ELECTRICAL MACHINES

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Electrical Machines
A Unified Treatment on a Physical Basis

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Preface

THE subject of Electrical Machinery, which once formed the core of the Electrical Engineering syllabus, must now constitute a more modest portion of the course so that more time may be devoted to covering the vast range of developments in other fields. The situation is not without advantages. It has stimulated thought on means of rationalising the theory to emphasise the unity underlying the behaviour of the numerous different types of machine. For those with a flair for mathematics, generalised treatments have been and are being developed which are very satisfying in their results. By considering the electrical machine as a combination of inductively coupled coils whose interconnection determines the characteristic machine behaviour, an all-embracing theory has been approached. This has been made possible by the work of Gabriel Kron who first developed and applied the techniques of Tensor Analysis to electrical circuits and machines. One book on this basis has already been published in this series.*

However, the problem still remains of conveying in a manner more concise than previously the essential details concerning the physical nature of machines. Further, there are many practical problems which will have to be faced by the electrical student in the laboratory, by the applications engineer and by the machines specialist for which the physical approach is more suitable.

In this volume an attempt has been made to survey practically the whole field of Electrical Machinery without recourse to advanced mathematics but with the intention of supporting any

* N. N. HANCOCK. *Matrix Analysis of Electrical Machinery.* Pergamon Press, Oxford (1964)

mathematical generalised treatment. The method adopted is to give most space to a careful explanation of four main electromagnetic devices in their basic form, viz. the two-coil transformer, the d.c. machine, the induction machine and the synchronous machine. A sound knowledge of these four will permit the ready understanding of all other machines, which for the most part are modifications of one or other type. Although there are distinguishing features between the main kinds of machine there are many similarities too; these are brought out wherever possible, and the treatment of each machine follows similar lines where this is practicable. The theory is extended to introduce other modes of operation in a manner adequate for general understanding but without detailed explanation in all cases. The text is supported by nearly fifty comprehensive worked examples which cover most of the theoretical points raised.

The first chapter is a brief review of the common principles underlying machine operation and their application to produce the different machine types. A minimal knowledge of electrical machinery is assumed. However, Chapters 2 and 3 revise the appropriate magnetic and circuit theory so that to some extent the book is self-contained. These two chapters can be skipped by the student who feels secure enough on their subject matter, but they will be referred to frequently either implicitly or explicitly in the later chapters. Towards the end of Chapter 3, a concentrated review of more advanced work is given which necessitates the use of simple differential equations. For example, transient behaviour is discussed and a few elementary problems are considered in the later text. The ideas which form the basis of generalised treatments are also explained briefly in order that reference may be made in Chapter 9 to certain possible uses of the Generalised Laboratory Machine.

Since machine windings are the means by which theory is translated into practice, it has been considered necessary to devote one short chapter to them. In themselves they form a unifying link between different machines. Although the treatment has been simplified, it is adequate for straightforward winding problems, and the material is useful support for the remaining chapters.

Following the bulk of the text which is devoted to the main machine types, the final chapter serves to consider the general relationships obtaining when both commutator and slip rings are provided on the armature. Practical applications of these principles to various commutator machines are described together with a discussion of the recently available generalised laboratory machines.

Although work for the final year of Electrical Engineering Degree courses is introduced, the main intention of the book is to meet the requirements of all courses dealing with Electrical Machinery up to this stage, where students take up a few specialised studies. However, the Machines portions of the Electrical Power and Machines syllabus for the External B.Sc. (Eng.) degree of London University and for Higher National Certificate and equivalent courses are all covered.

The author is grateful to his colleagues in the Department of Electrical Engineering for the assistance received in various ways when preparing the text. He is particularly indebted to Dr. N. N. Hancock, M.I.E.E. who gave generously of his time to discuss and clarify many aspects of machine theory. Much helpful advice was received from Professor P. Hammond, M.A., M.I.E.E., the consulting editor, and Mr. G. E. Middleton, M.A., M.I.E.E., read through the manuscript and made many useful suggestions. Various firms have taken considerable trouble to find suitable photographs and these are acknowledged where they appear in the book. Figure 2.7 was copied from a flux plot kindly loaned by Mr. F. J. Pepworth, A.M.I.E.E., who had spent many hours in its preparation.

J. H.

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List of Symbols

THE following list comprises those symbols which are used fairly frequently throughout the text. Other symbols which are confined to certain limited sections in the book and those which are in general use are not included; e.g. *R* for resistance; A, B, and C for 3-phase quantities.

Instantaneous values are given small letters, e.g. *e*, *i*, for e.m.f. and current respectively

R.m.s. and steady d.c. values are given large letters e.g. *E*, *I*

Maximum values are written thus: $\hat{E}$, $\hat{I}$

Bold face type is used for vector quantities, e.g. **E**, **I**

a	Number of pairs of parallel circuits in machine winding
AT	ampere-turns
C	number of coils or commutator bars
d	symbol for direct-axis quantities
D	armature diameter in metres
D	operator d/d*t*
e	base of natural logarithms
f	frequency in cycles per second (cycles/sec) (c/s)
F	magnetomotive force (m.m.f.) in ampere-turns; peak m.m.f. per pole per phase
F_a	peak armature m.m.f. per pole
F_a'	effective demagnetising m.m.f. per pole
F_f	peak field m.m.f. per pole
F_r	peak resultant m.m.f. per pole
I_0	current in magnetising branch
I_p	power component of I_0
I_m	reactive or magnetising component of I_0

k	coefficient of coupling; a constant
k_{pn}	coil pitch factor for the *n*th harmonic
k_{dn}	distribution factor for the *n*th harmonic
k_e	eddy current loss constant
k_h	hysteresis loss constant
k_f	generated volts per field ampère
k_N	generated volts per rev/min
k_v	generated volts per radian/sec or torque per ampere
l	conductor length; magnetic path length in metres
L	inductance, e.g. L_{11} self-inductance of coil 1
M	or L_{12}, L_{21}, etc., mutual inductance
n	rev/sec. n_s rev/sec synchronous $= f/p$
N	number of turns; rev/min. N_s rev/min synchronous $= 60f/p$
p	number of pole pairs
P	power per phase
q	slots per pole per phase; symbol for quadrature axis quantities
Q	slots per pole
s	fractional slip
S	number of slots; $(1 - s)$
T	torque developed electromagnetically in newton-metres
T_m	torque arising mechanically in newton-metres
v	velooity in metres per second
V_s	voltage rise at the terminals of an infinite busbar system
W	total power
x	Steinmetz index
x_m	magnetising reactance
X	inductive reactance; leakage reactance
X_a	armature reactance due to armature-produced mutual flux across air-gap
X_s	synchronous reactance $= X_a + X$
y_c	commutator bar or coil-end pitch
z_s	number of conductors in series per phase or per parallel path of a winding
Z	total armature conductors
Z_s	synchronous impedance

α	general angle; slot angle; impedance angle $\tan^{-1} X/R$
β	general angle; chording angle
δ	load angle
η	efficiency
λ	flux linkage
Λ	permeance
μ_0	magnetic constant $= 4\pi/10^7$
μ_r	relative permeability
μ	absolute permeability $= B/H = \mu_0\mu_r$
ϕ	flux per pole; instantaneous value of flux, in webers
ϕ_m	mutual flux in webers
$\mathbf{\Phi}'$	flux space vector
$\mathbf{\Phi}$	flux time vector
θ	general angle; power factor angle
τ	time constant
ω	angular velocity in radian/sec

Contents

1 · Introduction and Basic Ideas

1.1 AIM OF THE BOOK

This book is intended to explain the behaviour of certain electromagnetic devices which are able to convert energy from one form to another. Electrical, mechanical or both forms of energy may appear in either input or output. Energy may be stored and recovered during transient processes, in both the magnetic field and in the mechanical inertia. An appreciable amount of energy will be converted to heat and lost; this will be discussed when dealing with losses. Trivial amounts of energy will be converted to other forms but will be neglected here, notwithstanding their importance in other contexts, e.g. acoustic noise and radio noise. With the exception of the transformer, rotation will be involved in the conversion process, but the term Electrical Machinery will be used to cover all cases.

A remarkable number of different machines has been devised, and as many different methods of analysis have grown up around them. Until quite recently it was the custom to discuss each machine separately as if it had a unique existence, but the whole subject has become so vast in content that to teach it on this basis is unsatisfactory. It is possible with some thought to see in Electrical Machinery many common features, and while it is erroneous to pretend that all machines are really the same, much can be done to economise in general ideas. The present chapter is an attempt to bring out certain points of similarity.

At this stage, certain liberties as to the extent of the reader's knowledge will be taken. For example, it will be assumed that the electromagnetic principles behind the operation of Electrical

Machinery are not entirely unfamiliar. They will be discussed, however, in Chapters 2 and 3 with a view to emphasising their relative importance and consolidating a suitable foundation. Certain general ideas which are also discussed in these earlier chapters will be elaborated in the later text, so it is suggested that there should be no undue delay at this stage if a difficulty is encountered. The abilities of students to absorb generalities are best judged by the individual lecturer concerned and he may therefore wish either to defer some of this material or to extend it.

1.2 GENERATION OF ELECTROMOTIVE FORCE

A voltage may be generated in various ways, but for present purposes, only the e.m.f. of electromagnetic induction is of importance. One way of expressing the result of Faraday's famous experiment is by means of the equation

$$e = -N(\mathrm{d}\phi/\mathrm{d}t) \text{ volts} \tag{1.1}$$

Here is introduced the concept of magnetic flux lines, ϕ webers, and their linkage with the N turns of a coil. Not all of the flux lines link all of the turns and the meaning assigned to ϕ is given by the equation $\phi = \lambda/N$ where λ is termed the flux linkage. λ is obtained by considering the number of turns encircled by each flux line or group of lines having the same turn linkage. Examples will be considered in Chapter 2, but the point is demonstrated for a simple case in Fig. 1.1. The total flux linkage is $4 \times 3 + 2 \times 1 = 14$. Hence ϕ the mean flux per turn is 14/3.

The negative sign in eqn. (1.1.) is used for the following reason. The direction of the induced e.m.f. is such that if the circuit was closed it would tend to drive a current whose magnetic effect would oppose the change of flux. This is sometimes referred to as Lenz's law, but in fact the alternative would be unthinkable in a stable universe; if the induced current supported the flux change, both e.m.f. and current would increase indefinitely.

Electrical machines are provided with coils and flux changes are provoked by one means or another to produce an e.m.f. Consider, for example, a single-turn coil placed as in Fig. 1.2a. It

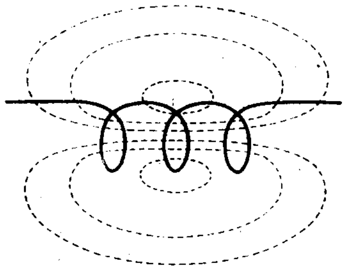

FIG. 1.1. Flux linkage with 3-turn coil.

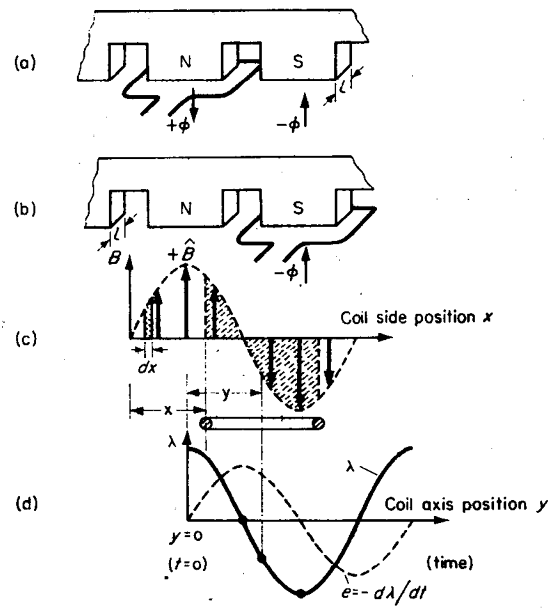

FIG. 1.2. Induced e.m.f.

embraces the whole of the flux $+\phi$ emanating from the north pole. If the poles are magnetised by alternating current such that after a time t the flux reverses to $-\phi$, the coil will experience a total change of 2ϕ webers and there will be an induced e.m.f. of average value $2\phi/t$ volts. The actual e.m.f. at any particular instant will depend on the time variation of ϕ. If, for example, $\phi = \hat{\phi} \sin 2\pi ft$, then the instantaneous e.m.f. $e = -N(\mathrm{d}\phi/\mathrm{d}t) = -1(2\pi f\hat{\phi}\cos 2\pi ft)$ which is another sinusoidal function delayed in time by $1/4f$ sec. Since this e.m.f. could provide electrical energy in the coil circuit, there could be an electrical/electrical energy "conversion" through the medium of the magnetic field, the input energy coming from the a.c. source magnetising the poles. This is called transformer action, and the resulting e.m.f. a *transformer e.m.f.*

If the poles are permanent magnets or d.c. excited, they require no electrical input other than that to sustain the I^2R loss in the exciting coils if provided. However, it is still possible, even though the flux is constant in time, to induce an e.m.f. If the coil is moved to the position of Fig. 1.2b in a time t seconds, it will now embrace a flux $-\phi$. The flux has changed, as far as the coil is concerned, from $+\phi$, through zero when the coil sides are midway under the poles and therefore there is no net flux linkage, to $-\phi$. As before, the average e.m.f. during the interval will be $2\phi/t$ volts, though it has been produced by different means. It is a *motional e.m.f.* and motion involves mechanical energy in the conversion process.

The instantaneous e.m.f. this time will depend on the way in which the flux is distributed in space since the distribution is "seen" by the coil as a time variation. Figure 1.2c shows a possible case where the spatial distribution of flux density is sinusoidal. The flux passing through any area $l.\mathrm{d}x$ is $B.l.\mathrm{d}x$ which is proportional to the area under the B curve over distance $\mathrm{d}x$. For any coil position then, the total flux through the coil is proportional to the area under the B curve within the coil sides. This is indicated by shading for a particular case. When the coil axis is midway between the poles the area will have equal positive and negative halves, flux entering into, and emerging from the top of the coil without linking it. With the coil centrally placed about any pole, the whole of the pole flux passes through thus giving maximum linkage. Salient points on a flux linkage curve can thus be plotted and since the area under a sine wave can be depicted by another sine wave, these points will lie on a curve as shown by Fig. 1.2d. Although this is a spatial variation, it can also represent the time variation of flux linkage when "viewed" from a coil moving with uniform velocity. On Fig. 1.2d $t = 0$ has been taken to correspond with the instant when the coil axis is under the middle of a north pole, i.e. $x = 0$ and $y = 0$. The coil e.m.f. is obtained by differentiating this curve, giving another sine wave which rises and falls in phase with the B variation as "seen" by the coil sides, e.g. at $t = 0$ the coil sides are under zero flux density and e is also zero.

The same e.m.f. could have been produced if with the coil stationary the whole pole system had been moved one pole pitch to the left in a time t sec. Viewed from the coil, the flux changes would have been identical. One could think of many interesting combinations of pole motion and coil motion which would give the same e.m.f. The important thing to realise is that as far as the coil is concerned, if at one instant of time t_1, it is linked by a flux ϕ_1 webers, and at a later instant t_2 it is linked by ϕ_2 webers, due account being taken of the sense or sign of the flux, then an average e.m.f. of value $N(\phi_2 - \phi_1)/(t_2 - t_1)$ volts will be induced during the time interval. Whether the coil moves is not the issue, it must just experience a change of flux. If, for example, the coil was moving but the poles too were moving at the same speed and in the same direction, no e.m.f. would be induced; it is just as if they were both stationary, as in fact they are, *relative to one another*.

Although it may be useful to distinguish between a *transformer e.m.f.* and a *motional e.m.f.* because of energy conversion considerations, the general statement is still true, that the total e.m.f. induced is due to the total flux change viewed from the coil. On some a.c. machines, both e.m.f. components are present since the coils are moving through a pulsating flux.