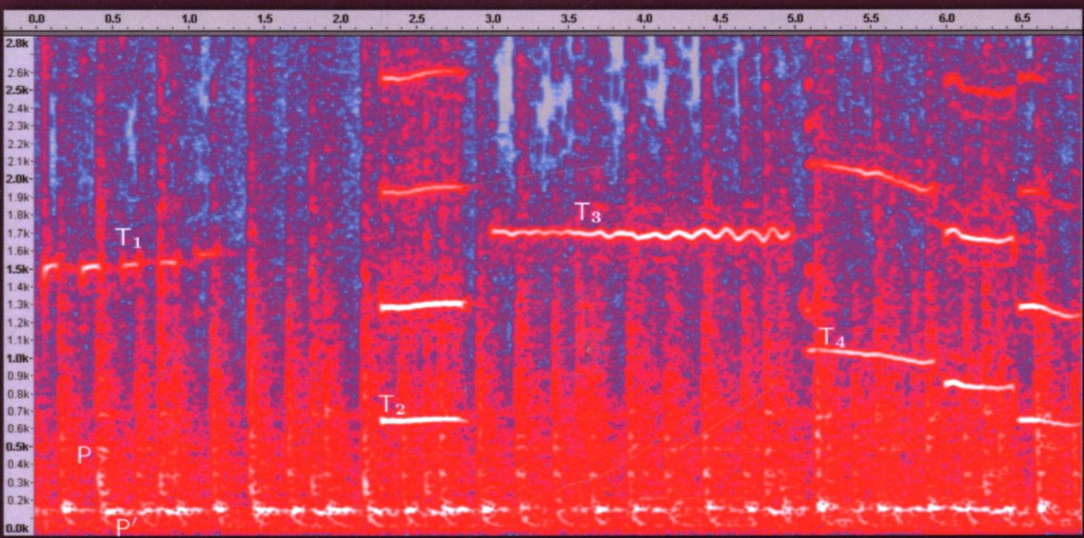
MATHEMATICS AND MUSIC

COMPOSITION, PERCEPTION, AND PERFORMANCE



JAMES S. WALKER
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General Mathematics

At first glance, mathematics and music seem to be from separate worlds—one from science, one from art. But in fact, the connections between the two go back thousands of years, such as Pythagoras's ideas about how to quantify changes of pitch for musical tones (musical intervals). **Mathematics and Music: Composition, Perception, and Performance** explores the many links between mathematics and different genres of music, deepening your understanding of music through mathematics.

In an accessible way, the text teaches the basics of reading music and explains how various patterns in music can be described with mathematics. The authors extensively use the powerful time-frequency method of spectrograms to analyze the sounds created in musical performance. Numerous examples of music notation assist you in understanding basic musical scores. The text also provides mathematical explanations for musical scales, harmony, and rhythm and includes a concise introduction to digital audio synthesis.

Features

- Provides thorough coverage of spectrograms for analyzing the sound of recorded music
- Gives equal emphasis to mathematics and music
- Requires only high school mathematics and no prior experience reading music
- Discusses harmony and rhythm, including the mathematical similarities of pitch and rhythm
- Presents an in-depth yet accessible treatment of the Geometry of Harmony as found in the *Tonnetz*
- Explains the principal methods of audio synthesis
- Offers video demos, links to articles, music software, and musical scores on the book's CRC Press web page

Along with helping you master some fundamental mathematics, this book gives you a deeper appreciation of music by showing how music is informed by both its mathematical and aesthetic structures.

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To Angela

Preface

No school would eliminate the study of language, mathematics, or history from its curriculum, yet the study of music, which encompasses so many aspects of these fields and can even contribute to a better understanding of them, is entirely ignored.

—Daniel Barenboim

The purpose of this book is to explore the connections between mathematics and music. This may seem to be a curious task. Aren't mathematics and music from separate worlds, mathematics from the world of science and music from the world of art? While mathematics does belong to the world of science, one of the goals of science is to understand everything that we experience, and music is no doubt an essential part of human experience. Mathematics has been described as the science of patterns, and we shall see that there are many patterns in music that can be described with mathematics. Mathematics has also been described as the language of the universe, and music itself has been described in such a poetic way. In fact, connections between these two subjects go back thousands of years. For example, the classical Greek mathematician, Pythagoras, contributed the essential ideas for how we quantify changes of pitch for musical tones (musical intervals). The connections between mathematics and music have grown enormously since those ancient days. We will try to explore as many of these connections as possible, in a way that presents both the mathematics and the music to as wide an audience as possible.

Summary of Chapters

Here is a brief description of the main topics covered in the book. For more details, please consult the Table of Contents.

Chapter 1 describes the scientific approach to musical pitch, first worked out by Helmholtz in the 19^{th} century. Helmholtz's theory, which relates pitch to frequency, provides a foundation for understanding different musical scales. One very distinctive aspect of our treatment of this material, is that we use the method of ***spectrograms***. A spectrogram is a graphical portrait of the tones within a musical passage, plotting these tones in terms of their frequencies and the time during which they are sounding. We believe that spectrograms are an important tool for understanding and appreciating music, and that they are not difficult to interpret correctly. So we introduce them before we describe the mathematics used to create them; we postpone that discussion to Chapter 4. Although some might object to using a mathematical technique before describing the details underlying it, we believe that the spectrogram examples described here are so compelling, and so dramatically illustrate this material, that we simply had to include them. In any case, they should provide a strong motivation for learning the mathematics of spectrograms described in Chapter 4. Chapter 2 provides a brief introduction to musical notation. It describes just enough notation so that all readers, even those who are not musicians, should be able to read the brief score excerpts that we include in the book. There are a number of such score excerpts in Chapter 3, where we provide some background in basic music theory. This basic music theory is surprisingly mathematical. We emphasize the different musical transformations—scale shiftings, transpositions, inversions—that composers have employed for centuries. These transformations do have a clear mathematical interpretation.

As described in the last paragraph, in Chapter 4 we discuss the mathematics of ***spectrograms***. In addition to the mathematics, we also provide some interesting musical illustrations, such as the phenomenon known as ***beating*** and its relation to musical consonance and dissonance. In Chapter 5 we demonstrate how spectrograms provide revealing insights into musical structure. These insights would be difficult if not impossible to obtain through listening alone, because listening involves mostly short-term memory, while spectrograms can display an analysis of several minutes of music. Furthermore, when videos of spectrograms are traced out as the music is played they allow us to see ahead what tones are to be played, thereby enhancing our anticipation of the music's development. Spectrograms also allow us to detect, and more deeply appreciate, subtle aspects of musical sound quality such as vibrato, dynamic emphasis, and percussion. All of these insights would be difficult, if not impossible, to gain if one only analyzed scores. Spectrograms provide a powerful tool for analyzing the music that we hear, rather than the notes prescribed for musicians to play. Having another tool for analyzing music, in addition to musical scores, is very valuable. One way that spectrograms and scores work together is that spectrograms reveal the overtone structure of the notes played from a musical score. This overtone structure is very important for understanding musical intervals, which are the building blocks of melody and harmony.

We have described some of the many valuable contributions that spectrograms make to the study and appreciation of music. Our students generally consider the material on spectrograms in Chapters 4 and 5 to be the highlight of the book. Following these chapters, we incorporate rhythm into our study of the mathematical aspects of music. In Chapter 6 we describe how pitch and rhythm share many of the same mathematical features. Most books on music, both in music theory and in mathematical treatments, focus exclusively on pitch and harmony. We believe our treatment of rhythm provides our book with a more complete description of music.

The six chapters just described form the core material of the book. The two chapters that follow them describe more advanced mathematical aspects of music. Throughout the book, we make use of geometrical diagrams to aid us in understanding the basic logic of pitch organization and harmony. Chapter 7 explores this connection of geometry with music theory more deeply. Chapter 8 describes some of the ways that computers can be used for synthesizing music. Electronically synthesized music is widely used, and we have tried to explain how it works without getting overwhelmed by technicalities.

Web site

To aid in the study of this book, there is an accompanying web site. To access this site, go to the CRC web site:

www.crcpress.com

and do a search using the book title:

Mathematics and Music: Composition, Perception, and Performance.

You will then arrive at the web page for our book, where you can click on a link to access the supporting web site. There are links at the book's web site for videos of many of the spectrograms we discuss in the book. You can also download the musical scores we examine in the book, playable with the free music software MUSESCORE. We have supplied an online bibliography with many links to free downloadable articles on math and music. Finally, there are links to other web sites related to math and music, including all the ones mentioned in the book.

Prerequisites

To read this book, one needs to have a good background in high school mathematics. We will not assume, however, the ability to read music. The book aims to teach some mathematics, so there are exercises at the end of each section. It also aims to teach how the mathematics relates to music, so many of the exercises involve musical examples. At the end we hope the reader will have a greater mastery of some fundamental mathematics, and a deeper appreciation of music. An appreciation of music made deeper because it is informed by both its mathematical and aesthetic structure.

Music Software

The world of recorded music has been enormously changed in the last three decades or so with the introduction of computer technology. In this book, we use computers to aid in applying mathematics to the analysis of music, and also to the creation of new music. Mostly, we use two *free* software programs. These two free programs are

1. AUDACITY. An audio editor. We have used it for creating and playing spectrograms.
2. MUSESCORE. A musical scoring program. We have used it to create brief passages of musical scores, which you can play on MUSESCORE when studying these passages in the text.

The book can be studied without working with these programs, although we encourage you to try them. We provide some tutorials on using these programs in Appendix B.

Order of Chapters

Chapters are mostly organized sequentially. Each chapter uses, to a degree, material from preceding chapters. Chapter 8 is an exception, as it can be read immediately following Chapter 4. Although chapters proceed sequentially, there is some flexibility in how they can be covered in a classroom setting. For example, in our Mathematics and Music course at UW-Eau Claire, we have successfully taught the material using the following sequence:

Chapter 1, Chapter 4, Chapter 5, Chapter 3, Chapter 6, Chapter 8, Chapter 7.

Since typically at least half of the class can play and read music, Chapter 2 is given as optional reading at the start of the class for those students who need to learn basic music notation. Having students work in groups on material, such as Chapter 3 with its emphasis on music theory, can be very helpful for those students who have a great interest in understanding music but lack performance ability. We have found, however, that even students who are not musicians can master the elementary material in Chapter 2 on their own, and then read the music theory in Chapter 3 with understanding.

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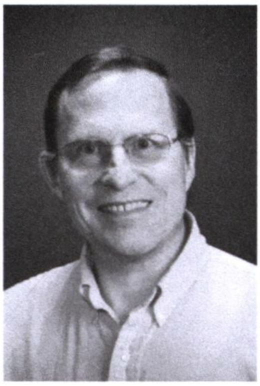
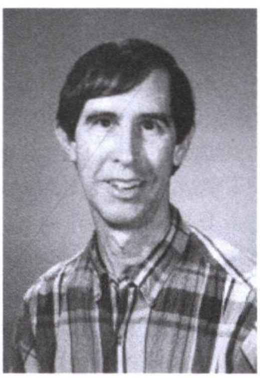
About the authors

Principal Author



James S. Walker received his doctorate from the University of Illinois, Chicago, in 1982. He has been a professor of Mathematics at the University of Wisconsin-Eau Claire since 1982. His publications include papers on Fourier analysis, wavelet analysis, complex variables, logic, and mathematics and music. He is also the author of five books on Fourier analysis, FFTs, and wavelet analysis.

Musical Consultant



Gary W. Don received his doctorate in music theory from the University of Washington in 1991. He teaches theory and aural skills, 20th-century techniques, counterpoint, and form and analysis as an associate professor at the University of Wisconsin-Eau Claire. The topics of his published articles include Goethe's influence on music theorists of the 19th and 20th centuries, overtone structures in the music of Debussy, and theory pedagogy.

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