

CIVIL ENGINEERING AND GEOMECHANICS SERIES



Equilibrium and Transfer in Porous Media 3

*Applications, Isothermal Transport
and Coupled Transfers*

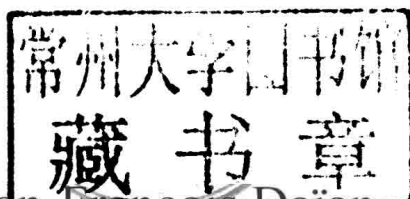
Jean-François Daïan

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Series Editor
Félix Darve

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Equilibrium and Transfer in Porous Media 3

Nomenclature

**Notations marked with asterisk (*) can have several different meanings. In general, their meaning will be clear from the context in which they are used.*

Notation	SI unit	Name	Reference section (volume) [Equation]
A, B, C, \dots		Arbitrary constants	
A_{jX}, A_{qT}	Variable	Principal transfer coefficients	2.1.2.2 [2.18]
c, \bar{C}_k	kg/kg	Concentration, mass fraction	1.2.2.3 (2)
C	$\text{J K}^{-1} \text{m}^{-3}$	Thermal capacity per unit volume	1.2.3 (2), 1.4.3
$\bar{\bar{C}}$	m^{-2}	Inverse of permeability tensor	1.5.4.3 (2)
C_{JT}, C_{qX}	Variable	Coupling coefficients	2.1.2.2
d	m	Pore diameter	
d_c	m	Characteristic (critical) pore diameter	2.1.2.2 (2)
d_h	m	Hydraulic diameter	1.4.2.2 (2), 1.3.6.2, [1.35]
$D_k, D_0, \bar{\bar{D}}_{ef}, D_{ef}$	$\text{m}^2 \text{s}^{-1}$	Diffusion coefficients or tensors	1.2.2.1, (2), 1.2.2.2, [1.7] (2) 1.4.3.2, [1.26] (2)
D_θ	$\text{m}^2 \text{s}^{-1}$	Hydric diffusivity	2.2.2.4, [2.26] (2), 2.2.3 (2)
D_T, D_T^*	$\text{m}^2 \text{s}^{-1}$	Thermal diffusivity (pure, apparent)	2.1.2.5, [2.22]

$\overline{\overline{D}}_k^{disp}$	$\text{m}^2 \text{s}^{-1}$	Dispersion tensor of the component k	1.4.3.4, [1.27] (2)
$\overline{\overline{D}}$	s^{-1}	Strain rate tensor	1.5.2.1 (2)
E	M	Thickness of the wall layer	1.2.1.1, [1.15]
E_k	$\text{m}^2 \text{s}^{-1}$	Effusion coefficient of the component k	2.3.2.5, [2.48] (2)
f	-	Resistance factor to diffusion	2.1.1.5, [2.11]
g_k, \vec{g}_k	$\text{mole m}^{-2} \text{s}^{-1}$	Density of molar flux*	1.2.2.1, [1.3] (2)
g	m s^{-2}	Graviational field*	
g	Variable	“Conductance”	1.5.7.2 (2)
h	Variable	Exchange coefficient	2.2.1.3, [2.14] (2)
h_ρ	m s^{-1}	Mass exchange coefficient	1.2.1.1, [1.15]
h_T	$\text{J m}^{-2} \text{s}^{-1} \text{K}^{-1}$	Heat exchange coefficient	1.2.1.4
\vec{J}_X	$\text{X m}^{-2} \text{s}^{-1}$	Flux density of the value X	1.4.1.2 (2)
$J, \vec{J}_k, \vec{J}_\alpha$	$\text{kg m}^{-2} \text{s}^{-1}$	Mass flux density, of the component k , of the α -phase	1.2.2.2 (2), 1.5.2.3, [1.48] (2)
\vec{j}_k	$\text{kg m}^{-2} \text{s}^{-1}$	Diffusive mass flux density of the component k	1.2.2.2 (2), 1.4.1.3, [1.21] (2)
$k, \overline{\overline{k}}$	m^2	Intrinsic permeability (tensor)	1.4.2.1, [1.22] (2)
K_f	$\text{m}^2 \text{Pa}^{-1} \text{s}^{-1}$	Hydraulic conductivity	2.2.2.2, [2.19] (2)
\hat{K}	m s^{-1}	Hydraulic conductivity	2.2.2.2, [2.21] (2)
K	$\text{m}^2 \text{Pa}^{-1} \text{s}^{-1}$	“Hydric conductivity”	2.2.2.3, [2.23] (2) 2.2.3, [2.30] (2)
l_m	M	Molecular mean free path	2.1.2.1 (2)
L_v	J kg^{-1}	Heat of vaporization	2.4.3.2 (1), 1.2.1.4
L_{sub}	J kg^{-1}	Heat of sublimation	1.2.2.3 and 1.4.4.1
L_{sl}	J kg^{-1}	Heat of liquefaction	2.3.1 (1)
M	kg mole^{-1}	Molar mass	
N, N_k	mole kg^{-1}	Moles per unit mass	1.5.2.3 (2)
n, n_k	mole m^{-3}	Molar concentration of a gas, of a component	2.4.1.1 (1), 1.2.2.1 (2)
\vec{n}	-	Normal unit vector	
Nu	-	Nusselt number	1.4.4.2, [1.20]
P	Pa	Pressure	

P_c	Pa	Capillary pressure	1.2.3, [1.1] (1)
Pe	-	Péclet number	1.4.3.4, [1.28] (2) 1.3.5.2, [1.50]
Q	$\text{m}^3 \text{s}^{-1}$	Volume flow rate	
q, \bar{q}	$\text{J m}^{-2} \text{s}^{-1}$	Density of conductive thermal flux	1.2.3, [1.9] (2)
q^*, \bar{q}^*	$\text{J m}^{-2} \text{s}^{-1}$	Apparent density of thermal flux Apparent density of thermal flux (freezing)	2.1.1.3, [2.8] 2.2.1.4, [2.34]
R	$\text{J mole}^{-1} \text{K}^{-1}$	Ideal gas constant	
R_c	M	Interfacial curvature radius	1.2.3, [1.1] (1)
Re	-	Reynolds number	1.2.1.3 (2), 1.4.4.2, [1.60]
S	$\text{m s}^{-1/2}$	Sorptivity*	1.1.1.1, [1.4]
S	$\text{J K}^{-1} \text{kg}^{-1}$	Specific entropy*	1.5.2.1 (2), footnote 32
S	m^2	Section*	
S_X	$\text{X m}^{-3} \text{s}^{-1}$	Volume source for the value X	1.4.1.2 (2)
s	$\text{J K}^{-1} \text{m}^{-3}$	Volume concentration of entropy*	1.5.2.2 (2)
t	s	Time, date	
T, \hat{T}	K	Kelvin temperature, Celsius temperature	
U	J kg^{-1}	Specific internal energy	1.5.2.1 (2), footnote 32
u	J m^{-3}	Volume concentration of internal energy*	1.4.1.2 (2), footnote 30
u	$\text{m s}^{-1/2}$	Boltzmann's variable*	1.1.1.1 and 1.4.1.1
V	m s^{-1}	Flow velocity, Darcian velocity	1.2.1, 1.4.1.3 (2)
V_f	m s^{-1}	Velocity of the fluid phase	1.5.3.1, [1.57] (2)
v_i	m s^{-1}	Cartesian coordinate of flow velocity	1.5.1.3 (2)
\bar{v}	m s^{-1}	Average molecular velocity	2.3.2.1 (2), [2.41]
x, y, z, x_k	m	Cartesian coordinates*	
x, x_i	-	Mole fraction*	2.4.4.1 (1), 2.3.2.7 (2)
X	X m^{-3}	Volume concentration of the value X	1.4.1.2 (2)
z	m	Descending vertical coordinate	

Greek Symbols			
α	K^{-1}	Coefficient of thermal variation*	
α_{vs}, β_{vs}	K^{-1}	Coefficient of thermal variation of the density, (of the pressure) of the saturated vapor	1.2.1.4 1.4.4.1
α	$m^2 s^{-1}$	Diffusivity (particularly thermal)*	1.2.3 (2), 1.4.1
β_D	-	Knudsen factor	2.1.2.2 (2)
β_K	-	Klinkenberg factor	2.1.2.3 (2)
β_X	Variable	Capacity coefficient	2.1.2.2, [2.15]
$\chi_\alpha, \gamma_\alpha, \zeta_\alpha$	-	EMT coefficients	1.5.7.1 (2), 2.3.4.1 (2)
$\bar{\delta}, \delta_{ij}$	-	Unit tensor, Kronecker symbol	
ε	-	Porosity	1.2.4.2 (1)
ε_α	-	Volume fraction of the phase α	1.3.2.3, [1.12] (2) 1.4.1.2, [1.17] (2)
φ	-	Vapor saturation level	1.3.2, [1.4] (1),
φ_H, φ_c	-	or relative humidity	2.1.3.1 (2) 2.3.1.1, [2.38]
Γ	Variable	Capacity	2.2.1.2, [2.13] (2)
Γ_c	Pa^{-1}	Capillary capacity	2.2.2.1, [2.17] (2)
$\hat{\Gamma}_c$	m^{-1}	Capillary capacity	2.1.2.2
Γ_φ	-	Hygroscopic capacity	2.2.2.4, [2.27] (2)
γ, γ_k	-	Capacity in relation to the component k	2.2.6.1, [2.37] (2)
$\Gamma_{\theta/T}$	K^{-1}	Cryogenic capacity	2.2.1.3, [2.30]
Γ_T	$J m^{-3} K^{-1}$	Thermal capacity	2.1.2.1, [2.14]
Γ_T^*	$J m^{-3} K^{-1}$	Apparent thermal capacity (freezing)	2.2.1.4, [2.34]
$\eta(d)$	-	Cumulative pore size distribution	3.1.1.2, 1.3.6.1
κ_v, κ_n	-	Relative permeabilities	2.1.1.3, [2.2] (2)
λ, Λ	$J m^{-1} s^{-1} K^{-1}$	Thermal conductivity	1.2.3, [1.9] (2)
Λ^*		Apparent thermal conductivity	2.1.1.3, [2.9], [2.10]
Λ^*		Apparent thermal conductivity (freezing)	2.2.1.4, [2.35]
μ	$Pa \cdot s$	Dynamic viscosity	1.2.1.1 (2)
μ_k	$J mole^{-1}$	Chemical potential of the component k	2.4.4.1 (1), [2.35] 1.5.2.3 (2)

θ	-	Liquid volume fraction, volume saturation	1.2.4.2 (1)
θ_s	-	volume fraction of ice (<i>freezing</i>)	2.2.1.1
$\theta_{r.}^w, \theta_{r.}^n$	-	Residual saturations	3.1.2.2 (1) 2.1.1 (2)
ρ	kg m^{-3}	Density, mass concentration	
ρ_s	kg m^{-3}	Density of ice (<i>freezing</i>)	
σ	N m^{-1}	Interfacial tension* (particularly capillary)	1.2.3 (1), [1.1] (1); 2.1.2 (1), 2.1.5.1 (1)
σ	Variable	Transport capacity, “conductivity”*	1.5.7.1 (2), 2.3.4.1 (2)
σ_X	$\text{X m}^{-2} \text{s}^{-1}$	Surface source of the value X	1.4.3.1
τ, τ, τ_{ij}	Pa	Constraint, constraint tensor*	1.2.1.1 (1)
τ, τ, τ_{ij}	-	Coefficient, tortuosity tensor*	1.4.3.2, [1.26] (2) 1.5.4.1, [1.68] (2) 1.3.6.2
Ω	m^3	Volume	
$\xi, \eta,$	Variable	Silent variable in an integration, Silent argument of a function	
Ψ	Pa	Capillary potential	1.2.3, [1.1] (1)
$\hat{\Psi}$	m	Capillary potential	2.1.2.2 (2)
Ψ_{sl}	Pa	Cryogenic depression (<i>freezing</i>)	2.3.1, [2.21] (2)
ζ_α	-	Thermal gradient factor relative to the phase α	2.1.1.2, [2.4]

Subscripts and superscripts	
a	Atmosphere, air
at	Standard atmosphere
c	Capillary
ef	Effective
eff	Effusion
f	Fluid
bd	Boundary
g	Gas
ini	Initial

$i, j \dots$	Index of direction in Cartesian coordinate system
$k, l \dots$	Index of a component in a mix
l	Liquid
m	Mean
s, sat	Saturated, at saturation*
s	Solid*
s	Ice (<i>freezing</i>)*
sub	Sublimation
v	Vapor
vs	Saturated vapor
v	Viscous
w, n	Wetting, non-wetting*
α	Index of a phase in composite medium or in porous medium

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Chapter 1

Isothermal Transport in Porous Media: Applications

The applications presented in this chapter are principally concerned with the isothermal transport in porous media of interstitial fluid, accompanied by the transport of its vapor by diffusion or filtration. The laws governing this transport have been explained in sections 2.2.2 and 2.2.3 in the classification proposed in section 2.2 of Chapter 2 (Volume 2). Two categories can be distinguished, relatively arbitrarily, depending on the importance of phase change in the process.

The processes dominated by the filtration of liquid have been categorized as *capillary transport* (section 1.1). The transport of vapor which accompanies capillary transport, whether by isobaric diffusion or filtration, plays a secondary, and usually negligible, role. In particular, the condition at the boundary of the porous body is presumed to be defined in terms of capillary potential controlled by the liquid phase, or liquid flux.

Section 1.2 is principally concerned with *drying* and also, as a related concept, with *sorption*. In these applications, phase change and transport in gas phase play an essential role. In the environment of the porous body, the vapor pressure is controlled, leading to the vaporization of the interstitial liquid at the boundary (or condensation of the vapor). Phase change at the boundary or in the neighboring body is likely to play an important role in the kinetics of the process. Thermal phenomena, which are inevitably associated with vaporization, must be evaluated, so as to justify or invalidate the approximation of *quasi-isothermal transport*.

In practice, these two categories of process are only separated in certain laboratory experiments that allow the control of boundary conditions, and in certain industrial processes. In contrast, in natural environmental conditions, the transport of water within soils under the influence of atmospheric stresses is essentially of a “capillary” nature in rainy conditions. However, when no liquid water is added, at least in the upper part of the soil, phase-change processes related to contact with the atmosphere are dominant. In this context, where the surface is exposed to the effect of the sun's rays and to the (somewhat periodic) variations of the ambient temperature, approximation of quasi-isothermal transport is hard to justify. It becomes necessary to take the effects of the thermal gradient on moisture transport into account. In Chapter 2, we will discuss this subject.

In the same way, in industrial drying processes, vapor transport controls the condition at the boundary and plays a dominant role in the superficial region of the body. However, within the body, transport remains capillary in nature, except during the final phase of the process.

Other processes of isothermal transport have been listed in the classification discussed in Chapter 2 (Volume 2): filtration of a non-condensable gas, and diffusion and dispersion in saturating liquid phase. They are more briefly addressed in sections 1.3 and 1.4, in the context of experimental processes and measurement methods.

1.1. Capillary transport

1.1.1. Isothermal transport without gravity

1.1.1.1. Imbibition or capillary drainage in a half-space

Here, we are concerned with isothermal transport (whether “capillary” or otherwise) in a semi-infinite porous body delimited by a planar boundary, in an initial state of uniform saturation defined by the liquid saturation θ_{mi} . In the absence of gravity, the most convenient constitutive equation for studying the transient processes is formulation [2.26] (Volume 2) in θ with a unique spatial variable x , where the domain is defined by $x > 0$.

$$\frac{\partial \theta}{\partial t} = \text{div} \left[D_{\theta}(\theta) \vec{\nabla} \theta \right] \quad \frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x} \left[D_{\theta}(\theta) \frac{\partial \theta}{\partial x} \right] \quad [1.1]$$

$$\frac{\partial \theta}{\partial t} = \text{div} \left[D_{\theta}(\theta) \vec{\nabla} \theta \right]$$

The boundary condition at $x = 0$ is Dirichletian in nature. This condition can be expressed in terms of ambient capillary pressure or the level of ambient vapor

saturation, but in both cases, its influence is seen by way of the saturation imposed at the boundary θ_a via the retention curve or the sorption isotherm. This condition represents *imbibition* under constant capillary pressure, or *sorption* under constant vapor pressure if $\theta_a > \theta_{mi}$, and conversely, *capillary drainage*, *drying* or *desorption*. We avoid the terms “infiltration” and “drainage”, which are used in everyday language for gravitational processes.

1.1.1.1.1. Integral mass balance

For this unidimensional configuration, the density of local and instantaneous volume¹ flux (or “velocity” in m/s) $V(t, x)$ can be expressed using a general balance. The liquid layer $e(t, x)$ (in m^3/m^2), which entered the region situated past the abscissa x at the time t (or in the case of drainage, was extracted from it), is expressed by the area comprised between the instantaneous saturation profile and the initial saturation level² (Figure 1.1(a)). The volume flow density is the temporal derivative of $e(t, x)$:

$$e(t, x) = \int_x^\infty [\theta(t, \xi) - \theta_{mi}] d\xi \quad \frac{J}{\rho_l} = V(t, x) = \frac{\partial e}{\partial t} \quad [1.2]$$

Another expression of the flux follows on from the transport law:

$$V = -K \frac{\partial \Psi}{\partial x} = -K \frac{d\Psi}{d\theta} \frac{\partial \theta}{\partial x} = -\frac{K}{\Gamma_c} \frac{\partial \theta}{\partial x} = -D_\theta \frac{\partial \theta}{\partial x} \quad [1.3]$$

1.1.1.1.2. Boltzmann solution

Although equation [1.1] is not linear, for this particular problem we may apply a process using the linear diffusion equation, which consists of searching for a solution as a function of the *Boltzmann variable* u (section 1.4.1.6). Equation [1.1] then takes on the form of a differential equation with a single variable:

$$\frac{d}{du} \left[D_\theta \frac{d\theta}{du} \right] = -\frac{u}{2} \frac{d\theta}{du} \quad \text{with } u = \frac{x}{\sqrt{t}} \quad \text{according to [1.55]}$$

1 Why not simply say Darcy’s velocity? Because this flux density can comprise, together with Darcian velocity of the liquid, a term that is representative of the coupled vapor flux, which may even dominate in the domain of the residual saturation.

2 In integral [1.2], and in general hereafter, the notations ξ, η are used as arguments of functions or as silent variables in the integrals. $x, t, u \dots$ are reserved for space, time and Boltzmann’s variable when they appear in integration limits.

and the solutions are sought in the form³:

$$\theta = \theta_{ini} + (\theta_a - \theta_{ini})f(u) \quad f(0) = 1 \quad f(\infty) = 0$$

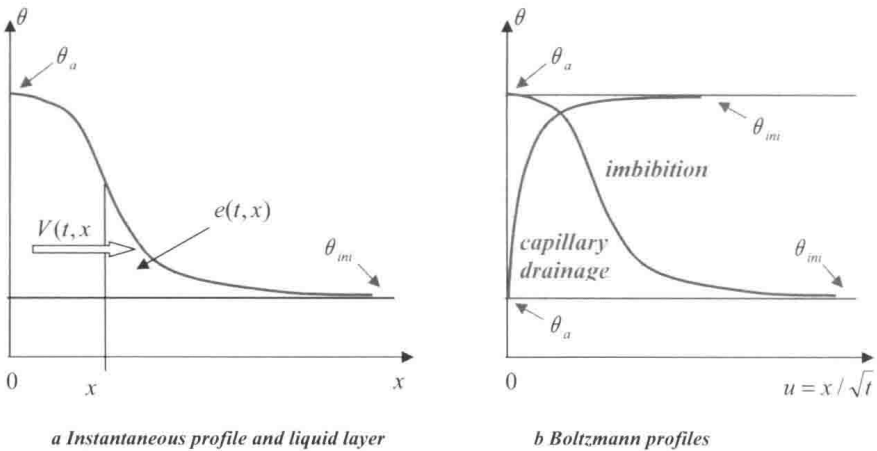


Figure 1.1. Instantaneous profile of saturation in imbibition and capillary drainage or drying

The function $f(u)$ is dependent on the law of diffusivity $D_\theta(\theta)$, but also on the initial and boundary conditions (Figure 1.1(b)).

The representation of $\theta(u)$ (Figure 1.1(b)) indicates the curve shape of all the instantaneous profiles of the saturation $\theta(x) = \theta(u\sqrt{t})$. These profiles are deformed over time by affinity along the x axis of a factor proportional to \sqrt{t} , and degenerate stepwise to $t = 0$ (see Figure 1.18).

With the same law of diffusivity $D_\theta(\theta)$ and the same saturation interval θ_{ini} , θ_a , the curves are clearly different during imbibition ($\theta_a > \theta_{ini}$) and drying ($\theta_a < \theta_{ini}$). Indeed, according to [1.3], the slope of an instantaneous profile is determined by:

³ This calculation may also be performed using the differential equation in Ψ or in φ as a basis. We arrive at the same equation [1A.15] governing the variable Ψ or φ in place of θ , with the same hydric diffusivity D_θ , which is a quotient of the coefficient of transport and of the corresponding capacity.