THE ELEMENTS OF LOGIC

FIFTH EDITION

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ABOUT
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PREFACE

This book offers a general introduction to logic, emphasizing the practical criticizing of reasoning. It deals with traditional logic, with modern symbolic logic, and with induction, and also it considers philosophical issues related to logic.

It has been my hope to make the exposition concise, accurate, and clear, and to provide accompanying exercises that really illustrate the points they are supposed to illustrate. To achieve such goals in full is difficult, but in this fifth edition I believe that some further progress is made toward them. In this edition the basic character of the book remains unchanged, but there have been many revisions in the wording and in details of exposition. Some revisions are for the sake of greater clarity and correctness; others are to make the prose more readable. A majority of the exercises are new for this edition, and their total number has been increased.

Some able teachers of logic disagree with the range of topics which this book covers. They consider that an introductory course which includes comparatively traditional topics such as the theory of the syllogism, fallacies, and induction, must become a mere "grab bag of topics," lacking formal rigor. They prefer a more highly unified approach that omits these topics and presents instead an abstract exposition of deduction, mathematical in style, with emphasis on the nature of formalized systems. By teaching introductory logic in this way, these instructors have the satisfaction of being able to deploy the knowledge of mathematical logic which they themselves have acquired during their own advanced studies, and they can regard themselves as imparting rigorous and up-to-date material.

A first course in logic along these lines can be a good course, I admit. However, it will appeal to only a minority of students—the ones who have a definitely mathematical turn of mind. As I see it, a course covering a more

varied range of topics and treating these in a less formal style will be better suited to the intellectual needs of most students. This more varied and less formal type of course has greater chance, I believe, of enhancing understanding of the actual reasoning that students find themselves and others engaging in. It has a greater chance of offering constructive guidance toward better reasoning, and better thinking generally. Moreover, it need not be an excessively easy or frivolous course, since many nonformal aspects of logic involve hidden difficulties and intellectual challenges which it is by no means child's play to master.

In recent years a new movement in the teaching of logic has been gaining momentum under the banner of "informal logic" or "critical thinking." This approach gives little or no attention to symbolic logic and instead seeks to train students to recognize arguments in ordinary language, to identify unstated premises, to avoid fallacies of ambiguity and irrelevance, and to deal with kindred informal matters.

This increased emphasis by teachers on nonformal aspects of logic is to be welcomed. These aspects indeed ought to be taught, for they are important to the evaluation of actual reasoning. However, I believe that any wholly nonformal presentation of logic will be somewhat unsatisfactory. The student who has been taught nothing but nonformal logic is unlikely to have gained a strong grasp of the distinction between valid and invalid reasoning. To learn to appreciate this distinction, some study of formal principles seems to me to be needed. Therefore, I continue to believe that the type of logic course which will be best for most students should combine both formal and nonformal elements. The formal and nonformal aspects should be presented in ways which connect them and show their relevance to actual reasoning.

This book contains more material than can conveniently be covered in an average one-term course. A teacher willing to omit one or more of the chapters can do so without appreciable loss of continuity, since the chapters are largely independent of one another. Chapter 5 can be omitted by instructors who do not want to go this far into the logic of quantification. Chapter 2 can be omitted by those who prefer to move directly into symbolic logic without studying traditional logic. Chapters 7 and 9 can be omitted by those who do not wish to deal with induction or philosophical questions about logic. The chapter on fallacies (Chapter 6) comes in the middle of the book following the chapters on deductive logic. I realize that some teachers of logic are accustomed to discussing fallacies near the very beginning of a course. Those who wish to do so can assign Chapter 6 before starting with deductive logic.

However, I would counsel against doing that. As I see it, the study of fallacies is not something easy, bland, and straightforward that belongs at the beginning. On the contrary, I consider this part of logic to be fraught with pitfalls. It is the part of logic from which students are most likely to carry away some terminology which they will try to apply to real-life reasoning which they encounter. And it is all too easy for them to carry away crudely oversimplified

notions about fallacies. This will encourage them to censure the remarks of others by crying out more or less at random the names of various fallacies—a procedure which can have the strong rhetorical effect of intimidating others who are unacquainted with the jargon of logic. Responsible instructors, however, will want to guard against encouraging sophistical habits. To learn to distinguish between arguments that are genuinely fallacious and ones that are not requires the development of mature judgment and sensitivity to varied forms of reasoning. The student who has mastered deductive principles will have acquired a notion of what logical validity is, and this will make it more likely that study of the treacherous topic of fallacies will be beneficial rather than harmful.

Chapter 8 may appear to be unimportant, but I believe it has a significant place in the presentation. It deals with the application of logic to actual reasoning and has some basic points to make about the application of formal rules. It also contains the discussion of enthymemes, which would traditionally have accompanied treatment of the syllogism, but is placed here because it is an informal matter that pertains to all reasoning, not to the syllogism alone.

Those who use the book should be aware that a study guide and other supplementary materials are available from the publisher. The study guide, by Christopher Dreisbach and Robert Cavalier, contains supplementary explanations and exercises for students to use. Also, it now contains answers to a sizable selection from among the exercises in this textbook.

My warm thanks are due to many people who have provided me with helpful suggestions. I am especially indebted to the publisher's several anonymous reviewers, who made many valuable comments. To Carolyn Loring I am grateful for her excellent assistance, especially regarding some exercises in this edition. Also, I am indebted to many others who have communicated with me about the book over the years, offering criticism and comments. I regret that these persons are too numerous for me to be able to list, but I extend to each of them my thanks and appreciation. Even though I have not adopted all their suggestions, I have benefitted from them. I shall be glad to hear from others who use the book and who care to offer criticisms or suggestions.

Stephen F. Barker

NOTE

From place to place in the following chapters arabic numbers appear as superscripts. The superscript numbers refer to notes that are found at the end of the book, after the Glossary of Symbols. The notes discuss controversial points and give supplementary information. Readers who prefer to concentrate only on the basic ideas will not need to consult the notes.

In the exercises, an indication of level is given. Groups of easier or more basic examples are marked with an asterisk (*). Groups of examples that are considerably more difficult or less essential are marked with a dagger (†). Examples of intermediate level are given neither sign.

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INTRODUCTION

1 LOGIC AND ARGUMENTS

Most courses in the curriculum of a college or university today are relatively new ones, which were not taught a few decades ago. Logic is an exception. Logic probably was taught regularly in the schools of ancient Greece, and certainly in Western Europe courses in logic have been offered to students ever since the first universities came into being some 800 or 900 years ago. What is there about logic that for so many centuries has made people regard it as deserving to be a part of higher education? The answer has two sides. Logic—the critical study of reasoning—is a subject having both theoretical interest and practical value.

On the one hand, the study of logic can be intellectually rewarding as knowledge for its own sake. This is because of the clear and systematic character of many of its principles and its close relations with basic philosophical questions and (in recent times) with the foundations of mathematics.

On the other hand, the study of logic is also of practical use. A mastery of its principles can help us to recognize and avoid mistakes in reasoning—both in the reasoning we do ourselves and in the reasoning that others use in trying to convince us of things. A person who can recognize and avoid logical mistakes in reasoning should be able to think more clearly and correctly, more soundly and surely, about any subject. To be sure, we probably should not expect the study of logic all by itself to make people who reason badly into good reasoners. Good reasoning is a very complex skill which requires sound judgment and broad knowledge concerning the subject matter about which one is going to reason. A single course in logic can hardly sup-

ply these. But it is to be expected that people who already have some skill at reasoning can improve and refine that skill through studying logic.

This book will deal with both theoretical and practical aspects of logic because both are important and both have educational value. Although it is not always easy to connect both these aspects closely together, ideally the theoretical principles of logic should be studied in living relationship with their application to actual reasoning.

Logic and Philosophy

Logic is a subject with a long history. Like so much of our intellectual heritage, it goes back to the ancient Greeks. Among the Greeks the formal study of logic began with Aristotle in the fourth century B.C. Aristotle's most important contribution to logic was his theory of the syllogism. (The nature of the syllogism will be discussed in Chapter 2.) Later, Stoic philosophers worked out some of the principles of truth-functional logic. (Truth functions will be explained in Chapter 3.)

Thinkers during the Middle Ages admired Aristotle's writings on logic far more than anyone else's, and so the medieval tradition came to regard the theory of the syllogism as the central and most important part of logic. This view persisted into the modern period. As late as the eighteenth century, the great German philosopher Kant reflected the prevailing opinion of that time when he declared that logic was a "completed science"—that is, a subject whose elements were fully understood, so that no new principles remained to be discovered in it.

Kant proved to be mistaken in this opinion, however. In the nineteenth century the Irish logician Boole showed that the field of truth functions was far richer than had previously been realized, and he devised powerful new methods for treating problems in that branch of logic as well as for generalizing the theory of the syllogism. Also, the German mathematician Frege originated the theory of quantification. (Quantification will be discussed in Chapter 4.) Then Whitehead and Russell systematized the new developments in logic in their famous work *Principia Mathematica*, written early in the twentieth century. In that work they presented the new logic in a comprehensive way, and they also tried to establish the controversial philosophical idea that the laws of pure mathematics can be derived from those of logic alone.

This modern logic does not in any way contradict the traditional aristotelian logic, when both are properly understood. However, modern logic differs from traditional aristotelian logic in two important ways. It is much more general, dealing with a far wider variety of forms of reasoning; and it uses more symbolism, its style and method being more akin to mathematics. In what follows, we shall be concerned both with the main ideas of traditional aristotelian logic and with some of the ideas of modern symbolic

logic. In studying these ideas, we shall be trying always to keep in view their application to ordinary reasoning.

The more advanced logical studies nowadays have taken on a character resembling that of pure mathematics, while elementary parts of logic have their special interest because of their practical value in helping to detect mistakes in reasoning. Thus it perhaps seems that logic does not have much relation to philosophy. Yet logic has always been regarded as a branch of philosophy, and there are good reasons for this. Let us briefly consider what some of the various branches of philosophy are, and what they have in common.

Moral philosophy, or *ethics*, is the branch of philosophy that investigates the notions of good and evil, right and wrong, duty and obligation, and the like. It tries to clarify the nature of these notions in order to answer general questions about their meaning. Are there objective standards of value and rightness? How can we determine what things are good or right? What general kinds of things are good or right? In dealing with such questions, moral philosophy seeks to analyze the critical standards used in making moral judgments.

Metaphysics is the branch of philosophy that tries to understand the nature of the real universe, considered in its most general aspect. It deals with questions about what kinds of things really exist. Is everything physical, or are there real nonphysical things? What is the nature of space and time? Does everything that happens have a cause? Metaphysics seeks to handle these questions by emphasizing the standards employed in judgments about reality.

The *theory of knowledge*, or *epistemology*, is the branch of philosophy that investigates the nature and scope of knowledge. It asks what it is genuinely to know something. Can we have knowledge of things outside our own minds? Does all knowledge depend upon sense experience? The theory of knowledge seeks to analyze the standards employed in judging the genuineness of claims to the possession of knowledge.

Aesthetics is the branch of philosophy that deals with the notions of beauty and ugliness and with the value of works of art. It asks what the nature of beauty is. Are there objective standards of beauty? Can experiencing beautiful things give us insight into the nature of reality? Aesthetics tries to deal with such questions by examining the critical standards used in making judgments about what is beautiful or ugly.

Four of the main branches of philosophy have been mentioned, and they are akin to one another in important ways. They deal with questions which are extremely general. Moreover, these are not questions that can be dealt with by the methods of the special sciences: we cannot settle these questions by scientific observations or laboratory experiments. These philosophical questions are ones with which we can make headway only by reflecting upon our own standards of various kinds (our moral standards, our stan-

dards of what counts as reality, our standards of what counts as knowledge, and so on). By obtaining a clearer view of these standards, we may be able to make progress toward unraveling philosophical questions—toward answering them in some cases, and in other cases toward clarifying the misconceptions that have given rise to the questions.

Although the study of logic differs in some ways from the pursuit of other branches of philosophy, it is no accident that in the past logic has always been classified as a branch of philosophy. Logic has a basic kinship with these other branches. Like them, it deals with some very general questions: questions about what good reasoning is and about the difference between correct and incorrect steps in thinking. Moreover, like other branches of philosophy, logic is a reflective study; experiments are not necessary, and no laboratory work is appropriate for verifying its principles. Like other branches of philosophy, logic involves the critical analysis of standards. In logic, standards of correctness in reasoning are central.

Someone might object that there is no need for a reflective, philosophical approach to reasoning, because reasoning is a phenomenon which can be studied empirically by the science of psychology. This objection rests on a misunderstanding. Of course, observations could be made and experiments conducted to find out how people reason and to discover some of the causes that make them reason as they do. But there is a difference between studying how people reason (a matter of psychology) and studying the nature of correct reasoning (a matter of logic). Logic does not undertake to describe or explain how people think; it has the different and more fundamental aim of analyzing what correct reasoning is, irrespective of whether people do, in fact, reason correctly.

Arguments

In studying logic we shall be studying the difference between good reasoning and bad. But what is reasoning? How shall we identify cases of reasoning? For the present, let us not worry about the difference between good and bad reasoning, but simply consider what reasoning is.

To start with, we can say that reasoning is a process of marshaling reasons. When one is reasoning, one is trying to put forward some things as good reasons for believing something else. Reasoning can take place when one is thinking privately to oneself, but also it can take place when one is trying to prove something to someone else. Let us look at an example of each kind.

Jane is thinking privately to herself. She remembers that last year her employer was in financial difficulty and had to cut her pay by 20 percent. Now business is better, and she is to receive a pay increase of 20 percent. This sounds good to her, but does it mean that her wages will be as large as before? She puts the facts together in her mind and realizes that the increase will be 20 percent of her present pay, and so her new wages will still be

only 96 percent of her original pay. "I'll have lower pay than before the cut," she concludes.

In thinking this out, Jane is *reasoning*; she is making an *inference*. She starts from some things which she believes to be true. Then she comes to accept a consequence because she regards it as something that *follows from* these beliefs—that is, as something these beliefs provide *good reason* for accepting. Jane makes a transition from these beliefs (or *premises*, as we shall call them) to the consequence (or *conclusion*, as we shall call it). She comes to believe the conclusion because she regards the premises as showing that it is true.

The situation is a little different when the person who is to come to believe the conclusion is a different individual from the person who presents the reasoning. Suppose that Bill has always rejected astrology, while Jim has been inclined to believe in it. Bill now tries to show Jim that astrology is unreliable. Bill argues that if a person's fate were determined by the positions of the stars and planets at the time and place of the person's birth, then any two people born at the same time and place would have the same destiny. But Bill says that twins born at the same time and place sometimes grow up to have very different destinies. He says that therefore it follows that astrology is unsound.

Here, Bill wants to start from premises which Jim will accept. Then he wants to get Jim to agree that from these premises the conclusion that astrology is unreliable does follow. Bill hopes in this way to get Jim to believe this conclusion. Bill's own belief is not going to change; he presents his reasoning merely in order to change Jim's belief. If Bill is candid and sincere, he will use only premises that he himself believes; if he is not, he may use premises that he thinks will help convince Jim, even though he, Bill, does not believe them.

To generalize, we may say that *reasoning* is a process of thinking which tries to show that a conclusion should be accepted (either by the reasoner or by those being addressed) because there are good reasons indicating it to be true.

When the reasoning is put into words, we call it an argument. An argument may have just one premise, or there may be two or more. But we shall say that each step of an argument has just one conclusion. Where several conclusions are drawn, either there are several separate arguments, or there is one longer chain of argument consisting of several shorter arguments as its steps.

In the examples considered so far, the person doing the reasoning actually comes to accept the conclusion, or actually tries to get a listener to do so. The person does not just suggest a possible conclusion which someone might want to reach—if that were all that was being done, there would be a *potential* argument, but not an *actual* argument.

To illustrate this, suppose that Clara thinks to herself, "If I get a 10 percent raise for next year, and if the consumer price index rises by only 5 per-