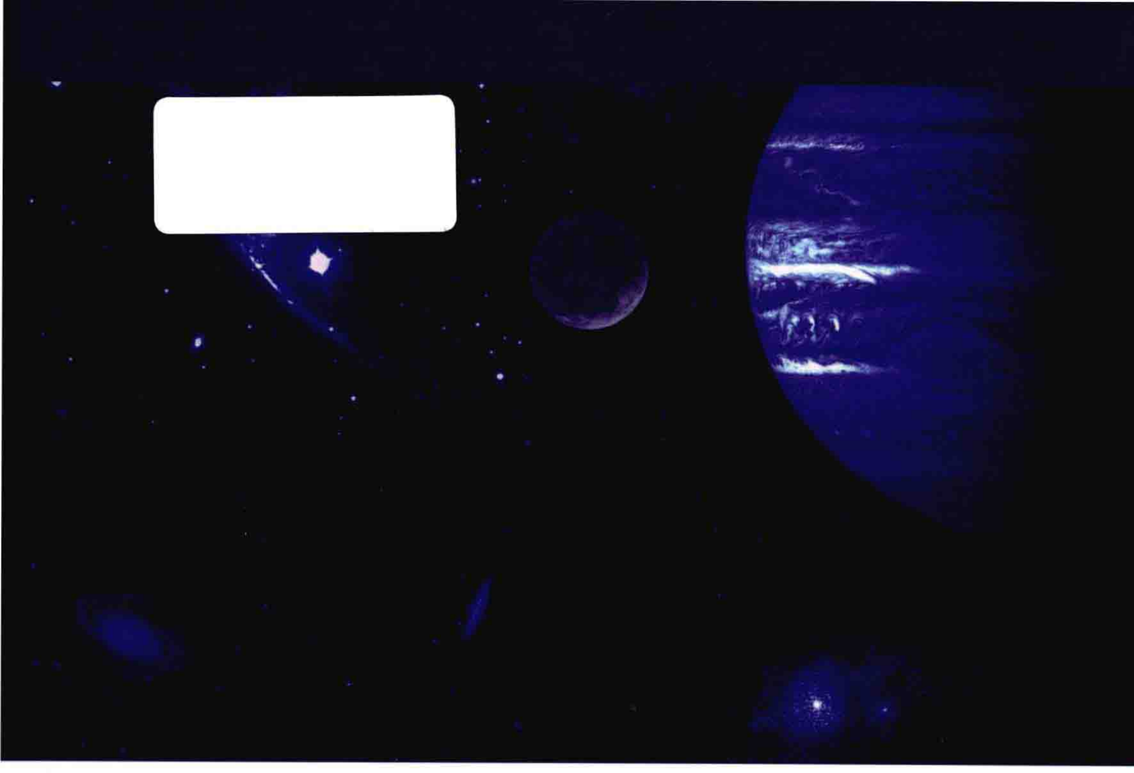

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Sepideh Stewart

Linear Algebra in Three Worlds of Mathematical Thinking

Understanding Linear Algebra Concepts Through the Embodied, Symbolic and Formal Worlds of Mathematical Thinking

Sepideh Stewart

28 August, 2008

A thesis submitted in fulfilment of the requirements for the degree of Doctor of Philosophy of Science in Mathematics Education, The University of Auckland, 2008.

To Wayne and Reuben-Wayne

Understanding Linear Algebra Concepts Through the Embodied, Symbolic and Formal Worlds of Mathematical Thinking

Sepideh Stewart, PhD. Thesis

Department of Mathematics, The University of Auckland

2008

Abstract

Linear algebra is one of the first advanced mathematics courses that students encounter at university level. The transfer from a primarily procedural or algorithmic school approach to an abstract and formal presentation of concepts through concrete definitions, seems to be creating difficulty for many students who are barely coping with procedural aspects of the subject. This research proposes applying APOS theory, in conjunction with Tall's three worlds of embodied, symbolic and formal mathematics, to create a framework in order to examine the learning of a variety of linear algebra concepts by groups of first and second year university students. The aim is to investigate the difficulties in understanding some linear algebra concepts and to propose potential paths for preventing them.

As part of this research project several case studies were conducted where groups of first and second year students were exposed to teaching and learning some introductory linear algebra concepts based on the framework and expressed their thinking through their involvements in tests, interviews and concept maps.

The results suggest that the students had limited understanding of the concepts, they struggled to recognise the concepts in different registers, and their lack of ability in linking the major concepts became apparent. However, they also revealed that

those with more representational diversity had more overall understanding of the concepts. In particular the embodied introduction of the concept proved a valuable adjunct to their thinking. Since difficulties with learning linear algebra by average students are universally acknowledged, it is anticipated that this study may provide suggestions with the potential for widespread positive consequences for learning.

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INTRODUCTION

" ...what we need is a better understanding of the problems, not just the recognition that they exist and that they are important."

William P. Thurston (1990, p. 844)

Linear algebra is an important subject in the sense that it appears in many mathematics courses, such as, abstract algebra, algebra, analytic geometry, calculus, chaos theory, cryptography, differential equations, fractal geometry, game theory, graph theory, linear programming and operations research. Linear algebra also has a multitude of applications in other fields, for example anatomy, chemistry, computer science, electrical engineering, economics, forest management, genetics, physics, statistics and population dynamics.

However, over the past few decades a small group of mathematics education researchers has been concerned with students' difficulties related to undergraduate linear algebra courses. There is agreement that teaching and learning this course can be a frustrating experience for both teachers and students, and despite all the efforts to improve the curriculum, it appears that linear algebra is a difficult subject for most students (Hillel & Sierpinska, 1993; Dorier & Sierpinska, 2001). The structure of linear algebra seems to be foreign to many first year students who have had no prior experience with the subject before coming to university. Under the traditional settings, students in linear algebra courses get introduced to many definitions and ideas very quickly before they have a chance to understand the previous ones. However at the end of the course they usually do reasonably well in their finals, since examination questions are mainly set on using techniques and following certain procedures, rather than understanding the concepts (Dorier, 1990).

At the personal level, while tutoring mathematics courses and interacting with students, I realised that some students were facing real difficulties with understanding linear algebra concepts. I soon discovered that although, some mathematics education researchers had been looking at the possibility of changing the curriculum, and had suggested new ways of teaching and handling the course, there was very little research evidence documenting students' difficulties with the linear algebra curriculum, textbooks and teaching methods. Hence, there were important issues to be considered, and strong reasons to bring the matter under scrutiny.

A preliminary study by Stewart and Thomas (2003) confirmed that the first year mathematics major students in this university were experiencing similar difficulties with understanding the linear algebra concepts and definitions, and this initiated the start of a thorough research project, and hence the beginning of this thesis.

Aim

The aim of this research was to investigate the level of conceptual and procedural understanding of linear algebra courses among first and second year university mathematics students, and to examine the outcomes from an alternate approach, namely incorporating embodied views in teaching the concepts. This research considers some major issues, such as understanding of definitions, and conceptual and procedural approaches to solving problems. It evaluates possible reasons for students' difficulties with linear algebra courses and seeks, through an intervention, to assist their conceptual understanding. The concepts to be considered include vector, scalar multiple, linear combinations, linear dependence/independence, span of vectors, subspace, basis, and eigenvectors and eigenvalues.

Moreover, the research addressed the gaps in current knowledge by constructing a framework for the learning of the above primary linear algebra concepts based on the vector space $\mathbb{R}^3$, their representational basis and a genetic decomposition into actions, processes, objects and schemas, in line with APOS and Tall's three worlds of mathematics learning theories. It also investigates students' understanding of different representations of the concepts of linear algebra, and their flexibility of movement between them and their facilitation of building appropriate schemas for solving problems.

In order to improve students' level of conceptual understanding of linear algebra, the following research questions provided a foundation for this study.

General Research Questions

- What is the level of conceptual and procedural understanding of the chosen linear algebra concepts among first and second year mathematics and science students at The University of Auckland?

- Are there any specific causes of any difficulties students have in understanding the above

linear algebra concepts? If so, in what areas do the major difficulties occur? Is there a role for definitions and representations in any difficulties?
- Does studying the above concepts, through the three worlds of embodied, symbolic and formal mathematics, help build linear algebra knowledge?

A more focused set of research questions is listed below:

Specific Research Questions

- Does a visual embodied approach to linear algebra concepts help/add to understanding, and if so, how?
- How well can first and second year students recognise and use the different representations of specific linear algebra concepts? Do students have the ability to move between different representations? Can representations other than natural language and symbolic algebra play a positive role in making definitions easier to understand?
- Can we construct a useful learning framework to emphasize the three worlds of mathematical thinking, and the transitions between action, process, object and schema conceptions in linear algebra? What variables influence the stability and usefulness of the framework, and how?

The Overview of the Thesis

The following is a brief description of the chapters of this thesis. The first chapter critically examines the related literature and the theories that have been considered as the core foundation of this thesis. This is followed by the discussion of the literature on the teaching and learning of linear algebra and related issues. Chapter 2 is devoted to the pilot study, examining students' understanding of a variety of concepts. The detailed description of the theoretical framework is presented in Chapter 3, which is based on the suggested theories and is used to illustrate the type of knowledge that students may need in order to build linear algebra knowledge for a variety of core concepts. The suggested approaches addressed in this chapter are intended to help inform teachers and instructors in charge of teaching linear algebra courses. The methodology and description of the data is addressed in Chapter 4 followed by the analysis of the results in Chapter 5. The discussion of the significant findings which are strengthened by the literature appear in Chapter 6. The thesis will conclude with the final remarks in Chapter 7.

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