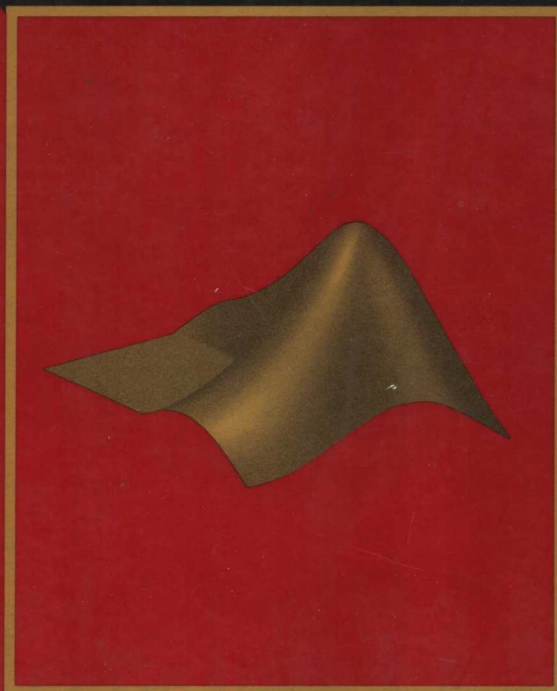


# Contemporary Communication Systems

Using MATLAB®



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**John G. Proakis    Masoud Salehi**



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# PREFACE

There are many textbooks on the market today that treat the basic topics in analog and digital communication systems, including coding and decoding algorithms and modulation and demodulation techniques. The focus of most of these textbooks is by necessity on the theory that underlies the design and performance analysis of the various building blocks, e.g., coders, decoders, modulators, and demodulators, that constitute the basic elements of a communication system. Relatively few of the textbooks, especially those written for undergraduate students, include a number of applications that serve to motivate the students.

## SCOPE OF THE BOOK

The objective of this book is to serve as a companion or supplement, to any of the comprehensive textbooks in communication systems. The book provides a variety of exercises that may be solved on the computer (generally, a personal computer is sufficient) using the popular student edition of MATLAB. The book is intended to be used primarily by senior-level undergraduate students and graduate students in electrical engineering, computer engineering and computer science. We assume that the student (or user) is familiar with the fundamentals of MATLAB. Those topics are not covered, because several tutorial books and manuals on MATLAB are available.

By design, the treatment of the various topics is brief. We provide the motivation and a short introduction to each topic, establish the necessary notation, and then illustrate the basic notions by means of an example. The primary text and the instructor are expected to provide the required depth of the topics treated. For example, we introduce the matched filter and the correlator and assert that these devices result in the optimum demodulation of signals corrupted by additive white Gaussian noise (AWGN), but we do not provide a proof of this assertion. Such a proof is generally given in most textbooks on communication systems.

## ORGANIZATION OF THE BOOK

The book consists of nine chapters. The first two chapters on signals and linear systems and on random processes treat the basic background that is generally required in the study of communication systems. There is one chapter on analog communication techniques, and the remaining five chapters are focused on digital communications.

### Chapter 1: Signals and Linear Systems

This chapter provides a review of the basic tools and techniques from linear systems analysis, including both time-domain and frequency-domain characterizations. Frequency-domain-analysis techniques are emphasized, since these techniques are most frequently used in the treatment of communication systems.

**Chapter 2: Random Processes**

In this chapter we illustrate methods for generating random variables and samples of random processes. The topics include the generation of random variables with a specified probability distribution function, the generation of samples of Gaussian and Gauss-Markov processes, and the characterization of stationary random processes in the time domain and the frequency domain.

**Chapter 3: Analog Modulation**

The performances of analog modulation and demodulation techniques in the presence and absence of additive noise are treated in this chapter. Systems studied include amplitude modulation (AM), such as double-sideband AM, single-sideband AM, and conventional AM, and angle-modulation schemes, such as frequency modulation (FM) and phase modulation (PM).

**Chapter 4: Analog-to-Digital Conversion**

In this chapter we treat various methods that are used to convert analog source signals into digital sequences in an efficient way. This conversion process allows us to transmit or store the signals digitally. We consider both lossy data compression schemes, such as pulse-code modulation (PCM), and lossless data compression, such as Huffman coding.

**Chapter 5: Baseband Digital Transmission**

The focus of this chapter is on the introduction of baseband digital modulation and demodulation techniques for transmitting digital information through an AWGN channel. Both binary and nonbinary modulation techniques are considered. The optimum demodulation of these signals is described, and the performance of the demodulator is evaluated.

**Chapter 6: Digital Transmission Through Bandlimited Channels**

In this chapter we consider the characterization of bandlimited channels and the problem of designing signal waveforms for such channels. We show that channel distortion results in intersymbol interference (ISI), which causes errors in signal demodulation. Then, we treat the design of channel equalizers that compensate for channel distortion.

**Chapter 7: Digital Transmission via Carrier Modulation**

Four types of carrier-modulated signals that are suitable for transmission through bandpass channels are considered in this chapter. These are amplitude-modulated signals, quadrature-amplitude-modulated signals, phase-shift keying, and frequency-shift keying.

**Chapter 8: Channel Capacity and Coding**

In this chapter we consider appropriate mathematical models for communication channels and introduce a fundamental quantity, called the channel capacity, that gives the limit on the amount of information that can be transmitted through the channel. In particular, we consider two channel models, the binary symmetric channel (BSC) and the additive white

Gaussian noise (AWGN) channel. These channel models are used in the treatment of block and convolutional codes for achieving reliable communication through such channels.

### **Chapter 9: Spread Spectrum Communication Systems**

The basic elements of a spread spectrum digital communication system are treated in this chapter. In particular, direct sequence (DS) spread spectrum and frequency-hopped (FH) spread spectrum systems are considered in conjunction with phase-shift keying (PSK) and frequency-shift keying (FSK) modulation, respectively. The generation of pseudonoise (PN) sequences for use in spread spectrum systems is also treated.

## **SOFTWARE**

The accompanying diskette includes all the MATLAB files used in the text. The files are in separate directories corresponding to each chapter of the book. In some cases a MATLAB file appears in more than one directory because it is used in more than one chapter. In most cases numerous comments have been added to the MATLAB files to ease their understanding. It should be noted, however, that in developing the MATLAB files, the main objective of the authors has been the clarity of the MATLAB code rather than its efficiency. In cases where efficient code could have made it difficult to follow, we have chosen to use a less efficient but more readable code.

In order to use the software, copy the MATLAB files to your personal computer and add the corresponding paths to your `matlabpath` environment (on an IBM PC compatible machine this is usually done by editing the `matlabrc.m` file). All MATLAB files have been tested using version 4 of MATLAB.

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# Chapter 1

## Signals and Linear Systems

### 1.1 Preview

In this chapter we review the basic tools and techniques from linear system analysis used in the analysis of communication systems. Linear systems and their characteristics in the time and frequency domains, together with probability and analysis of random signals, are the two fundamental topics whose thorough understanding is indispensable in the study of communication systems. Most communication channels and many subblocks of transmitters and receivers can be well modeled as linear and time-invariant (LTI) systems, and so the well-known tools and techniques from linear system analysis can be employed in their analysis. We emphasize frequency-domain analysis tools, since these are the most frequently used techniques. We start with the Fourier series and transforms; then we cover power and energy concepts, the sampling theorem, and lowpass representation of bandpass signals.

### 1.2 Fourier Series

The input-output relation of a linear time-invariant system is given by the convolution integral defined by

$$\begin{aligned} y(t) &= x(t) \star h(t) \\ &= \int_{-\infty}^{\infty} h(\tau)x(t - \tau) d\tau \end{aligned} \tag{1.2.1}$$

where  $h(t)$  denotes the impulse response of the system,  $x(t)$  is the input signal, and  $y(t)$  is the output signal. If the input  $x(t)$  is a complex exponential given by

$$x(t) = Ae^{j2\pi f_0 t} \tag{1.2.2}$$

then the output is given by

$$\begin{aligned} y(t) &= \int_{-\infty}^{\infty} A e^{j2\pi f_0(t-\tau)} h(\tau) d\tau \\ &= A \left[ \int_{-\infty}^{\infty} h(\tau) e^{-j2\pi f_0\tau} d\tau \right] e^{j2\pi f_0 t} \end{aligned} \quad (1.2.3)$$

In other words, the output is a complex exponential with the same frequency as the input. The (complex) amplitude of the output, however, is the (complex) amplitude of the input amplified by

$$\int_{-\infty}^{\infty} h(\tau) e^{-j2\pi f_0\tau} d\tau$$

Note that the above quantity is a function of the impulse response of the LTI system,  $h(t)$ , and the frequency of the input signal,  $f_0$ . Therefore, computing the response of LTI systems to exponential inputs is particularly easy. Consequently, it is natural in linear system analysis to look for methods of expanding signals as the sum of complex exponentials. Fourier series and Fourier transforms are techniques for expanding signals in terms of complex exponentials.

A Fourier series is the orthogonal expansion of periodic signals with period  $T_0$  when the signal set  $\{e^{j2\pi nt/T_0}\}_{n=-\infty}^{\infty}$  is employed as the basis for the expansion. With this basis, any periodic signal<sup>1</sup>  $x(t)$  with period  $T_0$  can be expressed as

$$x(t) = \sum_{n=-\infty}^{\infty} x_n e^{j2\pi nt/T_0} \quad (1.2.4)$$

where the  $x_n$ 's are called the *Fourier series coefficients* of the signal  $x(t)$  and are given by

$$x_n = \frac{1}{T_0} \int_{\alpha}^{\alpha+T_0} x(t) e^{-j2\pi nt/T_0} dt \quad (1.2.5)$$

Here  $\alpha$  is an arbitrary constant chosen in such a way that the computation of the integral is simplified. The frequency  $f_0 = 1/T_0$  is called the *fundamental frequency* of the periodic signal, and the frequency  $f_n = nf_0$  is called the *nth harmonic*. In most cases either  $\alpha = 0$  or  $\alpha = -T_0/2$  is a good choice.

This type of Fourier series is known as the *exponential Fourier series* and can be applied to both real-valued and complex-valued signals  $x(t)$  as long as they are periodic. In general, the Fourier series coefficients  $\{x_n\}$  are complex numbers even when  $x(t)$  is a real-valued signal.

---

<sup>1</sup>A sufficient condition for the existence of the Fourier series is that  $x(t)$  satisfy the Dirichlet conditions. For details see [1].

When  $x(t)$  is a *real-valued* periodic signal we have

$$\begin{aligned} x_{-n} &= \frac{1}{T_0} \int_{\alpha}^{\alpha+T_0} x(t) e^{j2\pi nt/T_0} dt \\ &= \frac{1}{T_0} \left[ \int_{\alpha}^{\alpha+T_0} x(t) e^{-j2\pi nt/T_0} dt \right]^* \\ &= x_n^* \end{aligned} \quad (1.2.6)$$

From this it is obvious that

$$\begin{cases} |x_n| = |x_{-n}| \\ \angle x_n = -\angle x_{-n} \end{cases} \quad (1.2.7)$$

Thus the Fourier series coefficients of a real-valued signal have *Hermitian symmetry*; i.e., their real part is even and their imaginary part is odd (or, equivalently, their magnitude is even and their phase is odd).

Another form of Fourier series, known as *trigonometric Fourier series*, can be applied only to real, periodic signals and is obtained by defining

$$x_n = \frac{a_n - jb_n}{2} \quad (1.2.8)$$

$$x_{-n} = \frac{a_n + jb_n}{2} \quad (1.2.9)$$

which, after using Euler's relation

$$e^{-j2\pi nt/T_0} = \cos\left(2\pi t \frac{n}{T_0}\right) - j \sin\left(2\pi t \frac{n}{T_0}\right) \quad (1.2.10)$$

results in

$$\begin{aligned} a_n &= \frac{2}{T_0} \int_{\alpha}^{\alpha+T_0} x(t) \cos\left(2\pi t \frac{n}{T_0}\right) dt \\ b_n &= \frac{2}{T_0} \int_{\alpha}^{\alpha+T_0} x(t) \sin\left(2\pi t \frac{n}{T_0}\right) dt \end{aligned} \quad (1.2.11)$$

and, therefore,

$$x(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(2\pi t \frac{n}{T_0}\right) + b_n \sin\left(2\pi t \frac{n}{T_0}\right) \quad (1.2.12)$$

Note that for  $n = 0$  we always have  $b_0 = 0$ , so  $a_0 = 2x_0$ .

By defining

$$\begin{cases} c_n = \sqrt{a_n^2 + b_n^2} \\ \theta_n = -\arctan \frac{b_n}{a_n} \end{cases} \quad (1.2.13)$$

and using the relation

$$a \cos \phi + b \sin \phi = \sqrt{a^2 + b^2} \cos \left( \phi - \arctan \frac{b}{a} \right) \quad (1.2.14)$$

Equation (1.2.12) can be written in the form

$$x(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} c_n \cos \left( 2\pi t \frac{n}{T_0} + \theta_n \right) \quad (1.2.15)$$

which is the third form of the Fourier series expansion for real and periodic signals. In general the Fourier series coefficients  $\{x_n\}$  for real-valued signals are related to  $a_n$ ,  $b_n$ ,  $c_n$ , and  $\theta_n$  through

$$\begin{cases} a_n = 2 \operatorname{Re}[x_n] \\ b_n = -2 \operatorname{Im}[x_n] \\ c_n = |x_n| \\ \theta_n = \angle x_n \end{cases} \quad (1.2.16)$$

Plots of  $|x_n|$  and  $\angle x_n$  versus  $n$  or  $nf_0$  are called the *discrete spectrum* of  $x(t)$ . The plot of  $|x_n|$  is usually called the *magnitude spectrum*, and the plot of  $\angle x_n$  is referred to as the *phase spectrum*.

If  $x(t)$  is real and even—i.e., if  $x(-t) = x(t)$ —then taking  $\alpha = -T_0/2$ , we have

$$b_n = \frac{2}{T_0} \int_{-T_0/2}^{T_0/2} x(t) \sin \left( 2\pi t \frac{n}{T_0} \right) dt \quad (1.2.17)$$

which is zero because the integrand is an odd function of  $t$ . Therefore, for a real and even signal  $x(t)$ , all  $x_n$ 's are real. In this case the trigonometric Fourier series consists of all cosine functions. Similarly, if  $x(t)$  is real and odd—i.e., if  $x(-t) = -x(t)$ —then

$$a_n = \frac{2}{T_0} \int_{\alpha}^{\alpha+T_0} x(t) \cos \left( 2\pi t \frac{n}{T_0} \right) dt \quad (1.2.18)$$

is zero and all  $x_n$ 's are imaginary. In this case the trigonometric Fourier series consists of all sine functions.

### ILLUSTRATIVE PROBLEM

**Illustrative Problem 1.1 [Fourier series of a rectangular signal train]** Let the periodic signal  $x(t)$ , with period  $T_0$ , be defined by

$$x(t) = A \Pi \left( \frac{t}{2t_0} \right) = \begin{cases} A, & |t| < t_0 \\ \frac{A}{2}, & t = \pm t_0 \\ 0, & \text{otherwise} \end{cases} \quad (1.2.19)$$

for  $|t| \leq T_0/2$ , where  $t_0 < T_0/2$ . The rectangular signal  $\Pi(t)$  is, as usual, defined by

$$\Pi(t) = \begin{cases} 1, & |t| < \frac{1}{2} \\ \frac{1}{2}, & t = \pm \frac{1}{2} \\ 0, & \text{otherwise} \end{cases} \quad (1.2.20)$$

A plot of  $x(t)$  is shown in Figure 1.1.

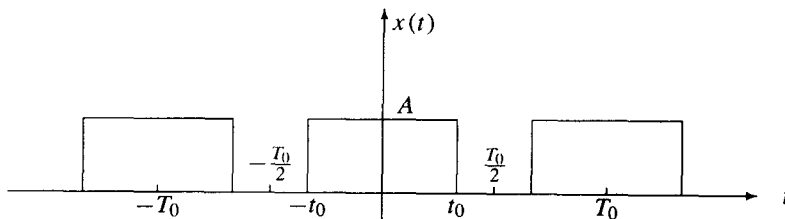


Figure 1.1: The signal  $x(t)$ , in Illustrative Problem 1.1.

Assuming  $A = 1$ ,  $T_0 = 4$ , and  $t_0 = 1$ :

1. Determine the Fourier series coefficients of  $x(t)$  in exponential and trigonometric form.
2. Plot the discrete spectrum of  $x(t)$ .

### SOLUTION

1. To derive the Fourier series coefficients in the expansion of  $x(t)$ , we have

$$\begin{aligned} x_n &= \frac{1}{4} \int_{-1}^1 e^{-j2\pi nt/4} dt \\ &= \frac{1}{-2j\pi n} \left[ e^{-j2\pi n/4} - e^{j2\pi n/4} \right] \end{aligned} \quad (1.2.21)$$

$$= \frac{1}{2} \operatorname{sinc}\left(\frac{n}{2}\right) \quad (1.2.22)$$

where  $\operatorname{sinc}(x)$  is defined as

$$\operatorname{sinc}(x) = \frac{\sin(\pi x)}{\pi x} \quad (1.2.23)$$

A plot of the sinc function is shown in Figure 1.2.

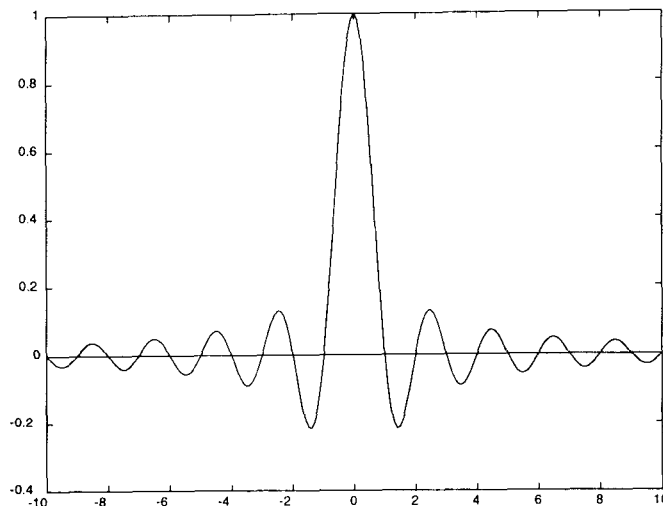


Figure 1.2: The sinc signal.

Obviously, all the  $x_n$ 's are real (since  $x(t)$  is real and even), so

$$\begin{cases} a_n = \text{sinc}\left(\frac{n}{2}\right) \\ b_n = 0 \\ c_n = \left|\text{sinc}\left(\frac{n}{2}\right)\right| \\ \theta_n = 0, \pi \end{cases} \quad (1.2.24)$$

Note that for even  $n$ 's,  $x_n = 0$  (with the exception of  $n = 0$ , where  $a_0 = c_0 = 1$  and  $x_0 = \frac{1}{2}$ ). Using these coefficients we have

$$\begin{aligned} x(t) &= \sum_{n=-\infty}^{\infty} \frac{1}{2} \text{sinc}\left(\frac{n}{2}\right) e^{j2\pi nt/4} \\ &= \frac{1}{2} + \sum_{n=1}^{\infty} \text{sinc}\left(\frac{n}{2}\right) \cos\left(2\pi t \frac{n}{4}\right) \end{aligned} \quad (1.2.25)$$

A plot of the Fourier series approximations to this signal over one period for  $n = 0, 1, 3, 5, 7, 9$  is shown in Figure 1.3. Note that as  $n$  increases, the approximation becomes closer to the original signal  $x(t)$ .

2. Note that  $x_n$  is always real. Therefore, depending on its sign, the phase is either zero or  $\pi$ . The magnitude of the  $x_n$ 's is  $\frac{1}{2} \left|\text{sinc}\left(\frac{n}{2}\right)\right|$ . The discrete spectrum is shown in Figure 1.4.



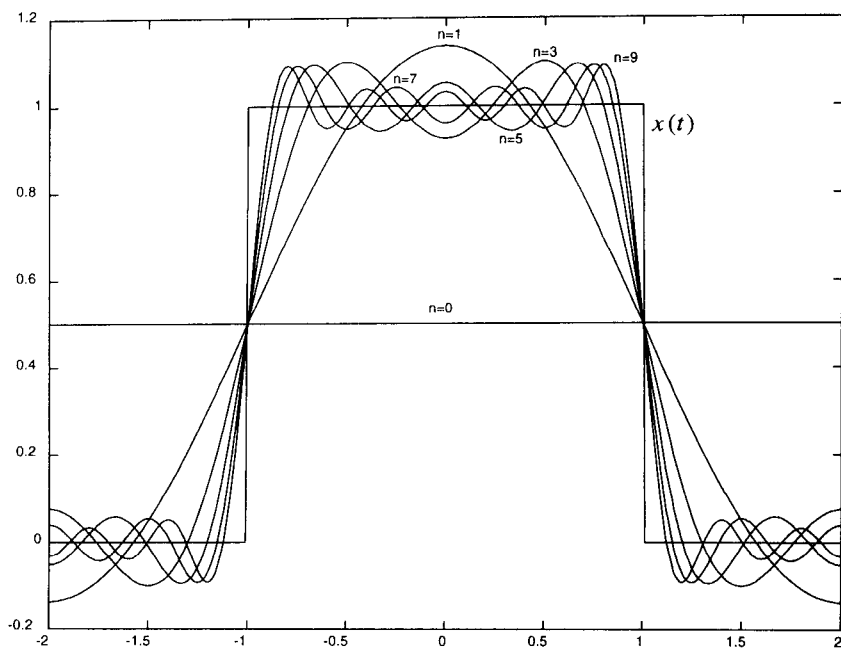


Figure 1.3: Various Fourier series approximations for the rectangular pulse in Illustrative Problem 1.1.

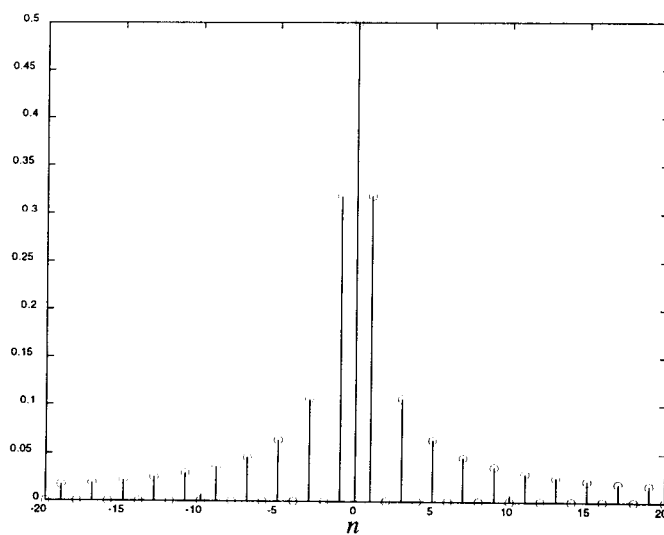


Figure 1.4: The discrete spectrum of the signal in Illustrative Problem 1.1.