Applications of Calculus to Biology and Medicine

Case Studies from Lake Victoria

Nathan C Ryan • Dorothy Wallace

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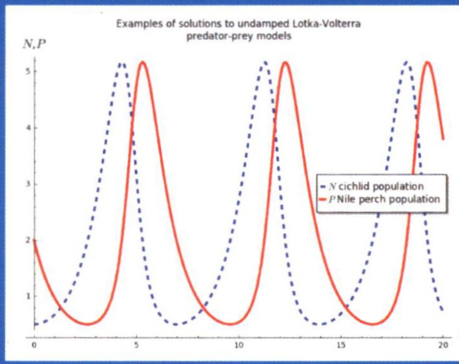

Biology majors and pre-health students at many colleges and universities are required to take a semester of calculus but rarely do such students see authentic applications of its techniques and concepts. *Applications of Calculus to Biology and Medicine: Case Studies from Lake Victoria* is designed to address this issue: it prepares students to engage with the research literature in the mathematical modeling of biological systems, assuming they have had only one semester of calculus. The text includes projects, problems and exercises: the projects ask the students to engage with the research literature, problems ask the students to extend their understanding of the materials and exercises ask the students to check their understanding as they read the text. Students who successfully work their way through the text will be able to engage in a meaningful way with the research literature to the point that they would be able to make genuine contributions to the literature.

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Preface

This book is about how quantitative reasoning plays out in simple biological examples. The quantitative reasoning the reader will engage in does not end with discussions of units and error margins — to us it is at its most useful when thoroughly integrated with traditional mathematical content such as calculus. The book, as we've conceived it, is meant to highlight the importance of calculus and other more advanced mathematical topics in the biological sciences in general.

Mathematics in the Biological Sciences

It is more important than ever that researchers and practitioners in biological fields know how to think quantitatively and use mathematical tools to their advantage. One can get a feel for the growth of mathematical tools in biology by comparing research articles found by Google Scholar, as in the table below.

On May 31, 2017 a brief Google Scholar search revealed the following number of items under these categories. Two searches were done on six terms, one for the whole database and the other for entries since 2016. In the percentages given below, we assume that the items found with the modifier "mathematical model" would be counted in the larger search.

Although this table represents only a small sample of the literature, it appears that the role of mathematics in the biological sciences has increased. This should not be a surprise, as mathematics gives extra predictive power beyond mere observations, and scientists want this predictive power. Researchers in these three aspects of mathematical biology (such as the book's second author) can vouch for the fact that many articles are written by biologists who use mathematics to explain and extend the power of their results, and many are written by mathematicians inspired to study a biological system. Many are written by interdisciplinary teams.

Google Scholar search term counts.

Search term No specified time period	Count	Mathematical search term	Count	Percent mathematical
"ecology"	3,540,000	"mathematical model ecology"	174,000	4.9
"malaria"	1,680,000	"mathematical model malaria"	55,000	3.3
"tumor"	3,730,000	"mathematical model tumor"	273,000	7.3
Since 2016:				
"ecology"	110,000	"mathematical model ecology"	27,000	24.5
"malaria"	56,100	"mathematical model malaria"	5,270	9.4
"tumor"	144,000	"mathematical model tumor"	17,500	12.2

Not all models are of equal quality of course. Some give predictions a practitioner might well believe, and others are no better than guesses.

Those who wish to read, understand and use the insights gained in this research need to think critically and be knowledgeable consumers of this quantitative information. This book is intended to equip such a person with the tools and intuition to be able to carry out this kind of work.

How we have used this book

Both authors have taught this material to a mixed class of biology students with little math background and also math majors who have taken at least linear algebra and often differential equations. We can verify that both groups come to the course with remarkably little recollection of what they learned in either their biology or math classes. Colleges and universities typically require math courses, pre-medical education usually includes calculus and biology, and more recently quantitative reasoning courses are required at the college level or within a discipline. Such courses will do little good in the long run if the material learned is not remembered or cannot be transferred to new problems. So an important pedagogical problem that this book addresses is this: How can we engage students from both of these groups in a way that fosters critical and creative thinking, and that sticks with the student after the course is over?

Most calculus books and courses help students do the following:

(1) learn some new technique,
(2) apply it to a series of practice problems,
(3) take a test on it, then
(4) go on to the next new technique,

Most biology books and courses help students do the following:

(1) read a huge amount of information,
(2) use some of it in labs,
(3) take a test on it, then
(4) learn more information

We wanted to do design quite a different class and to help both kinds of students have a more meaningful experience. The book (and the courses we have taught from it) shift the emphasis to different pedagogical approaches. So when we have used the book in class, our interest is in turning students into independent researchers and putting them in charge of posing and answering their own research problems.

After a lifetime of being handed math problems to solve, any student might be forgiven for experiencing alarm at the question, "What problem would you like to solve?" And yet forceful arguments have been made that learning that sticks is exactly the learning based on this question. Research experiences for undergraduates are known to improve learning and retention, but usually these are relegated to internships and summer programs. By creating a rich experience for students right in the classroom we hope to give them a chance to ask a question of interest to them, find their own unique answer to it, work with a team of enthusiastic peers, and write a paper of which they can be truly proud. All of these experiences are built to tie the mathematics and the biology to students' own emotions and motivations, thereby causing it to stick.

The roles of classroom actors

Actor	Role in traditional format	Role in research based format
Student	Absorbs assigned information, learns assigned computational techniques and reasoning. Takes tests.	Poses the research question and learns material necessary to solve problem. Writes research papers.
Teacher	Explains and describes, sets tasks to be completed, judges performance on intermediate tasks.	Asks additional questions, critiques thinking, helps modify approach, is a member of every team, does not judge the process of development but only the outcome.
Content delivered in class or textbook	Information to be tested on.	Ideas that might be helpful in students' own research.
Textbook	A resource for everything to be learned in the course. The whole mountain.	A platform from which students begin to form questions and strategies. Base camp.
Assessment	Tasks set by instructor in homework and examinations on course material.	Research papers by groups of students on just about anything to do with biology.
Research literature	Peripheral (to learning)	Central (to research)
Final grade	Examinations made with the express intention that not everyone will get the same grade.	Papers judged against a standard, not against each other.

Students as researchers

In our classrooms the roles played by the various actors differ considerably from traditional educational forms, summarized in the table below.

By concentrating on getting the best possible solution to their own research problem, students naturally encounter plenty of small problems in quantitative reasoning that arise naturally as they try to make the data from any experiment or field study relevant to their mathematical model. A deeper understanding of the meaning and importance of calculus happens equally naturally in the context of building systems of differential equations. They become critical evaluators of the published papers they are using to study their problem. They have an interdisciplinary research experience, without extra cost to them, a funding organization, or an institution. A course taught from this book can give a student with interests in biology a clear and tangible understanding of the value of quantitative reasoning, in general, and calculus, in particular, in the posing and solving of problems.

As an additional outcome, our attitude toward teaching is completely altered by this approach. It our privilege to work with these students. We nearly always learn something new from them. We get an overview of potential research areas we would not have thought about otherwise. We build a base of future colleagues with whom we may write papers in the future. All of these things compensate greatly for the additional time and attention this sort of teaching requires. We hope this book will lighten the burden of teaching this kind of course.

Some details about the book

The book is divided into five parts. The first part is some background on the Lake Victoria region and some review of calculus. The review of calculus focuses on terminology and physical intuition. The next four parts cover four general areas in biology in which biologists and mathematicians alike have posed and solved problems using the techniques of calculus. The four parts consist of three different kinds of chapters. Some chapters explain the biology and mathematics required to understand the kinds of problems posed in the fields covered by the book, some chapters discuss computational techniques used to solve problems, some chapters introduce the reader to what it is like to carry out research and some chapters develop the reader's ability to think like a mathematical modeler. The variety of fields within biology covered by the book as well as the variety of the kinds of things the reader is being asked to read and think about speak to the deeply interdisciplinary nature of the material and our approach to it.

Almost every chapter contains its own bibliography to allow the reader a springboard for finding related work that may be of interest. The book contains three different kinds of tasks (in addition to reading) for reader to engage in. Interspersed throughout the reading are short, contextualized questions to help the reader check their own understanding. If the reader cannot give a somewhat immediate answer to these "Exercises", then we recommend the reader back up and make sure that they have understood everything up to that point in the chapter. At the end of most chapters we have "Problems for Exploration," which, as in most textbooks serve as practice for the kind of tasks we want students to be able to do but, unlike most textbooks, the problems and tasks are practice for the carrying out of research. The questions are often open ended and require the application of the general intellectual and modeling skills we are trying to develop in the reader. Finally, at the end of each part, we present sample projects that the reader could undertake. In our experience, something interesting will result if the reader takes an existing paper in the literature and "tweaks" it in some small way. The projects we present are in that spirit and are provided to show what students in this class have been capable of.

The programming language highlighted in this book is Sage, but the computational methods required to carry out these projects is available in other computer algebra systems and mathematical programming languages. We chose Sage because it is a Python library and, as such, is fairly readable and intuitive. The sample code and all the plots in the book are available for download at

`https://github.com/nathancryan/math-bio-book`

Conclusion

We acknowledge that this book is unconventional but we believe that this kind of book and the kinds of things it asks of its readers can impact the way its readers connect with mathematics and biology. For readers who are biologists, they will be given tools to understand biological questions in ways that they otherwise would not be able to. For readers who are mathematicians, they will be shown a new landscape in which their way of understanding the world can have an impact. For readers who are neither, we hope that they will appreciate a guided and interactive tour through a truly interdisciplinary field.

All scientists have three main tasks: Problem Posing, Problem Solving and Peer Persuasion. We believe that this book and courses taught in the way that we have described will train students to carry out these three essential tasks.

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