ABERRATIONS OF THIN LENSES

AN ELEMENTARY TREATMENT FOR TECHNICIANS AND STUDENTS

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PREFACE

Because of the widespread use of optical methods and instruments in industry today there are technicians and executives in many industries who find it necessary at times to assemble, and maybe to design, experimental arrangements of thin lenses for some purpose arising in the factory or laboratory. Such people may have no desire to become expert optical designers and their mathematical ability may in some cases be limited. This small handbook has been written to help them with such optical problems.

It is hoped that the book may also be of service to optical students in bridging the gap between elementary geometrical and physical optics and the comprehensive or advanced treatises on optical design such as those by CONRADY and HOPKINS.* I appealed for the closing of this gap during the discussion of a paper† by R. KINGSLAKE before the Optical Society, London in March, 1926. In response to this appeal there appeared in 1927 an excellent monograph by I. C. GARDNER of the U.S. Bureau of Standards, entitled *Application of the Algebraic Aberration Equations to Optical Design*, which deals with optical systems composed of thin lenses, though without entering into the derivation of the equations from the basic geometrical laws.

CONRADY'S now famous treatise on *Applied Optics and Optical Design* was published in 1929 and, though there have been criticisms of his methods, the book has continued to exert a profound influence throughout the world.

It is assumed in this text that readers are familiar with the elementary groundwork of optics and lenses‡ and with mathematics of about matriculation standard. Numerous worked examples have been included; the book will not achieve its purpose unless these are carefully studied and as many as possible of the exercises worked out by the reader himself. These exercises are mainly practical; even those which may at a first glance appear to be somewhat academic have a practical bent.

The treatment and nomenclature are based on CONRADY and on the system introduced by AIRY and CODDINGTON, developed by

* See Appendix II for a short bibliography.

† *Trans. Opt. Soc*,. 27, 4, p. 233, 1925–26. R. KINGSLAKE is now Director of Optical Design, Eastman Kodak Company, Rochester, N.Y.

‡ Suitable textbooks are: *Optics*, by W. H. A. FINCHAM, Hatton Press, London, 1954; *Light for Students* by E. EDSER, Macmillan, London, 1915; *Mirrors, Prisms and Lenses* by J. P. C. SOUTHALL, Macmillan, London, 1933.

H. DENNIS TAYLOR and adopted with modifications by I. C. GARDNER. I acknowledge with gratitude my indebtedness to CONRADY and GARDNER.

The book deals only with the primary aberrations of thin lenses and systems composed of thin lenses; this is emphasised frequently in the text. In general, the results that emerge from the corresponding third order equations provide only a skeleton framework of a final design and must be completed either by the tracing of selected rays, using trigonometrical computation or calculating machine, or by having a trial lens or system made in the workshop; and by modifying curvatures, separations or glass types in the light of the performance of the system.

It may sometimes happen, however, when the lens is relatively thin or the instrument simple and of small aperture and field, or where only an experimental arrangement is required, that these third order equations provide all that is needed. It is for this reason that the equations have been given the form found in the text, based upon the AIRY-CODDINGTON nomenclature, since this form is simple and direct.

No one will gainsay the great advantage of high mathematical ability in dealing with a subject of this kind. Nevertheless, much useful practical work can be accomplished by those who are not so gifted. Technicians who have sufficient knowledge of principles to be entrusted with the less difficult aspects of design, and with applying the discoveries of the inventor or specialist designer, are extremely useful in the factory. To read CONRADY's book demands acquaintance, for the most part, only with ordinary geometry, algebra and trigonometry; and I well remember the accomplished DENNIS TAYLOR admitting at an Optical Society meeting in 1923 that his mathematical knowledge was limited to just those subjects.

The specialist who glances through these pages will probably discover defects. I shall be grateful for his observations and for information from any reader of errors that may be found in my calculations; for these have not been independently checked.

I am grateful to Professor Conrady's daughter, Mrs. R. Kingslake, and to Dennis Taylor's son, Mr. E. Wilfred Taylor, for so kindly helping me with the portraits.

H. H. EMSLEY

EVESHAM AND LETCHWORTH
January, 1956

NOTATION

Below are given the principal characters used in the text with brief explanations of their applications and meanings. The occasional uses of a few of the characters for incidental purposes are not mentioned in the table. In those cases where similar pairs of characters occur (for example : $f\,f'$; $h\,h'$; $l\,l'$; $\mathscr{L}\,\mathscr{L}'$), the unprimed characters refer to quantities in the object space of a lens or system and primed characters to the image space.

Two types of capital letters are used to denote quantities entering into various formulae, as described in § 3.2. The reader's attention is directed particularly to the distinction between L, L′ and $\mathscr{L}$, $\mathscr{L}'$; the former denote linear magnitudes and the latter the *reciprocals* of linear magnitudes.

Symbol	First appears in Section	Defined or illustrated by Equ.	Definition or Description
a	7.2	7.1	A dimensionless quantity expressing the ratio of two incidence heights of a ray from an axial object point ; namely, incidence height on lens (y) and incidence height on entrance pupil (p).
b	7.2	7.2	The incidence height (y_p) on the lens of a principal ray from an extra-axial object point divided by the tangent of the slope angle (u_p) of that ray.
b_0	4.9		The least distance of distinct vision, conventionally fixed at 25 cm. in front of the eye.
C	7.2	7.7	The coefficient of coma.
d	3.5		The axial distance between two optical components, e.g., two surfaces of a lens or two lenses of a system ; the distance may exist in glass or in air.
e	2.3		The eccentricity of a conic.
$e\,e'$	4.5	Table 6	The distances of the first and second principal points of a system from the first and last surfaces (or thin lenses) respectively.

Symbol	First appears in Section	Defined or illustrated by Equ.	Definition or Description
E	10.1 10.3	10.3	The illumination at a point of an illuminated surface.
f	1.2	1.1	The frequency, or number of oscillations per second, of a wave disturbance.
$f\ f'$	3.2 App. I	Table 6	The first (object space) and second (image space) focal lengths of a surface or lens or system ; in a system they are measured from the principal points to the focal points.
$f_v\ f'_v$	App. I	Table 6	The front (object space) and back (image space) vertex focusing distances ; measured from the first and last surfaces of a lens or from the first and last lenses of a system to the first and second focal points respectively.
F	3.2 4.2	Table 6 4.1	The focal power of a surface, lens or system ; often called the true or equivalent power.
$F_v\ F'_v$	4.5	Table 6 App. I	The front (object space) and back (image space) vertex powers of a lens or system ; measured from the first and last surfaces of a lens or from the first and last lenses of a system.
$F_S\ F_T$	8.3 8.4 9.4	8.1 8.1*a* 8.3 8.3*a* Table 8	(*A*) The oblique powers of a spherical surface or lens in the sagittal and tangential sections respectively. (*B*) The axial powers of an astigmatic lens in the sagittal and tangential sections respectively.
$F_{s_S}\ F_{t_S}$ $F_{s_T}\ F_{t_T}$	9.4	9.7 to 9.14	The oblique powers of an astigmatic surface or lens in the sagittal and tangential sections at points situated in the sagittal and tangential meridians respectively of the surface or lens.
F	10.3	10.2	Luminous flux.
g	4.9	App I	The separation of adjacent focal points of two lenses or systems ; in a microscope, the tube length.
G_1 to G_8	3.9	3.16 G	The G coefficients ; pure functions of refractive index.

Symbol	First appears in Section	Defined or illustrated by Equ.	Definition or Description
$h\ h'$	3.4	Table 6	The sizes, within the paraxial region, of object and image; or the lateral distance from the optical axis to an object point and image point respectively.
$H\ H'$	6.4 7.2		As for h and h' but relating to objects and images extending outside the paraxial region.
$i\ i'$ $I\ I'$	1.1 3.5		The angles of incidence and refraction of a ray at a refracting surface.
I	10.3	10.2	The luminous intensity of a source of light.
j	4.6 4.10	Table 6	The interstitium or distance PP' separating the principal points of a lens or system.
J	6.2	6.2a	The refraction invariant $=n \sin I = n' \sin I'$.
K_1 to K_6	3.10	3.19K	The K coefficients; pure functions of refractive index.
K	10.3		The luminous efficiency of radiation, or power-light equivalent; is a maximum (K_m) at wavelength 555 $m\mu$.
$l\ l'$	1.7 2.2		In paraxial imagery: the object and image distances; measured from the lens (thin) or from the principal points of the lens (thick) or system.
L L'	2.1 3.3		As for l and l' but relating to extra-paraxial imagery.
$LA\ LA'$	3.6	3.10	The longitudinal spherical aberration of the incident and refracted (or reflected) pencils respectively.
$LA_p\ LA'_p$	3.7	3.11 3.16	The primary longitudinal spherical aberration of the incident and refracted (or reflected) pencils respectively.
$l_p\ l'_p$	4.7		The distances of the entrance and exit pupils from the lens or system—Figs. 5.1 and 7.1.
$\mathscr{L}\ \mathscr{L}'$	3.2	3.5p Table 6	In paraxial imagery: the "vergence" of the light incident upon or leaving a lens, mirror or system after refraction (or reflection); for systems in air $\mathscr{L} = 1/l$ and $\mathscr{L}' = 1/l'$; usually expressed in dioptres. $\mathscr{L}$ and $\mathscr{L}'$ may also be considered as denoting the *nearness* of object and image.

Symbol	First appears in Section	Defined or illustrated by Equ.	Definition or Description
$\mathscr{L}_p\ \mathscr{L}'_p$	7.2		In paraxial imagery: the nearness of the entrance and exit pupils; for systems in air $\mathscr{L}_p=1/l_p$ and $\mathscr{L}'_p=1/l'_p$
$L_{ch}\ L'_{ch}$	5.2	5.1	The longitudinal chromatic aberration of the incident and refracted pencils respectively.
L	10.1 10.3	10.5	The luminance of a source or surface element.
m	3.4	Table 6	The lateral magnification; the ratio h'/h, which is equal to $\mathscr{L}/\mathscr{L}'$.
m_p	4.9	4.4	The lateral magnification at the pupils; the ratio p'/p which is equal to $\mathscr{L}_p/\mathscr{L}'_p$.
m_u	4.9		The convergence or slope ratio; the ratio between the slope angles u and u'; $m_u=u'/u$.
$\overline{m}$	4.3	Table 6	The longitudinal magnification; the ratio between a small axial displacement of the object and the corresponding axial displacement of the image; $\overline{m}=dl'/dl$.
M	4.9	4.5 4.6	Magnifying power or angular magnification.
$n\ n'$	1.3		Refractive index; n' refers to the image space of a refracting surface, lens or system.
N	2.2		The relative refractive index from one medium to the next; $N=n'/n$.
$n_D\ n_d$	5.1		Refractive index for light of the sodium D line or of the helium d line respectively.
$p\ p'$	4.7		The radii of the entrance and exit pupils of a system.
p	2.3	2.3	A constant in the equation of a conic.
$q\ q'$	4.9	4.3	The object and image distances referred to the pupils; measured respectively from entrance pupil to object and from exit pupil to image.
r	1.7 3.1		The radius of curvature of a lens or mirror surface or of a wavefront; the radii of the two surfaces of a lens are r_1 and r_2.

Symbol	First appears in Section	Defined or illustrated by Equ.	Definition or Description
R	3.2		The curvature of a lens or mirror surface or of a wavefront ; the curvatures of the two surfaces of a lens are R_1 and R_2 ; $R=1/r$, usually expressed in dioptres.
$r_s\ r_t$	9.3		The radii of curvature of a toroidal surface in the sagittal and tangential sections at any point in the sagittal meridian of the surface.
r'_s	9.3		The radius of curvature of a toroidal surface in the sagittal section at an extra-axial point in the tangential meridian of the surface.
$R_s\ R_t$ R'_s	9.5		The curvatures corresponding to r_s, r_t and r'_s.
$s\ s'$	3.3 6.2	3.6 6.1	The distances along the principal ray of a narrow oblique pencil from a refracting (or reflecting) surface or a lens to the sagittal object and image respectively.
S	3.10	3.19	The coefficient of spherical aberration.
$\mathcal{S}$	10.3	10.5	The area of a radiating surface.
$t\ t'$	6.2	6.1a	The distances along the principal ray of a narrow oblique pencil from a refracting (or reflecting) surface or a lens to the tangential object and image respectively.
t	7.8	7.I Pl.	The thickness of a parallel-sided plate or the length of path of an axial pencil through a reflecting prism ; t is the actual distance in the glass.
$T_{ch}\ T'_{ch}$	5.2	5.2	The transverse chromatic aberration of the incident and refracted pencil respectively.
$u\ u'$	3.3 3.5	3.1p	In paraxial imagery : the slope angles of two conjugate rays ; the angle between the optical axis and the ray in the object and image spaces respectively ; u is sometimes called the half aperture angle.
$U\ U'$	1.3 3.1	3.1	As for u and u' but relating to extra-paraxial imagery.

Symbol	First appears in Section	Defined or illustrated by Equ.	Definition of Description
u_p u'_p	4.7	7.2	The slope angles of the principal ray of a pencil at the entrance and exit pupils ; u_p is sometimes called the half field angle—Figs. 5.1 and 7.1.
v	4.6	Table 6 4.7	The total axial distance BB' from axial object B to conjugate image B'.
V	5.1	Table 7	The reciprocal of the dispersive power of a medium ; sometimes called the constringence ; $V=(n_d-1)/(n_F-n_C)$.
V_0 V	1.2 1.6	1.1 1.4	The velocity of light in vacuo and in a medium.
w w'	4.2	4.1	w is the angle subtended by an object at a given point ; the point is usually the axial point of a lens (thin) or the first principal point of a thick lens or system ; w' is the conjugate angle in the image space—Fig. 4.4.
x x'	4.9	Table 6	The focal point distances of object and image respectively ; the distance from first focal point F to object and from second focal point F' to image.
y Y	1.7 7.2	1.6 7.1	The height of incidence on a lens (or mirror) of a ray from an axial object point.
y_p	5.2 7.2	5.2 7.2	The height of incidence on a lens (or mirror) of the principal ray of a pencil from an extra-axial object point ; $y_p=l_p \,.\, \tan u_p$—Figs. 5.1 and 7.1.
z	8.6		The radius of a blur patch formed in a given plane or on the retina due to the pencil being out of focus or afflicted with aberration.

Symbol	First appears in Section	Defined or illustrated by Equ.	Definition or Description
			Greek Characters
α	4.10	4.7	(*A*) When a lens of thickness d images an object B at B' then $\alpha = BB' - d = v - d$.
	10.3	10.1	(*B*) The half plane angle of a cone.
β	3.10	3.17	The form factor or bending factor of a thin lens.
γ	3.10	3.18	The conjugates factor in imagery by a thin lens.
δ	4.1	Table 6	The deviation of a ray when refracted by a surface or lens.
ϵ	4.10	4.7	A dimensionless constant $= 1 - \beta^2$.
λ	1.2	1.1	The wavelength of light in air.
	1.6	1.4	
μ	4.10	4.7	A dimensionless constant $\doteq -(1-m)^2/m$.
ρ	10.3		The reflection factor of a surface.
ϕ	2.4	3.3	The angle subtended at the centre of a refracting or reflecting surface by a given extent AD of the surface; sometimes called the central angle—Figs. 2.7 and 3.1.
ω ω'	10.3 10.6	10.1 10.11	The solid angle of a cone; ω' applies to the cone of light in the image space of a lens or system.

CONTENTS

ERRATUM

Page 337, *line 5, should read:*

$a^2 = b^2 + c^2 - 2bc \cos A$

CHAPTER I

GEOMETRICAL OPTICS

1.1. The Laws of Geometrical Optics

In this small volume we are to be concerned almost entirely with matters that are investigated by the methods of geometrical optics. This subdivision of optics may be broadly defined as the theory of mirrors, lenses and optical instruments and is founded on four laws or postulates ; these are :

1. The law of rectilinear propagation of light ; in a homogeneous and isotropic medium light travels in straight lines.

2. The mutual independence of different portions of a beam of light ; when two rays of light intersect they continue along their original paths as though each ray existed separately.

3 and 4. The laws of reflection and refraction ; when light is incident on a boundary surface between two optical media of refractive indices n and n', the reflected and refracted rays remain in the plane of incidence ; the angles of incidence (I) and reflection (R) are equal ; the angle of refraction (I') is given by $n' \sin I' = n \sin I$.

In ordinary everyday experience and indeed in the exact measurements made in surveying and astronomy and in the use of sighting instruments, these postulates are accepted as true facts and lead to accurate results. They *are* true up to a point, but they are not the whole truth. We use them in spite of this because the study of a very large class of optical phenomena, particularly those upon which the design and construction of optical instruments chiefly depend, is rendered thereby much simpler ; so much so that the laws of geometrical optics are indispensable in the practical study of optics. But they are limited in their scope and in the last analysis a " ray " of light is an abstraction which does not actually exist ; and so it is necessary to establish a justification for the use of geometrical optics and to enquire into the conditions under which its laws can be accepted as valid, and useful. This requires a short review of the manner of wave propagation.

1.2. Waves and Rays. Huygens-Fresnel Principle

When energy is discharged from a source, it continues to propagate itself outwards in all directions as radiant energy at a speed depending on the nature of the surrounding medium and at the same speed whether the source is weak or strong. Since we have insufficient knowledge about the nature of light we cannot know precisely how it is propagated; but many of the effects it produces (interference, diffraction) force us to the view that whilst it is travelling through space and through media that are transparent to it, it possesses wave-like properties. The disturbance set up in the carrying medium is electromagnetic in character. At any point of the medium both the electric field and the magnetic field vary periodically in magnitude and direction; they grow to a certain maximum value in one direction, fall to zero, grow to an equal maximum in the opposite direction and again fall to zero; and so on. These periodic changes constitute what we call electric and magnetic oscillations at that place, just as the swinging of a pendulum constitutes a mechanical oscillation. These periodic changes at one place induce (MAXWELL) corresponding changes in neighbouring places, which in turn induce the same sequence at points further outwards; thus the same sequence of values of field strengths appear ranged periodically along straight lines radiating outwards from each element of the source. This is an **electro-magnetic wave.**

The disturbance that travels outwards in all directions over the surface of a sheet of water when it is agitated at some point has just the same periodic qualities and we may use these transverse water waves to supply a simplified mental picture of the electro-magnetic waves of light. Whatever the *nature* of the waves, they can all be represented graphically and treated mathematically in exactly the same way; we need postulate nothing more than that the disturbance is a periodically varying quantity acting transversely to the direction of propagation. Thus Fig. 1.1, exhibiting the conditions

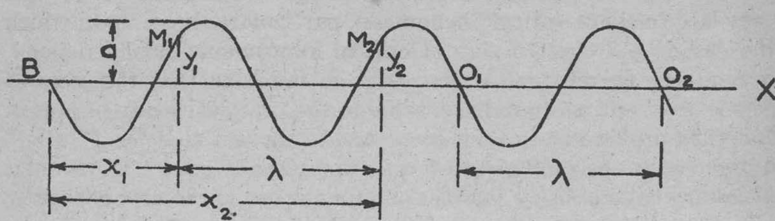

FIG. 1.1.—Graphical representation of simple transverse wave.

along one direction from the source at B, may be considered as representing the rise and fall of the water level in water waves, or the vibrations of the supposed ether particles in the early mechanical wave theory of HUYGENS or the periodic changing of the field strengths from positive to negative in the electromagnetic light wave.

The diagram shows the simplest form of wave motion that can be represented mathematically. It may be considered as a picture of the carrying medium (e.g. the water surface) at a given instant of time ; it could alternatively be regarded as representing the manner in which the electric (or magnetic) field strength varies with time at a given point of the medium.

λ = wavelength ; i.e. distance separating two points of the medium that are in the same condition or phase.

a = amplitude of wave.

T = period, or time taken for one complete oscillation.

f = frequency, or number of oscillations per second $= \frac{1}{T}$.

V_0 = wave velocity, with which the disturbance travels along the direction BX in vacuo.

Since the disturbance will have travelled a distance λ, say from M_1 to M_2 or from O_1 to O_2, in time T we can write

$$V_0 = \frac{\lambda}{T} = \lambda f \quad \ldots \ldots \quad (1.1)$$

which applies to any form of wave motion.

It can be shown that the rate at which energy is carried along by a wave is proportional to the square of the amplitude. Since the energy flow, or flux, varies as the inverse square of the distance from the point source, it follows that the amplitude varies inversely as the distance.

Light is propagated in a three-dimensional medium and it is hardly possible to visualise the mode of propagation of the electromagnetic wave. HUYGENS (1678) proposed a simple conception ; though it is necessarily incomplete since he was thinking of longitudinal vibrations in an elastic solid ether,* it still serves adequately, as extended by FRESNEL, to explain the formation of optical images. Suppose the point source† B in Fig. 1.2 is situated in a uniform medium so that the velocity of propagation is the same everywhere and in all directions. At successive intervals of time the light of any particular wavelength reaches spherical surfaces W_1, W_2, W_3 etc. which we may call lightfronts or wavefronts. A wavefront is the continuous locus of points *which are in the same phase of vibration* —a surface of equal phase—and the energy is spread out uniformly over such a surface. The wavefronts shown are supposed to be successively $\frac{1}{2}\lambda$ apart. At a certain moment the disturbance has

* That is, a progressive series of collisons between the ether particles of the same type as those observed during shunting operations on a railway siding.

† Actually there is no such thing as a *point* source, but we will use this term to signify a very small area of a luminous surface ; similarly in later chapters we speak of a point object or a point of an object.

reached the wavefront W_5; all the paths *Ba*, *Bb*, *Bc* etc. have been traversed in the same time; and all such points as *a*, *b*, *c* ... are in the same phase. According to the principle enunciated by HUYGENS, the formation of a further wavefront such as W_6 can be visualised by assuming that each of these points now acts as a centre of a new disturbance emanating from that point; just as if it were an independent source except that no energy is propagated backwards.

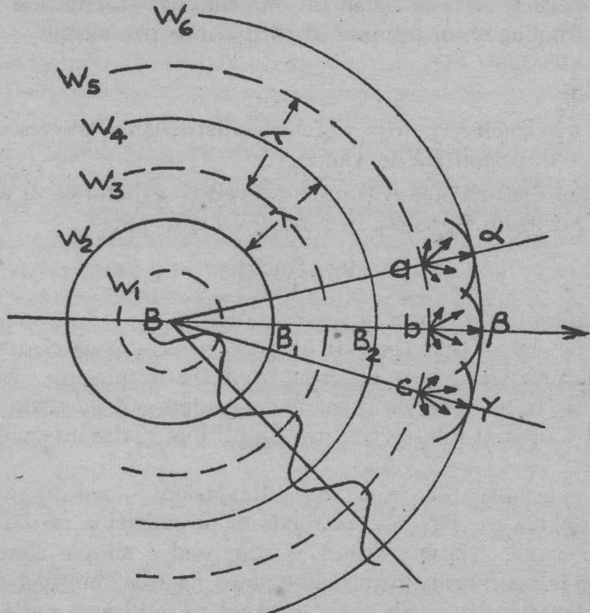

FIG. 1.2.—Waves and rays. Effect at any point β due mainly to energy travelling along ray $B\beta$.

The energy travels in *all* forward directions from each point *a*, *b*, *c*, etc.; but the intensity of this energy is greatest along the directions $Ba\alpha$, $Bb\beta$, $Bc\gamma$, etc. which radiate from the source and are normal to the wave-fronts; the intensity along oblique directions from these points, along $a\beta$ or $c\beta$ for example, decreases progressively with increasing obliquity. The total energy arriving at any point such as β further out in the medium is thus made up, not only of the energy reaching it along the one normal direction $Bb\beta$ from its "pole" *b*, but also of energy reaching it along innumerable other directions such as $a\beta$, $c\beta$, etc., from a large area of the spreading wavefront. These oblique contributions arrive at β in various phases and intensities and the resultant effect at β is the sum of them all.

To find this sum requires the application of appropriate mathematics, involving integration. This has been carried out by KIRCHHOFF, FRESNEL and others, using the principle of interference propounded by YOUNG,*, the result being that, provided there is no obstacle to prevent the disturbances from an appreciable area of the advancing wave reaching β, the net effect at β is such as would be produced by an element of the wavefront at its pole b acting alone. It can indeed be appreciated without formal proof that all the components of the electrical forces perpendicular to the normal direction $Bb\beta$ from points such as a on one side of this normal will cancel the similar transverse forces from points such as c on the other side. Thus if the wave is unrestricted as stipulated we may *assume* for all practical purposes that the effective light energy reaches β sensibly along the line $Bb\beta$. Such a line is called a **ray** of light; it is normal to the wavefront. We see that there is justification for the first geometrical law, that light travels in straight lines.

The advancing wave being unrestricted by obstacles or apertures as in Fig. 1.2, the conditions are the same at points such as α and γ as they are at β, the effects at α and γ being considered as due solely to the small elements of the wavefront at the points a and c respectively; just as if the light-streams or rays $Ba\alpha$, $Bb\beta$ and $Bc\gamma$ were independent of one another as stated in the second geometrical law. Thus the sphere W_6 which envelops all the wavelets of equal radii $a\alpha$, $b\beta$ and $c\gamma$ is a new wavefront. All points on it have been reached in the same time and are in the same phase. The light energy is spread uniformly over it and the process of advancing and spreading continues.

1.3. Diffraction. Image of Self-luminous Object Point by Perfect Optical System

The effect (illumination) at any point such as β will be altered if any kind of obstacle is interposed in the region between B and the area $\alpha\beta\gamma$ surrounding β since it will cut off from β some of the light-streams which would otherwise reach it. Because of the smallness of the wavelength of light the change of effect will not be visually appreciable, however, unless the obstacle is very close to the line $B\beta$ joining β to the source. In practical optics we are mainly concerned with the effects of apertures and in Fig. 1.3 a diaphragm with a circular aperture SS of finite area has been inserted in the path of the advancing wave. The effect at any point V, W, X, Y or Z depends, as always, upon the sum total of all the disturbances reaching the point from *all* elements a, b, c, etc. of the wavefront to which it is exposed. In symmetrical cases, such as a circular aperture, the effect can be calculated mathematically. In the region of

* See FINCHAM'S *Optics*, Chapter XV.