ZHANG WOHUA

# Fundamentals ELASTICITY MECHANICS and Finite Element Method

浙江大學出版社

# FUNDAMENTALS
# ELASTICITY MECHANICS
# and
# FINITE ELEMENT METHOD

ZHANG WOHUA

浙江大學出版社

**图书在版编目（CIP）数据**

弹性力学及有限元基础＝Fundamentals Elasticity Mechanics and Finite Element Method / 张我华编著. 杭州：浙江大学出版社，2003. 3
ISBN 7-308-03265-5

Ⅰ. 弹… Ⅱ. 张… Ⅲ. ①弹性力学—高等学校—教材—英文 ②有限元法—高等学校—教材—英文
Ⅳ. ①O343 ②O241.82

中国版本图书馆 CIP 数据核字（2003）第 013659 号

| | |
|---|---|
| 责任编辑 | 樊晓燕 |
| 出版发行 | 浙江大学出版社 |
| | （杭州浙大路 38 号 邮政编码 310027） |
| | （E-mail：zupress@mail.hz.zj.cn） |
| | （网址：http：//www.zjupress.com） |
| 排　　版 | 浙江大学出版社电脑排版中心 |
| 印　　刷 | 浙江大学印刷厂 |
| 开　　本 | 850mm×1168mm 1/32 |
| 印　　张 | 11.5 |
| 字　　数 | 362 千 |
| 版 印 次 | 2003 年 3 月第 1 版 2003 年 3 月第 1 次印刷 |
| 印　　数 | 0001—1000 |
| 书　　号 | ISBN 7-308-03265-5/O・289 |
| 定　　价 | 20.00 元 |

# PREFACE

*"Some books are to be tasted, others to be swallowed, and some few to be chewed and digested: that is, some books are to be read only in parts; others to be read but not curiously; and some few to be read wholly and with diligence and attention."*

*Francis Bacon (1561—1626)*

This textbook is intended primarily for the senior undergraduate course in elastic mechanics and finite element fundamentals teaching in English for Chinese students majoring in Civil Engineering. Thus, this book has been written bearing in mind the definition of books by Francis Bacon as quoted above; it is needless to say that this book belongs to the third category for senior undergraduate Chinese students.

It is generally accepted that the fundamental concepts of elastic mechanics have been widely used at various stages by engineers in the courses of solid mechanics, structural mechanics, geo-mechanics and materials engineering. The basic concepts of elastic mechanics are used in their simpler form for the initial analysis before preliminary design of engineering structures and later, more advanced principles are being used for complex analysis before final design. Thus, elastic mechanics plays an important role in the curriculum for engineers. Many of the undergraduate and graduate courses employ the concepts of elastic mechanics as the basis for further development of engineering principles. Therefore, it is necessary as a first step to understand clearly the basic principles and the corresponding mathematical expressions involved in elastic mechanics. So, it is assumed that the reader has taken general physics and has a mathematical background, which includes some familiarity with algebra and a working knowledge of differential and integral calculus.

Since the solutions to most of the engineering problems are obtained by solving the governing differential equations, it is imperative that one should learn the origin of such equations and the basis from which they have been derived. In addition, it is recommended that an introductory course in ordinary differential equations or a course in advanced mathematics including differential equations be taken prior to or concurrently with this course in civil engineering. For this reason, attention has been given in this book to present the derivations of various fundamental equations in an extensive manner.

No mathematical theory can completely describe the complex world around us. Every theory is aimed at a certain class of phenomena, formulates their essential features, and disregards what is of minor importance. The theory meets its limits of applicability where a disregarded influence becomes important. Thus, elastic mechanics describes in many cases the stresses, strains and displacements of actual solid bodies with high accuracy, but it fails to produce more than a few general statements in the case of finite deformation, because non-linear elasticity or anelasticity, no matter how local or how small, attains a dominating influence.

The finite element method is the best approach available for the numerical analysis of engineering. Although the method has been applied to many mechanics fields, this book is devoted to the fundamental analysis of two-dimensional elastic mechanics. The theory requires discretization of the elastic mechanics problem into a numerical network of finite elements and implementation of the analysis on a digital computer. Because this book is intended for the beginner, we have emphasized clarity in the presentation. The user should find the explanations in essentials to read, either as a student or as an engineer. The techniques and computer programs described here are fundamentals and used in an undergraduate-level course for Chinese students at Zhejiang University for many years.

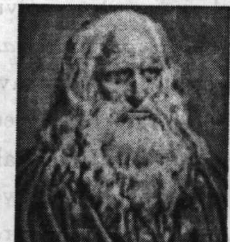
The subject matter has been presented in seven chapters. No specific comments are warranted regarding chapter 1 which introduces the concept of elastic mechanics. Chapters 2 and 3 deal respectively with the concept of stress and strain at a point in a more general sense. The discussion also includes the development of equilibrium equations, compatibility conditions and strain-displacement relations, which are an essential part of elasticity governing differential equation. In chapter 4, the generalized constitutive relationship for a linearly elastic material is presented both in isotropic and anisotropic cases. A brief discussion on non-linear behaviours of materials is also included in Chapter 4. Chapter 5 deals with the derivation of fundamental equations of elasticity for solids based on the principles discussed in chapters 2 — 4 of equations for solving their problems. Chapter 6 presents solutions to a number of simple problems in elasticity mechanics. The finite element formulation to solve two dimensional elastic problems is presented in chapter 7. In chapter 7, the discussion involves the theories and the FORTRAN and C++ language programming of finite element method as well as the user constructions of the programs for analysis of two dimensional elastic mechanics problems and corresponding numerical examples.

S. VALLIAPPAN
Ph. D., D. Sc., F. ASCE, F. IACM,
Professor of Civil Engineering,
President Australian Association;
Vice President International Association for Computational Mechanics

# CONTENTS

# Chapter 1 INTRODUCTION

*"Mechanics is the paradise of mathematical science because here we come to the fruits of mathematics."*
*Leonardo da Vinci (1452—1519), ITALY*



## 1.1 BASIC CONCEPTS

The rapid progress in various fields of technology has created the need for new types of structures and structural materials, which, in turn, has involved the search for more rational shapes and economy of materials. This, in fact, has motivated the engineers to seek for various methods capable of evaluating the strength and deformation characteristics of complex structures and new materials. The branch of science, which deals with these methods, is known as "*Strength of Materials*". Through this branch of science, engineers can obtain simple formulae in order to arrive at a sufficiently safe solution for a simple practical problem. However, this simplified approach is not quite sufficient for dealing with many practical problems involving complex material behaviour and loading conditions. Such problems can be solved using the more general theory of "*Elastic Mechanics*". The Elastic Mechanics can be considered as the theoretical basis for estimating the elastic stress and deformation of any solid structure or structural material under the action of any general loading, whether it is static or dynamic.

The objective of elastic mechanics is to deal with the stress, strain and displacement of solid materials based on a general three-dimensional situation and on their linear behavior. While techniques for solving problems in *Elastic Mechanics and Strength of Materials* may be different, the basic principles underlying the behavior of solids and structures are the same. Therefore, the subject matter covered in the following chapters consists of general states of stresses and strains and displacements as well as constitutive relationships such as isotropic and anisotropic elasticity pertaining to linear range of the solid strength. The linear elasticity concept regards matter as infinitely divisible. Thus it is valid to accept the idea of an infinitesimal volume of materials referred to as a particle within the continuum. The founders of the theory of elasticity, Navier (1821) and Cauchy (1822) developed their theories on the basis of molecular nature of matter (Ref. [1]). It is not quite convenient in dealing with the displacements of each particular particle in the continuum or to determine the forces or interaction between each pair of molecules. However, it is more appropriate to consider the material of the body as distributed continuously throughout the whole volume filling the spaces completely without gaps or empty spaces and express the field quantities such as deformation and stress as piecewise continuous functions of space and time.

The elastic mechanics deals with three kinds of quantities: stresses, strains and displacements (Ref. [4~7]).

*Stresses* describe forces acting inside a body. Usually they are defined as forces (or force components) per unit of area of an infinitesimal section. However, the bending and twisting moments in a plane, the membrane forces in a shell, the bending moment and the shear force in a beam, and the torque in a shaft are also quantities of the same kind. They all describe forces or moments transmitted from one side of a section to the other, and they all come in pairs equal in magnitude but opposite in sense, as they are acting on both parts of the body separated by the section.

*Strains* describe local deformations, for example, the increase in length of a line element divided by its original length (tensile strain), or the decrease of the right angle between two line elements (shear strain). There are more sophisticated definitions of strain quantities, like the tensorial strain derived from the change of square of the line element, or the logarithmic strain. On the other hand, the curvature of a bent beam or plate and the twist of a shaft are also strain quantities, since they describe local change of form without reference to an external coordinate system.

***Displacements*** describe the movement of a point or a line element during the process of deformation, with reference to a fixed coordinate system outside the deformable body. The displacements of the theory of elasticity and the deflection of a beam or a plate are linear displacements; the rotation of a beam element (the "slope" of the textbooks) is an angular displacement.