
LINEAR AND NONLINEAR CIRCUITS

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PREFACE

Electrical engineering is a discipline driven by inventions and technological breakthroughs. To mention one: 20 years ago, engineers barely knew how to produce an IC chip; now some chips have one million devices; it is expected that with foreseeable developments in silicon technology during the next decade chips will have 10^8 devices. Also the ubiquitous presence of the computer terminal reminds us of the enormous impact of computers on engineering design. Clearly such tremendous changes would have considerable influence on engineering education.

In teaching a course on introductory electrical circuits, the traditional approach has been to teach exclusively linear time-invariant passive RLC circuits. Admittedly they constitute a good vehicle to learn the dynamics of such simple circuits. Clearly such an approach is obsolete.

It is clear that circuit theory is one of the basic disciplines of electrical engineering; a well designed circuit theory course should cover the basic concepts and the basic results used in circuit design. It should serve as a foundation course to be followed by courses in various fields of electrical engineering, e.g., communication and signal processing, electronic devices and circuits, control and power systems, microwaves and optoelectronics, etc. . . . The concept of device modeling and its applications to currently used devices are crucial in a course on linear and nonlinear circuits: many examples of device modeling are given in the text. Furthermore, the course should be designed so that the graduate from such a curriculum knows how to approach the devices and circuits yet to be invented but that he or she will encounter, say, 10 to 15 years from now. With these goals in mind, the present book presents material with sufficient breadth, depth, and rigor to give a solid foundation to the student's future professional life.

At the University of California, Berkeley, as in most American engineering schools, there is a sophomore 45-lecture-hour course called Introductory Electrical Engineering. Its purpose is to give a broad introduction to most of the aspects of electrical engineering.

This book is intended as a textbook for the junior course that follows.

Since it is a junior course, it takes advantage of the greater competence and maturity of the students: in particular, physics, linear algebra, and differential equations. This course is the electrical analog of the typical junior physics course in say, mechanics, electromagnetism, and so on.

This book differs from many other texts on circuit theory by the following features:

1. Due to the ubiquitous op amp and similar devices, it views a circuit as an interconnection of *multiterminal* elements rather than of *two-terminal* elements.
2. *Active* and *passive* circuits are given equal emphasis.
3. *Linear* and *nonlinear* elements are treated together. (Note that computers simulate nonlinear circuits almost as easily as linear ones.)
4. The concept of *operating point* and the topic of *small-signal analysis* are covered thoroughly.
5. Switching, triggering, and memory circuits as well as oscillators are illustrated with first-order and second-order examples.
6. Tableau analysis is used to greatly simplify the proof of many network theorems in linear and nonlinear circuits.
7. Modified node analysis is introduced in view of its complete generality and importance in the design of computer circuit simulators, such as SPICE.
8. Some numerical methods are introduced and implemented via equivalent circuits: in particular, solution of nonlinear algebraic equations (Newton-Raphson) and integration of the circuit differential equations (forward and backward Euler method).
9. Stability issues are met head on; in particular oscillators are analyzed and an elementary version of the Nyquist criterion (useful in the design of op amps circuits) is introduced.

CONTENTS OF THE BOOK

Chapter One treats Kirchhoff's Laws and Tellegen's theorem. The next four chapters introduce two-terminal and multiterminal *resistive* elements and *resistive* circuits; linear, nonlinear, passive, and active circuits; op amp circuits with linear and nonlinear models; operating points and small signal analysis; and network theorems and the Newton-Raphson procedure for solving nonlinear dc resistive circuits.

Chapters Six and Seven cover first- and second-order linear and nonlinear dynamic circuits: our goal is to exhibit their properties and illustrate them by numerous examples, including flip flops and oscillators. General dynamic circuits, analyzed by Tableau or modified node analysis, are covered in Chapter Eight.

The next three chapters build up the fundamentals of linear time-invariant

circuits: sinusoidal steady-state analysis; very brief treatment of Laplace transforms; properties of natural frequencies and network functions such as poles, zeros, stability, and convolution; Nyquist criterion; and stability of terminated one-ports.

A brief Chapter Twelve broadens the background on network topology and treats the usual general circuit analysis methods.

Chapter Thirteen covers two-ports, n -ports, and their properties—reciprocity in particular.

The last chapter brings out design issues such as the approximation problem, design of active Butterworth filters, and sensitivity analysis.

We believe that the topics covered in this text constitute an excellent background for further education in electronic circuits, computer-aided design, communications, control, and power.

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Even though this book is a systematic introduction to circuit theory, it uses many concepts and techniques which were developed by people doing research in circuits, communications and control. In fact, without our own deep involvement in research, this book could not have been written. It is a pleasure to publicly acknowledge the research support of the University of California, the National Science Foundation, the Department of Defense, the State of California, and the support of industry.

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