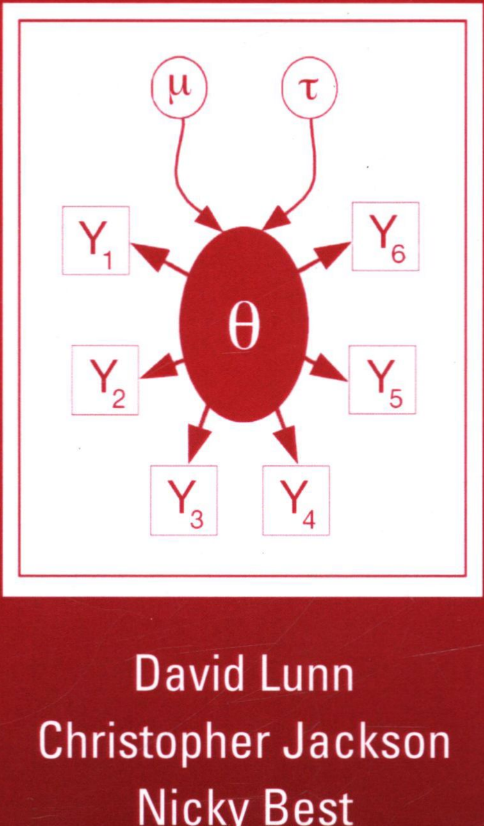
Texts in Statistical Science

The BUGS Book

A Practical Introduction to Bayesian Analysis



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Statistics

Bayesian statistical methods have become widely used for data analysis and modelling in recent years, and the BUGS software has become the most popular software for Bayesian analysis worldwide. Authored by the team that originally developed this software, **The BUGS Book** provides a practical introduction to this program and its use. The text presents complete coverage of all the functionalities of BUGS, including prediction, missing data, model criticism, and prior sensitivity. It also features a large number of worked examples and a wide range of applications from various disciplines.

The book introduces regression models, techniques for criticism and comparison, and a wide range of modelling issues before going into the vital area of hierarchical models, one of the most common applications of Bayesian methods. It deals with essentials of modelling without getting bogged down in complexity. The book emphasises model criticism, model comparison, sensitivity analysis to alternative priors, and thoughtful choice of prior distributions—all those aspects of the "art" of modelling that are easily overlooked in more theoretical expositions.

More pragmatic than ideological, the authors systematically work through the large range of "tricks" that reveal the real power of the BUGS software, for example, dealing with missing data, censoring, grouped data, prediction, ranking, parameter constraints, and so on. Many of the examples are biostatistical, but they do not require domain knowledge and are generalisable to a wide range of other application areas.

Full code and data for examples, exercises, and some solutions can be found on the book's website.

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Thomas, and Spiegelhalter

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Preface

History Markov chain Monte Carlo (MCMC) methods, in which plausible values for unknown quantities are simulated from their appropriate probability distribution, have revolutionised the practice of statistics. For more than 20 years the BUGS project has been at the forefront of this movement. The BUGS project began in Cambridge, United Kingdom, in 1989, just as Alan Gelfand and Adrian Smith were working 80 miles away in Nottingham on their classic Gibbs sampler paper (Gelfand and Smith, 1990) that kicked off the revolution. But we never communicated (except through the intermediate node of David Clayton) and whereas the Gelfand–Smith approach used image processing as inspiration, the philosophy behind BUGS was rooted more in techniques for handling uncertainty in artificial intelligence using directed graphical models and what came to be called Bayesian networks (Pearl, 1988). Lunn et al. (2009b) lay out all this history in greater detail.

Some people have accused Markov chain Monte Carlo methods of being slow, but nothing could compare with the time it has taken for this book to be written! The first proposal dates from 1995, but things got in the way, as they do, and it needed a vigorous new generation of researchers to finally get it finished. It is slightly galling that much of the current book could have been written in the mid-1990s, since the basic ideas of the software, the language for model description, and indeed some of the examples are unchanged. Nevertheless there have been important developments in the extended gestational period of the book, for example, techniques for model criticism and comparison, implementation of differential equations and nonparametric techniques, and the ability to run BUGS code within a range of alternative programs.

The BUGS project is rooted in the idea of generic reusable components that can be put together as desired, like a child's construction set but not quite as colourful. In this book we typically tackle each of these components one by one using deliberately simplified examples, but hopefully it will be clear that they can be easily assembled into arbitrarily complex models. This flexibility has enabled BUGS to be applied in areas that we had never dreamed about, which is gratifying. But it is also important to note that in many situations BUGS may not be the most efficient method, and there are many things it cannot do. Yet...

What's in the book? Perhaps we should start by saying what is *not* in the book. First, there is minimal statistical theory, neither of statistical inference nor of Markov chain Monte Carlo methods (although a presumption of some familiarity with probability theory is made). This is partly to keep the book to a manageable length, but also because the very way in which BUGS works removes the need for much of the theory that is taught in standard Bayesian statistics courses. Second, we do not cover decision theory, as BUGS has been designed for handling Bayesian inferences expressed as an appropriate posterior distribution. Finally, we take it for granted that a Bayesian approach is desired, and so barely bother to lay out the reasons why this may be appropriate.

A glance at the chapter contents will reveal that we introduce regression models, techniques for criticism and comparison, and a wide range of modelling issues before going into the vital and traditional Bayesian area of hierarchical models. This decision came after considerable thought and experimentation, and was based on the wish to deal with the essentials of modelling without getting bogged down in complexity. Our aim is to bring to the forefront model criticism, model comparison, sensitivity analysis to alternative priors, and thoughtful choice of prior distributions — all those aspects of the "art" of modelling that are easily overlooked in more theoretical expositions. But we have also really enjoyed going systematically through the large range of "tricks" that reveal the real power of the BUGS software: for example, dealing with missing data, censoring, grouped data, prediction, ranking, parameter constraints, and so on.

Our professional background has meant that many of the examples are biostatistical, but they do not require domain knowledge and hopefully it will be clear that they are generalisable to a wide range of other application areas. Full code and data for the examples, exercises, and some solutions can all be found on the book website: www.mrc-bsu.cam.ac.uk/bugs/thebugsbook.

The BUGS approach clearly separates the model description from the "engine," or algorithms and software, used to actually do the simulations. A brief introduction to WinBUGS is given in Chapter 2, but fully detailed instructions of how to run WinBUGS and similar software have been deferred to the final chapter, 12, and a reference guide to the modelling language is given in the appendices. Since BUGS now comes in a variety of flavours, we have tried to ensure that the book works for WinBUGS, OpenBUGS, and JAGS, and any differences have been highlighted. Nevertheless the software is constantly improving, and so in some areas the book is not completely prescriptive but tries to communicate possible developments.

Finally, we acknowledge there are many shades of Bayesianism: our own philosophy is more pragmatic than ideological and doubtless there will be some who will continue to spurn our rather informal attitude. An example of this informality is our use of the term 'likelihood', which is sometimes used when referring to a sampling distribution. We doubt this will lead to confusion.

How to use the book. Our intended audience comprises anyone who would like to apply Bayesian methods to real-world problems. These might be practising statisticians, or scientists with a good statistical background, say familiarity with classical statistics and some calculus-based probability and mathematical statistics. We do not assume familiarity with Bayesian methods or MCMC. The book could be used for self-learning, for short courses, and for longer courses, either by itself or in combination with a textbook such as Gelman et al. (2004) or Carlin and Louis (2008).

Chapters 1 to 6 provide a basic introduction up to regression modelling, which should be a review for those with some experience with Bayesian methods and BUGS. Beyond that there should be new material, even for experienced users. For a one-semester course we would recommend Chapters 1 to 6, most of Chapter 8 on model criticism and comparison, and Chapter 10 on hierarchical models. A longer course could select from the wide range of issues and models outlined in Chapters 7, 9 and 11, depending on what is most relevant for the audience.

Whether studying on your own or as part of a course, instructions for running the WinBUGS software are given briefly in Chapter 2 and fully in Chapter 12. A full explanation of BUGS model syntax and a list of functions and distributions are given in the appendices. Chapter 12 explains how OpenBUGS and JAGS differ from WinBUGS and gives examples of how all varieties of BUGS can be conveniently run from other software, in particular from R.

Other sources. If an accompanying text on the underlying theory of Bayesian inference is required, possibilities include Gelman et al. (2004), Carlin and Louis (2008) and Lee (2004), with Bernardo and Smith (1994) providing a deeper treatment. Other books focus explicitly on BUGS: Ntzoufras (2009) provides a detailed exposition of WinBUGS with accompanying theory, Gelman and Hill (2007) explore both standard and hierarchical regression models using both R and BUGS, while the texts by Congdon (2003, 2005, 2006, 2010) explore a staggering range of applications of BUGS that we could not hope to match. Jackman (2009) covers both theory and BUGS implementations within social science, ecology applications are covered by Kéry (2010) and Kéry and Schaub (2011), while Kruschke (2010) gives a tutorial in Bayesian analysis and BUGS with applications in psychology. Expositions on MCMC theory include Gamerman and Lopes (2006) and Brooks et al. (2011), while Gilks et al. (1996) is still relevant even after many years.

Finally, there are numerous websites that provide examples and teaching material, and when tackling a new problem we strongly recommend trying to find these using appropriate search terms and adapting someone else's code. We have always been impressed by the great generosity of BUGS users in sharing code and ideas, perhaps helped by the fact that the software has always been freely available.

A suggested strategy for inference and reporting. Rather than leaving it until later in the book, it seems appropriate to lay out at an early stage the approach to modelling and reporting that we have tried to exemplify. Bayesian analysis requires a specification of prior distributions and models for the sampling distribution for the data. For prior distributions, we emphasise that there is no such the thing as the "correct" prior, and instead recommend exploring a range of plausible assumptions and conducting sensitivity analysis. Regarding assumptions for the sampling distribution, throughout this book we try to exemplify a reasonably consistent approach to modelling based on an iterative cycle of fitting and checking. We recommend starting with fairly simple assumptions, cross-checking with graphics and informal checks of model fit which can then suggest plausible elaborations. A final list of candidate models can then be compared using more formal methods.

There have been limited "guidelines" for reporting Bayesian analyses, e.g., Spiegelhalter et al. (2004), Sung et al. (2005), and Johnson (2011) in a medical context, and also BaSiS (2001). Naturally the data have to be summarised numerically and graphically. We need to acknowledge that Bayesian methods tend to be inherently more complex than classical analyses, and thus there is an additional need for clarity with the aim that the analysis could be replicated by another investigator who has access to the full data, with perhaps full details of computational methods and code given online.

If "informative" priors are included, then the derivation of the prior from an elicitation process or empirical evidence should be detailed. If the prior assumptions are claimed to be "non-informative," then this claim should be justified and sensitivity analysis given. The idea of "inference robustness" (Box and Tiao, 1973) is crucial: it would be best if competing models with similar evidential support, or alternative prior distributions, gave similar conclusions, but if this is not the case then the alternative conclusions must be clearly reported. Where possible, full posterior distributions should be given for major conclusions, particularly for skewed distributions.

Finally. We would like to thank, and apologise to, our publishers for being so patient with the repeatedly deferred deadlines. Special thanks are extended to Martyn Plummer for his contributions to the book and for keeping us on our toes with his persistent efforts at doing everything better than us. Special thanks also to Simon White for his contribution, and to four reviewers, whose comments were extremely helpful. Thanks also to our friends, colleagues, and families for their support and words of encouragement, such as "Have you not finished that bloody book yet?" Many thanks to the (tens of) thousands of users out there, whose patience, enthusiasm, and sense of humour are all very much appreciated. And finally, we are deeply grateful to all those who have freely contributed their knowledge and insight to the BUGS project over the years. We shall be thinking of you when we get to share out whatever minimal royalties come our way!

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