Probability and Measure

概率与测度

第 3 版

Third Edition

Patrick Billingsley

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Third Edition

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前 言

Edward Davenant said he "would have a man knockt in the head that should write anything in Mathematiques that had been written of before." So reports John Aubrey in his *Brief Lives*. What is new here then?

To introduce the idea of measure the book opens with Borel's normal number theorem, proved by calculus alone, and there follow short sections establishing the existence and fundamental properties of probability measures, including Lebesgue measure on the unit interval. For simple random variables—ones with finite range—the expected value is a sum instead of an integral. Measure theory, without integration, therefore suffices for a completely rigorous study of infinite sequences of simple random variables, and this is carried out in the remainder of Chapter 1, which treats laws of large numbers, the optimality of bold play in gambling, Markov chains, large deviations, the law of the iterated logarithm. These developments in their turn motivate the general theory of measure and integration in Chapters 2 and 3.

Measure and integral are used together in Chapters 4 and 5 for the study of random sums, the Poisson process, convergence of measures, characteristic functions, central limit theory. Chapter 6 begins with derivatives according to Lebesgue and Radon–Nikodym—a return to measure theory—then applies them to conditional expected values and martingales. Chapter 7 treats such topics in the theory of stochastic processes as Kolmogorov's existence theorem and separability, all illustrated by Brownian motion.

What is new, then, is the alternation of probability and measure, probability motivating measure theory and measure theory generating further probability. The book presupposes a knowledge of combinatorial and discrete probability, of rigorous calculus, in particular infinite series, and of elementary set theory. Chapters 1 through 4 are designed to be taken up in sequence. Apart from starred sections and some examples, Chapters 5, 6, and 7 are independent of one another; they can be read in any order.

My goal has been to write a book I would myself have liked when I first took up the subject, and the needs of students have been given precedence over the requirements of logical economy. For instance, Kolmogorov's existence theorem appears not in the first chapter but in the last, stochastic processes needed earlier having been constructed by special arguments which, although technically redundant, motivate the general result. And the general result is, in the last chapter, given two proofs at that. It is instructive, I think, to see the show in rehearsal as well as in performance.

The Third Edition. The main changes in this edition are two. For the theory of Hausdorff measures in Section 19 I have substituted an account of L^p spaces, with applications to statistics. And for the queueing theory in Section 24 I have substituted an introduction to ergodic theory, with applications to continued fractions and Diophantine approximation. These sections now fit better with the rest of the book, and they illustrate again the connections probability theory has with applied mathematics on the one hand and with pure mathematics on the other.

For suggestions that have led to improvements in the new edition, I thank Raj Bahadur, Walter Philipp, Michael Wichura, and Wing Wong, as well as the many readers who have sent their comments.

Envoy. I said in the preface to the second edition that there would not be a third, and yet here it is. There will not be a fourth. It has been a very agreeable labor, writing these successive editions of my contribution to the river of mathematics. And although the contribution is small, the river is great: After ages of good service done to those who people its banks, as Joseph Conrad said of the Thames, it spreads out "in the tranquil dignity of a waterway leading to the uttermost ends of the earth."

PATRICK BILLINGSLEY

Chicago, Illinois
December 1994

目　录

* 表示初学者相关内容可以略去。

Contents

*Stars indicate topics that may be omitted on a first reading.

第一章

概 率

1 Borel 的正轨数定理

Although sufficient for the development of many interesting topics in mathematical probability, the theory of discrete probability spaces† does not go far enough for the rigorous treatment of problems of two kinds: those involving an infinitely repeated operation, as an infinite sequence of tosses of a coin, and those involving an infinitely fine operation, as the random drawing of a point from a segment. A mathematically complete development of probability, based on the theory of measure, puts these two classes of problem on the same footing, and as an introduction to measure-theoretic probability it is the purpose of the present section to show by example why this should be so.

The Unit Interval

The project is to construct simultaneously a model for the random drawing of a point from a segment and a model for an infinite sequence of tosses of a coin. The notions of independence and expected value, familiar in the discrete theory, will have analogues here, and some of the terminology of the discrete theory will be used in an informal way to motivate the development. The formal mathematics, however, which involves only such notions as the length of an interval and the Riemann integral of a step function, will be entirely rigorous. All the ideas will reappear later in more general form.

Let Ω denote the unit interval $(0,1]$; to be definite, take intervals open on the left and closed on the right. Let ω denote the generic point of Ω. Denote the length of an interval $I=(a,b]$ by $|I|$:

$$|I| = |(a,b]| = b - a. \tag{1.1}$$

†For the discrete theory, presupposed here, see for example the first half of Volume I of Feller. (Names in capital letters refer to the bibliography on p. 581.)