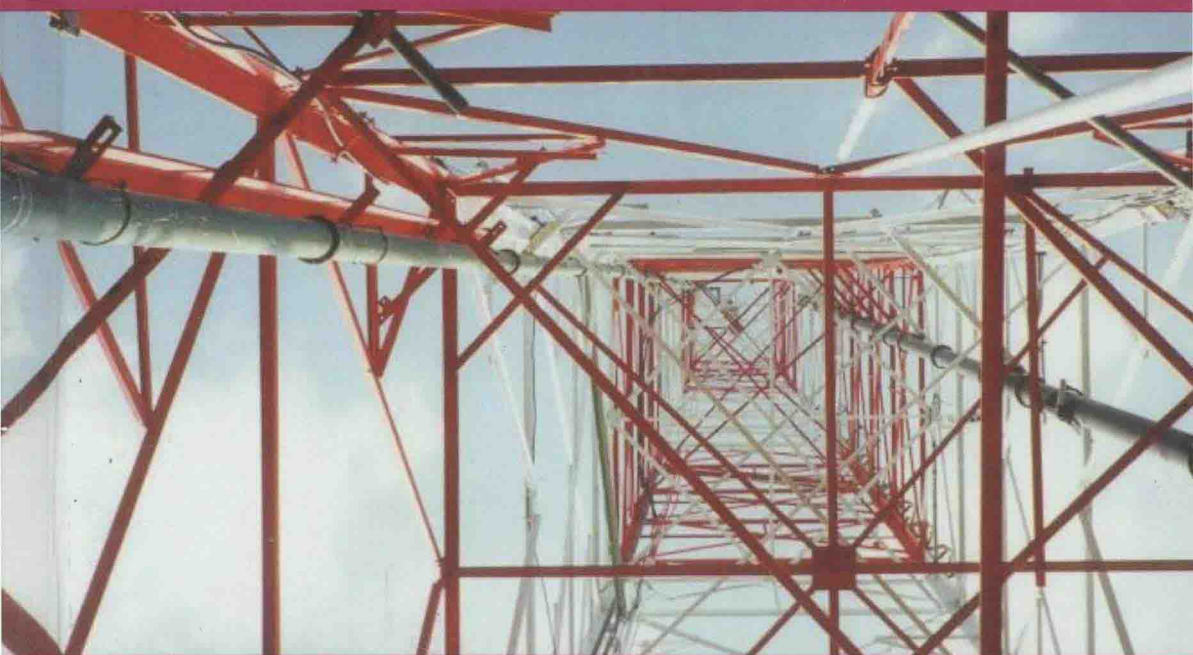


FOUNDATIONS OF CODING

Compression, Encryption, Error Correction



*Jean-Guillaume Dumas • Jean-Louis Roch
Éric Tannier • Sébastien Varrette*

WILEY

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Encryption,
Error Correction**

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Université de Grenoble

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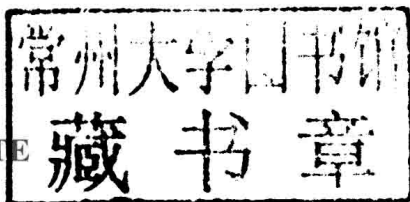
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FOUNDATIONS OF CODING

FOREWORD

This work has been initiated in spring 2000 with the creation of the joint ENSIMAG-ENSERG Telecommunication department at the National Polytechnic Institute of Grenoble (INPG – France) and the setting up of a general course (last year French Licence level in the Licence-Master-Doctorate scheme) providing an introduction to codes and their applications.

Although it was initially published as a handout, it evolved and became reference material for several courses in Grenoble universities – both in the INPG and in the Joseph Fourier University (UJF) at the master level.

We take this occasion to thank our colleagues who participated in these courses and helped us improve our material with their remarks: Gilles Debunne, Yves Denneulin, Dominique Duval, Grégory Mounié and Karim Samaké.

In 2007, a book was published in French by Dunod editions in their mathematics and computer science collection. It was then reprinted, with a few amendments, in the beginning of 2009, and edited in an augmented version in 2013.

Éric Bourre, Cécile Canovas-Dumas, Mélanie Favre, Françoise Jung, Madeline Lambert, Benjamin Mathon, Marie-Aude Steineur, and Antoine Taveneaux participated to these first two editions by reading the drafts and spotting some mistakes.

This English edition was started in 2009, when our colleagues Rodney Coleman and Romain Xu undertook the task of translating the 352 pages of the French edition in English. Let them be gratefully thanked here.

Compared to this translation, this book has been revised and significantly augmented (20% additional pages and 27 new exercises). We now cover modern and frequently used techniques, as elliptic curves, low density codes, or matrix bar-codes as well as new standards like the eSTREAM portfolio, Galois hashing and counter mode, and the new standard hashing algorithm 3, Keccak. In addition, we

have updated several parts, including steganography and watermarking, maximum likelihood decoding, saturation attacks (on AES), and postquantum cryptography.

“Foundations of Coding: Compression, Encryption, Error Correction” comes now with a companion website <http://foundationsofcoding.imag.fr>. This web site provides access to tools, resources, and news about the book, and, more generally, about security. In particular, we propose interactive solutions to several exercises via worksheets, using the free mathematical software Sage.

GRENOBLE, LYON, LUXEMBOURG
JEAN-GUILLAUME DUMAS, JEAN-LOUIS ROCH,
ÉRIC TANNIER, SÉBASTIEN VARRETTE.

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Acronyms

ADSL	Asymmetric Digital Subscriber Line	276
AES	Advanced Encryption Standard	vi
AKS	Agrawal–Kayal–Saxena	45