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Inverse Source Problems in Optics

Editor: H. P. Baltes

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Foreword

Puis, lorsque j'ai voulu descendre à celles (les choses) qui étaient plus particulières, il s'en est tant présenté à moi de diverses, que je n'ai pas cru qu'il fût possible à l'esprit humain de distinguer les formes ou espèces de corps qui sont sur la terre, d'une infinité d'autres qui pourraient y être si c'eût été le vouloir de Dieu de les y mettre, ni par conséquent de les rapporter à notre usage, si ce n'est qu'on vienne au-devant des causes par les effets, et qu'on se serve de plusieurs expériences particulières. [*R. Descartes: Discours de la méthode (Librairie Ch. Delagrave, Paris 1877) Part 6, p.65*]

It is an interesting fact that text-book physics is characterized by a strong predilection for direct problems, that is predicting physical effects on the basis of known physical causes. The complex mathematical apparatus involved in solving inverse problems, especially the type covered in this book, presents serious difficulties to the beginner in the field. He might also be surprised that a purely industrial need is at least partially responsible for the writing of this book, a hint that the field of inverse problems in optical physics has grown beyond mathematical art.

What is it then which makes, for example, the apparatus industry take a close look at the inverse problem in optical physics? Let me give one example from the market of high- and low-speed banknote testing equipment. The function of such machines is to test a graphic product, the banknote, for its genuineness with the highest degree of security and reliability. The introductory chapter gives a direct link to the type of problem encountered in optical authenticity checking.

However, this book will not reveal practical approaches to the solution of technical problems. The road leading to the technical implementation of the results achieved so far is long and not easy.

Chapter 1 attempts a brief systematic survey of the inverse problems in optical physics, together with a discussion of the role of prior knowledge. The chapter presents a tentative list of more than 20 specific inverse optical problems (including those not covered by this volume). The agreed size of the Topics in Current Physics series volumes imposed the selection of not more than the following five chapters.

Chapter 2 presents a state-of-the-art review of the phase reconstruction problem for wave amplitudes, as well as coherence functions with application to both light and electron optics.

Chapter 3 is devoted to the problem of reconstructing a scattering object or potential from scattered field amplitudes with emphasis on the question of uniqueness and nonradiating sources. The reconstruction of the field up to the surface of the scatterer, as well as the reconstruction of the object from the field outside the object, are described in detail.

Chapter 4 reports recent work toward solving a superresolution problem, namely the reconstruction of the near field of very small localized sources from far-field data. As a by-product, this chapter contains a comprehensive study of nonuniform plane waves.

Chapter 5 aims at the new field where coherence and radiometry overlap. The relationship between far-zone and source coherence functions is discussed along with new radiometric concepts for sources of any state of coherence. Both amplitude and intensity correlations are considered. A brief survey of the history of radiometry is also given.

Finally, Chapter 6 reviews the retrieval of statistical features of random phase screens from scattering data in terms of correlation functions and photon statistics. Higher-order statistical properties of the scattered field are emphasized. Nonchaotic scattered radiation due to a small number of scatterers is discussed.

We believe this is the first book written on the subject; too few people have been taken this road. May this book invite others to join in the effort to widen the potentials of this particular field.

Zug, Switzerland
July 1978

J.-F. Moser

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Contents

1. Introduction

H. P. Baltes

We begin the introductory chapter with a general definition of the inverse optical problem. Next, we discuss the role of prior knowledge and the questions of uniqueness and stability. We then review the various specific inverse problems in optics as well as the contents of Chapters 2 to 6. Finally, we summarize the notation in coherence theory.

1.1 Direct and Inverse Problems in Optical Physics

The "direct" or "normal" problem in optical physics is to predict the emission or propagation of radiation on the basis of a known constitution of sources or scatterers. The "inverse" or "indirect" problem is to deduce features of sources or scatterers from the detection of radiation. An intuitive solution of the optical inverse problem is commonplace: we infer the size, shape, surface texture, and material of objects from their scattering and absorption of light as detected by our eyes. Intuition has to give way to mathematical reconstruction as soon as we wish to analyze optical data beyond their visual appearance. Examples are the extrapolation and deblurring of optical images, the reconstruction from intuitively inaccessible data such as defocused images and interferograms, or the search for information that is "lost" in the detection process such as the phase.

Following CHADAN and SABATIER [1.1], a general definition of inverse optical problems can be attempted as follows. We describe the sources and scatterers by the set

$$G = \{g_1, g_2, \dots, g_n\} \tag{1.1}$$

of space-time functions g_j which we call the *source functions* (scatterers being included as indirect or secondary sources). The resulting propagation of radiation is described by the set

$$F = \{f_1, f_2, \dots, f_m\} \tag{1.2}$$

of space-time functions f_i called *results* or *data*, which can be checked by measurement. From the source functions g_j, we can derive unique data f_i by virtue of the *direct relations*

$$f_i = E_i(g_1,g_2,\dots,g_n) \quad , \tag{1.3}$$

where the set E of operators E_i provides a mapping of G into F, viz.

$$E : G \to F \quad . \tag{1.4}$$

In coherent optics, for example, the E_i correspond to certain integral transformations and the g_j and f_i to source and, for example, far-zone amplitudes and their correlations.

Solving the direct problem in optical physics means computing the data f_i from known source functions g_j using the direct relations (1.3). *Solving the inverse problem* means finding source functions g_j which

1) correspond to given data f_i by virtue of the prescriptions (1.3) and

2) are consistent with the physical information coming from general principles or other experiments, the so-called *prior knowledge*.

The prior knowledge reduces the set of possible source functions. For example, we can often take for granted that the source has a finite volume.

Apparently there are two opposite approaches to the above problem.

1) We establish formulas or algorithms which allow the reconstruction of the source functions by inversion of the mapping (1.3,4), viz.

$$E^{-1} : F \to G \quad . \tag{1.5}$$

The name "inverse problem" is usually reserved for this approach.

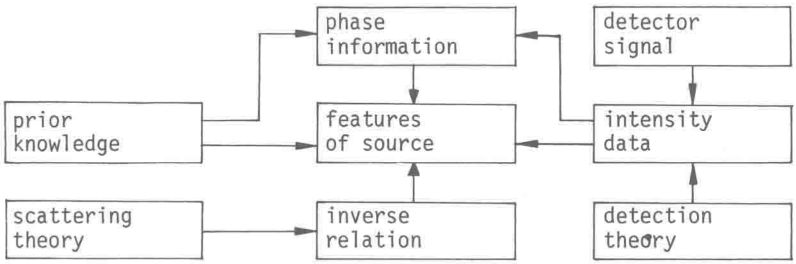
2) We find specific model source functions by trial and error and fit free parameters from the experimental data. This approach brings us back to the direct problem, since we have to check the models by (1.3). The notion of inverse problem in the strict sense is usually understood to exclude such fitting procedures. In practice there is, however, a more or less continuous transition from "inverse" to "direct" procedures, the inverse character of the problem becoming less pronounced with increasing prior knowledge (see Sect.1.2).

It is well known that an inversion of mappings as indicated by (1.5) involves the mathematical questions of the *existence*, *uniqueness*, and *stability* of the solution. For example, the extrapolation of optical image data [1.2,3] belongs to the class of problems (usually called "ill-posed" or "improperly posed problems"), in which the solution depends uniquely, but not continuously, on the data (see, e.g., [1.4]). Small errors in the data can lead to large errors in the solution unless suitable stabilizing constraints are imposed, i.e., unless additional prior knowledge can be taken for granted. Of course, errors and noise are inevitable in experimental data.

As for historical remarks, we refer to the introduction of Chapter 3 (see also [1.1]).

1.2 Role of Prior Knowledge

Let us now attempt to re-collect the prerequisites for obtaining information on an optical source or a scatterer (or the propagating medium) from experimental data. The following scheme may be helpful.



The sought features of the source or the scatterer are, in principle, inferred from intensity and phase data by virtue of the appropriate inverse relation, and accounting for the available prior knowledge. The inverse relation or inversion algorithm is, of course, based on the pertinent theory of propagation and scattering of radiation. Detectors provide intensity data. We thus have to deduce the necessary phase information from intensity distributions. This problem of *phase reconstruction* is the objective of Chapter 2. Intensity data can also include quantities from coherent and quantum optics such as the modulus of the degree of coherence, the intensity autocorrelation, and photon statistics. The correct evaluation of detector signals requires knowledge of the *theory of photodetection* [1.5] and involves another inverse problem, namely how to reconstruct the statistics of the incident radiation from that of the photoelectrons [1.6].

Prior knowledge means simply any knowledge about the source functions available prior to the experiment in which we are interested, but, of course, not prior to the development of the plan of observation [1.7]. Such knowledge is inferred from general principles, hypotheses [1.8], the result of other experiments, and the constraints imposed by the planned experimental procedure. The notion of prior knowledge used here is distinct from epistemological or a priori knowledge in the strict Kantian sense (see [1.7]). Prior knowledge is crucial for achieving uniqueness and stability of the solution of the inverse problem. Moreover, the nature of the prior knowledge largely determines the character of the problem (inverse or direct). If sufficient prior knowledge allows us to infer specific source models, we get away with solving the direct problem and fitting the parameters, as is illustrated by the following example.

1) We begin the well-known determination of stellar diameters from measurements of the modulus $|\mu|$ of the degree of coherence μ as a function of angular spacing (see, e.g., [Ref.1.2, Sect.2]). An enormous amount of prior knowledge is taken for granted here: we assume that the source is a uniformly bright, circular disk with zero coherence area. Applying the Van Cittert-Zernike theorem to this model, we learn how the angular diameter of the source is found from the first zero of $|\mu|$. We had, therefore, to solve a "weak" inverse problem, i.e., to establish and evaluate nothing but a direct relation.

2) Let us now drop the prior knowledge on the shape of the source. By the Van Cittert-Zernike theorem, μ is the Fourier transform of the intensity profile I_0 in the source plane. Thus the shape and size of the source can, in principle, be determined by inversion of the Fourier transform relationship. However, we now have to measure $|\mu|$ over a large range, and must possibly reconstruct the phase of μ. Moreover, we are faced with a serious extrapolation problem if we ask for small details of the source intensity distribution.

3) If neither the intensity distribution I_0 nor the degree of spatial coherence μ_0 in the source plane is known, we have a still more complicated inverse problem involving the convolution of μ_0 with the autocorrelation of $I_0^{1/2}$ (see Sect.5.4.2). Without further data (e.g., the radiant intensity), the measurement of $|\mu|$ is not sufficient for disentangling the information on I_0 and μ_0.

Concluding this section we emphasize that the consideration of stability questions and the exact specification of the prior knowledge are indispensable.

1.3 Survey of Specific Inverse Problems

This volume presents only a small number of selected topics out of the many (20 or more) specific inverse problems of optical physics. In this section we attempt to list the various problems, including those not to be covered in this book (and a few that have hardly been attacked yet). Some readers may be concerned about what is *not* to be found in this book. A selection (five out of ten originally planned chapters) was imposed by the agreed size of the Topics in Current Physics series volumes.

Perhaps a future complementary volume will improve the situation.

In principle, we can distinguish two classes of inverse optical problems, namely

1) problems aiming at information on *spatial* variations of the source functions (spatial frequency spectra), such as the intensity profile or the degree of spatial coherence and other space correlations, and

2) problems aiming at information on *time* variations (dynamics) of the source functions (time frequency spectra) such as the spectral density or the degree of temporal coherence and other time correlations.

In the present volume we consider mainly inverse problems of the type 1). We notice, however, that speckle patterns in polychromatic light [1.9] and scattering by moving diffusers (see, e.g., [1.10]) involve time *and* space variation and are included in Chapter 6. Another inverse problem combining spectral and spatial aspects is the reconstruction of the shape of a cavity resonator from the eigenvalue spectrum (or temporal coherence function) mentioned in Chapter 5.

Another possible classification of inverse problems can be based on the statistical aspect of the radiation. Thus we have inverse problems in classical radiative transfer ("transport of intensity" in the limit of poor spatial coherence), wave optics (coherence limit), and coherent and quantum optics. This volume presents a selection of inverse problems with wave amplitudes (Chaps.3 and 4, first part of Chap.2) and coherence functions (Chaps.5 and 6, second part of Chap.2). Inverse radiative transfer is not studied in this book.

Including related mathematical questions, as well as a number of "applied problems", we arrive at the following, probably incomplete, *list of inverse problems in optics*. (The asterisk indicates that the problem is treated in this volume.)

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1. Intensity propagation
   - 1.1 Inverse radiative transfer (inverse transport theory)
2. Wave amplitudes
   - 2.1 *Phase reconstruction
   - 2.2 *Inverse diffraction (from surface to surface)
   - 2.3 *Inverse scattering (determination of scattering object or potential)
   - 2.4 *Reconstruction of source fields or scattering objects beyond diffraction limit (superresolution problems)
   - 2.5 Extrapolation of images beyond borders
   - 2.6 Computational reconstruction from holographic data
   - 2.7 Reconstruction of optical cavity from the eigenvalue spectrum
   - 2.8 Inverse problems in ellipsometry
3. Coherence functions
   - 3.1 *Phase reconstruction for spatial coherence functions
   - 3.2 *Phase reconstruction for temporal coherence functions
   - 3.3 *Inversion of radiometric data for planar sources (2D)
   - 3.4 Inverse diffraction and scattering of coherence functions for 3D sources
   - 3.5 Extrapolation and superresolution problems for partially coherent light
4. Statistical states
   - 4.1 Reconstruction of radiation field statistics from detector signals
   - 4.2 Determination of statistical field operators from moments or correlations
   - 4.3 Maximum entropy image restoration (photon statistical aspect)