

RELATIVITY

THE SPECIAL AND GENERAL THEORY

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PREFACE

THE present book is intended, as far as possible, to give an exact insight into the theory of Relativity to those readers who, from a general scientific and philosophical point of view, are interested in the theory, but who are not conversant with the mathematical apparatus¹ of theoretical physics. The work presumes a standard of education corresponding to that of a university matriculation examination, and, despite the shortness of the book, a fair amount of patience and force of will on the part of the reader. The author has spared himself no pains in his endeavour to present the main ideas in the simplest and most intelligible form, and on the

¹ The mathematical fundamentals of the special theory of relativity are to be found in the original papers of H. A. Lorentz, A. Einstein, H. Minkowski¹ published under the title *Das Relativitätsprinzip* (The Principle of Relativity) in B. G. Teubner's collection of monographs *Fortschritte der mathematischen Wissenschaften* (Advances in the Mathematical Sciences), also in M. Laue's exhaustive book *Das Relativitätsprinzip* — published by Friedr. Vieweg & Son, Braunschweig. The general theory of relativity, together with the necessary parts of the theory of invariants, is dealt with in the author's book *Die Grundlagen der allgemeinen Relativitätstheorie* (The Foundations of the General Theory of Relativity) — Joh. Ambr. Barth, 1916; this book assumes some familiarity with the special theory of relativity.

whole, in the sequence and connection in which they actually originated. In the interest of clearness, it appeared to me inevitable that I should repeat myself frequently, without paying the slightest attention to the elegance of the presentation. I adhered scrupulously to the precept of that brilliant theoretical physicist, L. Boltzmann, according to whom matters of elegance ought to be left to the tailor and to the cobbler. I make no pretence of having withheld from the reader difficulties which are inherent to the subject. On the other hand, I have purposely treated the empirical physical foundations of the theory in a "step-motherly" fashion, so that readers unfamiliar with physics may not feel like the wanderer who was unable to see the forest for trees. May the book bring some one a few happy hours of suggestive thought!

A. EINSTEIN

December, 1916

NOTE TO THE THIRD EDITION

IN the present year (1918) an excellent and detailed manual on the general theory of relativity, written by H. Weyl, was published by the firm Julius Springer (Berlin). This book, entitled *Raum — Zeit — Materie* (Space — Time — Matter), may be warmly recommended to mathematicians and physicists.

BIOGRAPHICAL NOTE

ALBERT EINSTEIN is the son of German-Jewish parents. He was born in 1879 in the town of Ulm, Württemberg, Germany. His schooldays were spent in Munich, where he attended the *Gymnasium* until his sixteenth year. After leaving school at Munich, he accompanied his parents to Milan, whence he proceeded to Switzerland six months later to continue his studies.

From 1896 to 1900 Albert Einstein studied mathematics and physics at the Technical High School in Zurich, as he intended becoming a secondary school (*Gymnasium*) teacher. For some time afterwards he was a private tutor, and having meanwhile become naturalised, he obtained a post as engineer in the Swiss Patent Office in 1902, which position he occupied till 1909. The main ideas involved in the most important of Einstein's theories date back to this period. Amongst these may be mentioned: *The Special Theory of Relativity*, *Inertia of Energy*, *Theory of the Brownian Movement*, and the *Quantum-Law of the Emission and Absorption of Light* (1905). These were followed some years later by the

Theory of the Specific Heat of Solid Bodies, and the fundamental idea of the *General Theory of Relativity*.

During the interval 1909 to 1911 he occupied the post of Professor *Extraordinarius* at the University of Zurich, afterwards being appointed to the University of Prague, Bohemia, where he remained as Professor *Ordinarius* until 1912. In the latter year Professor Einstein accepted a similar chair at the *Polytechnikum*, Zurich, and continued his activities there until 1914, when he received a call to the Prussian Academy of Science, Berlin, as successor to Van't Hoff. Professor Einstein is able to devote himself freely to his studies at the Berlin Academy, and it was here that he succeeded in completing his work on the *General Theory of Relativity* (1915-17). Professor Einstein also lectures on various special branches of physics at the University of Berlin, and, in addition, he is Director of the Institute for Physical Research of the *Kaiser Wilhelm Gesellschaft*.

Professor Einstein has been twice married. His first wife, whom he married at Berne in 1903, was a fellow-student from Serbia. There were two sons of this marriage, both of whom are living in Zurich, the elder being sixteen years of age. Recently Professor Einstein married a widowed cousin, with whom he is now living in Berlin.

R. W. L.

TRANSLATOR'S NOTE

IN presenting this translation to the English-reading public, it is hardly necessary for me to enlarge on the Author's prefatory remarks, except to draw attention to those additions to the book which do not appear in the original.

At my request, Professor Einstein kindly supplied me with a portrait of himself, by one of Germany's most celebrated artists. Appendix III, on "The Experimental Confirmation of the General Theory of Relativity," has been written specially for this translation. Apart from these valuable additions to the book, I have included a biographical note on the Author, and, at the end of the book, an Index and a list of English references to the subject. This list, which is more suggestive than exhaustive, is intended as a guide to those readers who wish to pursue the subject farther.

I desire to tender my best thanks to my colleagues Professor S. R. Milner, D.Sc., and Mr. W. E. Curtis, A.R.C.Sc., F.R.A.S., also to my friend Dr. Arthur Holmes, A.R.C.Sc., F.G.S.,

of the Imperial College, for their kindness in reading through the manuscript, for helpful criticism, and for numerous suggestions. I owe an expression of thanks also to Messrs. Methuen for their ready counsel and advice, and for the care they have bestowed on the work during the course of its publication.

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RELATIVITY

PART I

THE SPECIAL THEORY OF RELATIVITY

I

PHYSICAL MEANING OF GEOMETRICAL PROPOSITIONS

IN your schooldays most of you who read this book made acquaintance with the noble building of Euclid's geometry, and you remember—perhaps with more respect than love—the magnificent structure, on the lofty staircase of which you were chased about for uncounted hours by conscientious teachers. By reason of your past experience, you would certainly regard every one with disdain who should pronounce even the most out-of-the-way proposition of this science to be untrue. But perhaps this feeling of proud certainty would leave you immediately if some one were to ask you: "What, then, do you mean by the assertion that these propositions are true?" Let us proceed to give this question a little consideration.

Geometry sets out from certain conceptions such as "plane," "point," and "straight line," with

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which we are able to associate more or less definite ideas, and from certain simple propositions (axioms) which, in virtue of these ideas, we are inclined to accept as "true." Then, on the basis of a logical process, the justification of which we feel ourselves compelled to admit, all remaining propositions are shown to follow from those axioms, *i.e.* they are proven. A proposition is then correct ("true") when it has been derived in the recognised manner from the axioms. The question of the "truth" of the individual geometrical propositions is thus reduced to one of the "truth" of the axioms. Now it has long been known that the last question is not only unanswerable by the methods of geometry, but that it is in itself entirely without meaning. We cannot ask whether it is true that only one straight line goes through two points. We can only say that Euclidean geometry deals with things called "straight lines," to each of which is ascribed the property of being uniquely determined by two points situated on it. The concept "true" does not tally with the assertions of pure geometry, because by the word "true" we are eventually in the habit of designating always the correspondence with a "real" object; geometry, however, is not concerned with the relation of the ideas involved in it to objects of experience, but only with the logical connection of these ideas among themselves.

It is not difficult to understand why, in spite of this, we feel constrained to call the propositions of geometry "true." Geometrical ideas correspond to more or less exact objects in nature, and these last are undoubtedly the exclusive cause of the genesis of those ideas. Geometry ought to refrain from such a course, in order to give to its structure the largest possible logical unity. The practice, for example, of seeing in a "distance" two marked positions on a practically rigid body is something which is lodged deeply in our habit of thought. We are accustomed further to regard three points as being situated on a straight line, if their apparent positions can be made to coincide for observation with one eye, under suitable choice of our place of observation.

If, in pursuance of our habit of thought, we now supplement the propositions of Euclidean geometry by the single proposition that two points on a practically rigid body always correspond to the same distance (line-interval), independently of any changes in position to which we may subject the body, the propositions of Euclidean geometry then resolve themselves into propositions on the possible relative position of practically rigid bodies.¹

¹ It follows that a natural object is associated also with a straight line. Three points A , B and C on a rigid body thus lie in a straight line when, the points A and C being given, B is chosen such that the sum of the distances AB and BC is as short as possible. This incomplete suggestion will suffice for our present purpose.

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Geometry which has been supplemented in this way is then to be treated as a branch of physics. We can now legitimately ask as to the "truth" of geometrical propositions interpreted in this way, since we are justified in asking whether these propositions are satisfied for those real things we have associated with the geometrical ideas. In less exact terms we can express this by saying that by the "truth" of a geometrical proposition in this sense we understand its validity for a construction with ruler and compasses.

Of course the conviction of the "truth" of geometrical propositions in this sense is founded exclusively on rather incomplete experience. For the present we shall assume the "truth" of the geometrical propositions, then at a later stage (in the general theory of relativity) we shall see that this "truth" is limited, and we shall consider the extent of its limitation.

II

THE SYSTEM OF CO-ORDINATES

ON the basis of the physical interpretation of distance which has been indicated, we are also in a position to establish the distance between two points on a rigid body by means of measurements. For this purpose we require a "distance" (rod S) which is to be used once and for all, and which we employ as a standard measure. If, now, A and B are two points on a rigid body, we can construct the line joining them according to the rules of geometry; then, starting from A , we can mark off the distance S time after time until we reach B . The number of these operations required is the numerical measure of the distance AB . This is the basis of all measurement of length.¹

Every description of the scene of an event or of the position of an object in space is based on the specification of the point on a rigid body (body of reference) with which that event or object coin-

¹ Here we have assumed that there is nothing left over, *i.e.* that the measurement gives a whole number. This difficulty is got over by the use of divided measuring-rods, the introduction of which does not demand any fundamentally new method.