everyman's Library. Apologies are due to certain unknown or anonymous authors from whom I have quoted without acknowledgment.

I wish to express my thanks to all those who lent photographs for reproduction, and in particular to Dr. F. W. Aston, Mr. P. M. S. Blackett, Drs. Eisenhut and Kaupp (of Ludwigshafen), Prof. A. Fowler, Dr. D. Jack, Dr. A. Müller, Prof. G. P. Thomson, and Prof. C. T. R. Wilson. For the photographs of sand figures on vibrating plates I am indebted to the late Dr. C. R. Gibson of Glasgow, two of whose plates are now in the St. Andrews laboratory. I am also indebted for the use of blocks to

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Finally, I wish to tender my sincere thanks to Sir Richard Gregory for his invaluable assistance and advice whilst the book was passing through the press.

H. S. A.

December, 1931.

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CHAPTER I

ATOMICITY AS THE BASIS OF PHYSICS: DEMOCRITUS TO DALTON

Some twenty-five centuries ago, before the close of the lyric period in Greek history, certain philosophers on the shores of the Mediterranean were already teaching that changeful matter is made up of indestructible particles in constant motion ; atoms which chance or destiny has grouped in the course of ages into the forms or substances with which we are familiar. But we know next to nothing of these early theories, of the works of Moschus, of Democritus of Abdera, or of his friend Leucippus. No fragments remain that might enable us to judge of what in their work was of scientific value. And in the beautiful poem, of a much later date, wherein Lucretius expounds the teachings of Epicurus, we find nothing that enables us to grasp what facts or what theories guided Greek thought.

JEAN PERRIN, *Atoms* (1923).

NUMBER

IN *Just So Stories* Rudyard Kipling has told how the first letter was written and how the alphabet was made. He explains how the art of picture-writing developed into the symbolic representation of sounds by conventional signs. A similar romance might be woven round the birth of number and the evolution of counting. Primitive man in very early times must have made

some progress in arithmetic, and by using pebbles or shells as "counters" learnt to perform simple addition or subtraction. By counting on his fingers and toes he became acquainted with groups of five, ten, and twenty. Our decimal system no doubt had its origin in this fact; and we still find traces of groups of twenty in the French names of numbers, and in the "score" a word which recalls an early method of counting by scoring notches on a stick.

An interesting illustration of the survival of a scale of five was given by Sir Richard Gregory in his presidential address to the Decimal Association in 1925. "Primitive people," he said, "could count only in fives, this being the number of fingers on one hand, so that six was described as five and one; seven as five and two; ten as two fives; and twenty-three as four fives and three. The extension of this system to the number ten, or the number of fingers on two hands, came later in the history of the human race. Reckoning in fives is still followed by old shepherds in many parts of the country in counting sheep; and there is usually a pause at every fifth word used. In Lincolnshire the 'shepherd's score' is:

'Yan, tan, tethera, pethera, pimp;
sethera, lethera, hovera, covera, dik;
yan-a-dik, tan-a-dik, tethera-dik, pethera-dik, bumfit;
yan-a-bumfit, tan-a-bumfit, tethera-bumfit, pethera-
bumfit, figgit.'

These words are corruptions of ancient Celtic speech, and are easily traced in the Welsh language, in which, for example, the first five numerals are named 'Un, dau, tri, pedwar, pump'; fifteen is 'pymtheg,' and numbers following it are called one-and-fifteen, two-and-fifteen, up to twenty.'

The idea of number involves that of distinct entities which can be in some way distinguished from one another, but yet can be regarded as sufficiently alike to allow the process of counting to be applied to them. Pressing this conception to the extreme limit of subdivision we arrive at the hypothetical "atoms" of Democritus, uncreatable, unchangeable, indestructible, and indivisible.

Professor Whitehead * tells us that "the science of Pure Mathematics in its modern developments may claim to be the most original creation of the human spirit." This claim he bases on the fact that "in mathematical science connections between things are exhibited which, apart from the agency of human reason, are extremely unobvious." "For example, take the question of number. We think of the number 'five' as applying to appropriate groups of any entities whatsoever, to five fishes, five children, five apples, five days. Thus in considering the relations of the number 'five' to the number 'three,' we are thinking of two groups of things, one with five members and the other with three members." We may dissociate our thought from any consideration of the particular entities, or even of any particular kinds of entities, which go to make up membership of the two groups. "This is a very remarkable feat of abstraction; and it must have taken ages for the human race to rise to it. During a long period groups of fishes will have been compared to each other in respect to their multiplicity, and groups of days to each other. But the first man who noticed the analogy between a group of seven fishes and a group of seven

* A. N. Whitehead, *Science and the Modern World*, Chapter II. (1926).

days made a notable advance in the history of thought. He was the first man who entertained a concept belonging to the science of pure mathematics."

PHYSICAL MEASUREMENTS

From the subject of arithmetic we pass by a natural transition to the subject of measurement. As Sir James Jeans * has said : "To our primitive ancestors measuring was probably much the same thing as counting. The content of a flock of sheep was estimated by counting heads, and the length of a journey was recorded as being so many days' marches, the method used being perhaps that of cutting a notch in a stick at the close of each day. It must soon have emerged that a measurement by integral numbers, while adequate for a flock of sheep, was quite unsuitable for the length of a journey ; the reason, stated in modern scientific phraseology, being that sheep are ' atomic,' while a journey is not."

In passing we may notice that the term " atomic " must now be interpreted in a relative sense ; to the shepherd his sheep are atomic ; to the butcher such a term, if it implied that a sheep could not be divided, would be absurd !

In most of the familiar physical measurements we have to measure not by integers but by continuously changing quantities. If, for example, we attempt to measure the edge of a table by means of a footrule, we find that in general it is not possible to say that the length is an integral number of times the rule : the two quantities are *incommensurable*.

If we measure the diagonal of a square which has its

* J. H. Jeans, *Atomicity and Quanta*, Cambridge, 1925.

side of unit length, we find that we cannot represent the length of the diagonal by an integer or even by a vulgar fraction, which may be regarded as *rational* numbers. But the length of the diagonal may be represented by $\sqrt{2}$, which is an *irrational* number. Its value cannot be expressed either as a vulgar fraction or as a finite decimal, and yet we can calculate a value differing from it by less than any assigned fraction however small.

Again, there are numbers which are not rational and do not satisfy algebraic equations with rational coefficients. Such a number is $\pi = 3.141592\dots$, representing the circumference of a circle of unit diameter. Its value cannot be calculated exactly, though by a process of approximation the value may be found with any desired degree of closeness. Here we have an example of a *transcendental* number, which cannot be expressed by a simple algebraic formula.

CONTINUITY

Opposed to the concept of number is that of continuity exemplified in the "void" of Democritus and Lucretius. The sands of the seashore are composed of visible grains, but the waters of the ocean suggest the idea of a continuous medium. In the classical atomic theory we have to do with two antagonistic images, the void and the atom, continuity and discontinuity, concepts which have provided material for age-long discussions by the metaphysicians. The paradoxes of Zeno, described and examined in interesting fashion by Wildon Carr in his book on *The Principle of Relativity*, may be regarded as examples of the philosophical antinomy

between continuity and discontinuity. Conclusions which are discrepant, though apparently logical, may be drawn from these opposed ideas, so that the very possibility of motion may be denied.

When we sharpen our pencil and rule a fine line on a smooth sheet of paper we produce what at first sight appears a fair approximation to a Euclidean line ; but the trace only requires examination by means of a low-power microscope to reveal the presence of irregularities and discontinuities. The Euclidean point has *no* parts and *no* magnitude, and the ideal line of geometry may be regarded as formed by the motion of such an infinitely small tracing point.

In a similar way a surface may be regarded as formed by a line moving in a direction perpendicular to its length, and a solid as formed by a surface which sweeps through space in a direction normal to its area. Thus in the use of a cyclostyle an inked roller leaves a thin surface film of ink on the paper over which it passes, and working in three dimensions the pastry-cook forces icing sugar from the shaped orifice of a large syringe to form a solid pattern on his confectionery.

A geometrical line may be used as an illustration of what the mathematician calls a "continuum of points"—it is, in fact, a one-dimensional continuum. This term "continuum" deserves some further consideration. As applied to a line, it means that there must be no gaps in the line. If we place a centimetre scale on a sheet of paper and mark the points corresponding to the integral distances 1 cm., 2 cm., 3 cm., etc., from the end, it is clear that we get a series of disconnected points and not a continuous line on the paper. The gaps between the points are large compared with the comparatively few

points that are present. The points may be made more numerous and the gaps smaller by marking points corresponding to $\frac{1}{10}$ cm. ; or, going a step further, points corresponding to all proper or improper fractions such as $\frac{1}{13}$ or $\frac{23}{17}$. It might at first be thought that in this way we would eventually obtain a continuous line, but this idea is erroneous. "Certainly we cannot stand now at one point of the line and name the 'next' point, as we could a moment ago. There is no 'next' rational number to $\frac{116}{125}$ for instance ; $\frac{115}{124}$ comes before it, and $\frac{117}{128}$ after it, but between it and either, or between it and any other rational number we might name, lie many others of the same sort. Yet in spite of the fact that the line containing all these rational points is now 'dense,' it is still not continuous." * Numbers can easily be defined that are not represented at all, irrational numbers like the square root of 2, or transcendental numbers like π , the ratio of the circumference of a circle to its diameter.

In the same sort of way we might discuss a two-dimensional continuum corresponding to a surface, either plane or curved. If the surface is to be continuous there must be no holes or gaps in it ; or, in mathematical language, the surface must contain a point for every possible pair of numbers, x and y , defining the position of a point upon it. The conception may be extended to space of three dimensions, and such a space will form a three-dimensional continuum when there are no holes of any kind in it, and it contains a point for every possible set of three numbers, x , y and z defining such a point, as can be named. It is possible to extend this idea of a continuum, and it may be applied to elements of any

* J. M. Bird, *Relativity and Gravitation* (Methuen).

sort or of any number ; but for our immediate purpose this generalisation is not required.

ATOMIC THEORIES

Early Greek philosophers were familiar with the concepts of "fulness" (or continuity), and "emptiness" (or discontinuity). We know very little of their theories of the universe, but we do know that certain thinkers adopted an atomic theory, and were opposed by Anaxagoras, who championed the cause of the homogeneity and continuity of bodies. "Surely Democritus was the greatest of them all," exclaimed Arrhenius as he walked past the Museum hard by his house with D'Arcy Thompson, and looked up at the great names inscribed upon its walls.

"It was the mantle of Democritus which Epicurus wore, and it was at Epicurus's feet that Lucretius sat to learn the story of the atoms ; but who it was that had told it to Democritus we do not know. Democritus was a rich man's son in a provincial town, about the time when Xerxes drew his broken fleet and army home from Salamis ; he has left nothing of his very own, save a few hearsay fragments and broken sentences out of his many books. Yet at this day a great physicist looks on him with something like awe, and speaks of him with reverent admiration ; for he was the father, so far as all our pedigrees can go, of the most fundamental postulate of all physical science." *

In the Latin poem of Lucretius,† written about 50 B.C.,

* D'Arcy W. Thompson, *Nature*, Vol. 121, p. 566, 1928.

† Lucretius, *De Rerum Natura*, with notes and a translation by H. A. J. Munro (G. Pell & Sons, Ltd.). Metrical translation by W. E. Leonard (J. M. Dent, Everyman's Library, 1921).

more than three hundred years after the time of Epicurus, we find an account of the theory of the hard atom—strong in solid singleness—a theory which may be assumed to represent the fruit of the earlier speculations.

Amongst the principles of this theory as quoted by Tyndall we find the following :

“ The only existing things are the atoms and empty space ; all else is mere opinion.

“ The atoms are infinite in number and infinitely various in form ; they strike together, and the lateral motions and whirlings which thus arise are the beginnings of worlds.

“ The varieties of all things depend upon the varieties of their atoms, in number, size, and aggregation.”

Thus far we have statements which, with some slight modifications, might be accepted by the majority of physicists or chemists of the present day. We might, however, find some difficulty in subscribing to the further principle :

“ The soul consists of fine, smooth, round atoms like those of fire. These are the most mobile of all. They interpenetrate the whole body and in their motions the phenomena of life arise.”

The ancient atomic theory was a philosophical theory—“ an effort of the mind to conceive the ultimate constitution of matter based on deductions from the logical principle of non-contradiction.” Atoms are falling in an infinite void, but from time to time they show a slight inclination, named by Lucretius their *clinamen*. This brings them into collision, so that they are able to form masses.

“ This is that *exiguum clinamen*, by which the atoms swerve ever so little, and only now and then, from their