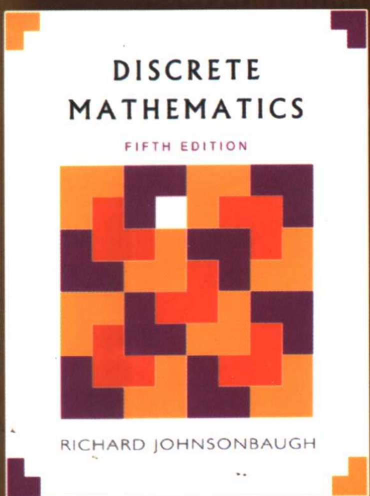
国外计算机科学教材系列

离散数学

（第五版）

Discrete Mathematics, Fifth Edition



英文版

［美］Richard Johnsonbaugh 著

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北京 · BEIJING

内容简介

本书从算法分析和问题求解的角度，全面系统地介绍了离散数学的基础概念及相关知识。书中通过大量实例，深入浅出地讲解了数理逻辑、组合算法、图论、布尔代数、网络模型、形式语言与自动机理论等与计算机科学密切相关的前沿课题，既着重于各部分内容之间的紧密联系，又深入探讨了相关的概念、理论、算法和实际应用。本书内容叙述严谨、推演详尽，各章配有相当数量的习题，并且在书后提供了相应的提示和答案，为读者迅速掌握有关知识提供了有效的帮助。

本书既可作为计算机科学及计算数学等专业的本科生和研究生教材，也可作为工程技术人员和相关人员的参考书。

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出 版 说 明

21世纪初的5至10年是我国国民经济和社会发展的重要时期，也是信息产业快速发展的关键时期。在我国加入WTO后的今天，培养一支适应国际化竞争的一流IT人才队伍是我国高等教育的重要任务之一。信息科学和技术方面人才的优劣与多寡，是我国面对国际竞争时成败的关键因素。

当前，正值我国高等教育特别是信息科学领域的教育调整、变革的重大时期，为使我国教育体制与国际化接轨，有条件的高等院校正在为某些信息学科和技术课程使用国外优秀教材和优秀原版教材，以使我国在计算机教学上尽快赶上国际先进水平。

电子工业出版社秉承多年来引进国外优秀图书的经验，翻译出版了“国外计算机科学教材系列”丛书，这套教材覆盖学科范围广、领域宽、层次多，既有本科专业课程教材，也有研究生课程教材，以适应不同院系、不同专业、不同层次的师生对教材的需求，广大师生可自由选择和自由组合使用。这些教材涉及的学科方向包括网络与通信、操作系统、计算机组织与结构、算法与数据结构、数据库与信息处理、编程语言、图形图像与多媒体、软件工程等。同时，我们也适当引进了一些优秀英文原版教材，本着翻译版本和英文原版并重的原则，对重点图书既提供英文原版又提供相应的翻译版本。

在图书选题上，我们大都选择国外著名出版公司出版的高校教材，如Pearson Education培生教育出版集团、麦格劳－希尔教育出版集团、麻省理工学院出版社、剑桥大学出版社等。撰写教材的许多作者都是蜚声世界的教授、学者，如道格拉斯·科默（Douglas E. Comer）、威廉·斯托林斯（William Stallings）、哈维·戴特尔（Harvey M. Deitel）、尤利斯·布莱克（Uyless Black）等。

为确保教材的选题质量和翻译质量，我们约请了清华大学、北京大学、北京航空航天大学、复旦大学、上海交通大学、南京大学、浙江大学、哈尔滨工业大学、华中科技大学、西安交通大学、国防科学技术大学、解放军理工大学等著名高校的教授和骨干教师参与了本系列教材的选题、翻译和审校工作。他们中既有讲授同类教材的骨干教师、博士，也有积累了几十年教学经验的老教授和博士生导师。

在该系列教材的选题、翻译和编辑加工过程中，为提高教材质量，我们做了大量细致的工作，包括对所选教材进行全面论证；选择编辑时力求达到专业对口；对排版、印制质量进行严格把关。对于英文教材中出现的错误，我们通过与作者联络和网上下载勘误表等方式，逐一进行了修订。

此外，我们还将与国外著名出版公司合作，提供一些教材的教学支持资料，希望能为授课老师提供帮助。今后，我们将继续加强与各高校教师的密切联系，为广大师生引进更多的国外优秀教材和参考书，为我国计算机科学教学体系与国际教学体系的接轨做出努力。

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PREFACE

This book is intended for a one- or two-term introductory course in discrete mathematics, based on my experience in teaching this course over a 20-year period. Formal mathematics prerequisites are minimal; calculus is *not* required. There are no computer science prerequisites. The book includes examples, exercises, figures, tables, sections on problem-solving, section reviews, notes, chapter reviews, self-tests, and computer exercises to help the reader master introductory discrete mathematics. In addition, an *Instructor's Guide* and World Wide Web site are available.

The main changes in this edition (discussed in more detail later) are an expanded discussion of logic and proofs, the addition of two sections on discrete probability, a new appendix that reviews basic algebra, many new examples and exercises, section reviews, and computer exercises.

OVERVIEW

In the early 1980s there were almost no books appropriate for an introductory course in discrete mathematics. At the same time, there was a need for a course that extended students' mathematical maturity and ability to deal with abstraction and also included useful topics such as combinatorics, algorithms, and graphs. The original edition of this book (1984) addressed this need. Subsequently, discrete mathematics courses were endorsed by many groups for several different audiences, including mathematics and computer science majors. A panel of the Mathematical Association of America (MAA) endorsed a year-long course in discrete mathematics. The Educational Activities Board of the Institute of Electrical and Electronics Engineers (IEEE) recommended a freshman discrete mathematics course. The Association for Computing Machinery (ACM) and IEEE accreditation guidelines mandated a discrete mathematics course. This edition, like its predecessors, includes topics such as algorithms, combinatorics, sets, functions, and mathematical induction endorsed by these groups. It also addresses understanding and doing proofs and, generally, expanding mathematical maturity.

ABOUT THIS BOOK

This book includes

- Logic (including quantifiers), proofs, proofs by resolution, and mathematical induction (Chapter 1).

- Sets, sequences, strings, sum and product notations, number systems, relations, and functions, including motivating examples such as an application of partial orders to task scheduling (Section 2.4), relational databases (Section 2.7), and an introduction to hash functions and pseudorandom number generators (Section 2.8).
- A thorough discussion of algorithms, recursive algorithms, and the analysis of algorithms (Chapter 3). In addition, an algorithmic approach is taken throughout this book. The algorithms are written in a flexible form of pseudocode. (The book does not assume any computer science prerequisites; the description of the pseudocode used is self-contained.) Among the algorithms presented are the Euclidean algorithm for finding the greatest common divisor (Section 3.3), tiling (Section 3.4), the RSA public-key encryption algorithm (Section 3.7), generating combinations and permutations (Section 4.3), merge sort (Section 5.3), Dijkstra's shortest-path algorithm (Section 6.4), backtracking algorithms (Section 7.3), breadth-first and depth-first search (Section 7.3), tree traversals (Section 7.6), evaluating a game tree (Section 7.9), finding a maximal flow in a network (Section 8.2), finding a closest pair of points (Section 11.1), and computing the convex hull (Section 11.3).
- A full discussion of the "big oh," omega, and theta notations for the growth of functions (Section 3.5). Having all of these notations available makes it possible to make precise statements about the growth of functions and the complexity of algorithms.
- Combinations, permutations, discrete probability, and the Pigeonhole Principle (Chapter 4).
- Recurrence relations and their use in the analysis of algorithms (Chapter 5).
- Graphs, including coverage of graph models of parallel computers, the knight's tour, Hamiltonian cycles, graph isomorphisms, and planar graphs (Chapter 6). Theorem 6.4.3 gives a simple, short, elegant proof of the correctness of Dijkstra's algorithm.
- Trees, including binary trees, tree traversals, minimal spanning trees, decision trees, the minimum time for sorting, and tree isomorphisms (Chapter 7).
- Networks, the maximal flow algorithm, and matching (Chapter 8).
- A treatment of Boolean algebras that emphasizes the relation of Boolean algebras to combinatorial circuits (Chapter 9).
- An approach to automata emphasizing modeling and applications (Chapter 10). The *SR* flip-flop circuit is discussed in Example 10.1.11. Fractals, including the von Koch snowflake, are described by special kinds of grammars (Example 10.3.19).
- An introduction to computational geometry (Chapter 11).
- An appendix on matrices, and another that reviews basic algebra.
- A strong emphasis on the interplay among the various topics. As examples, mathematical induction is closely tied to recursive algorithms (Section 3.4); the Fibonacci sequence is used in the analysis of the Euclidean algorithm (Section 3.6); many exercises throughout the book require mathematical induction; we show how to characterize the components of a graph by defining an equivalence relation on the set of vertices (see the discussion following Example 6.2.13); and we count the number of n-vertex binary trees (Theorem 7.8.12).
- A strong emphasis on reading and doing proofs. Most proofs of theorems are illustrated with annotated figures. Ends of proofs are marked with the

symbol ■. Separate sections (*Problem-Solving Corners*) show students how to attack and solve problems and how to do proofs.

- Numerous worked examples throughout the book. (There are over 500 worked examples.)
- A large number of applications, especially applications to computer science.
- Over 3500 exercises, with answers to about one-third of them in the back of the book. (Exercises with numbers in color have an answer in the back of the book.)
- Figures and tables to illustrate concepts, to show how algorithms work, to elucidate proofs, and to motivate the material. Several figures illustrate proofs of theorems. The captions of these figures provide additional explanation and insight into the proofs.
- Section reviews.
- Notes sections with suggestions for further reading.
- Chapter reviews.
- Chapter self-tests.
- Computer exercises.
- A reference section containing 150 references.
- Front and back endpapers that summarize the mathematical and algorithm notation used in the book.

CHANGES FROM THE FOURTH EDITION

- The first chapter on logic and proofs is considerably enhanced. Several new motivating examples have been added. A *logic game*, which offers an alternative way to determine whether a quantified propositional function is true or false, is discussed in Example 1.3.17. Section 1.4 now includes rules of inference for both propositions and quantified statements. The number of exercises in this chapter has been increased from 232 to 391.
- *Arrow diagrams* have been added to give a pictorial view of the definition of a function, one-to-one functions, onto functions, inverse functions, and the composition of functions (see Section 2.8).
- Graphs of functions have been added to give yet another view of functions (see Section 2.8).
- Two optional sections (Sections 4.4 and 4.5) have been added on discrete probability. We discuss the fundamental terminology (e.g., experiment, event), the use of counting techniques to compute probabilities, basic formulas (e.g., $P(E) + P(\overline{E}) = 1$), mutually exclusive events, conditional probability, independent events, and Bayes' Theorem and its use in pattern recognition.
- The setting for the Problem-Solving Corner in Chapter 5 has been changed to a more inviting and contemporary setting: sorting in a spreadsheet.
- The fourth edition's Section 8.5 on Petri nets has been moved to the Web site that accompanies this book.
- Appendix B, which reviews basic algebra, has been added. The topics treated are rules for combining and simplifying expressions, fractions, exponents, factoring, quadratic equations, inequalities, and logarithms.
- A number of computer examples now show actual computer screens to help connect the theory to practical applications.

- Several new examples have been added dealing with
  - Searching the World Wide Web, with a real example using the AltaVista search engine and Boolean expressions (Example 1.1.14)
  - A logic game (Example 1.3.17)
  - Using the matrix of a relation to determine whether the relation is transitive (Example 2.6.7)
  - Pseudorandom number generators (Example 2.8.14)
  - The Melissa virus (as an example of combinatorial explosion) (Example 4.1.2)
  - The birthday problem (Example 4.5.7)
  - Telemarketing (Example 4.5.21)
  - Detecting the HIV virus (Example 4.5.22)
  - Computer file systems (Example 7.1.6).
- The new section reviews, which precede the exercises in *every* section, consist of exercises with answers in the back of the book. These exercises review the key concepts, definitions, theorems, techniques, and so on, of the section. Although intended for reviews of the sections, section reviews can also be used for placement and pretesting.
- Computer exercises have been added to the end of every chapter. Although there is no programming prerequisite for this book and no programming is introduced in the book, these exercises are provided for those readers who want to explore discrete mathematics concepts with a computer.
- The definition of "bipartite graph" (Definition 6.1.11) has been corrected. (To see what the problem is, use an old definition to check whether a one-vertex graph is bipartite.) This book is now probably the only one in which this definition is correct!

- The icon shown, which occurs throughout the book, indicates that more explanation, examples, and so on about a particular topic are available at the Web site that accompanies this book.

- The icon shown, which also occurs throughout the book, signals that a link posted at the Web site that accompanies this book points to another Web site that contains additional information about a particular topic.
- A number of recent books and articles have been added to the list of references. Several book references have been updated to current editions.
- The number of worked examples has been increased to over 500. (There were approximately 430 in the fourth edition.)
- The number of exercises has been increased to over 3500. (There were approximately 2400 in the fourth edition.)
- The World Wide Web site has been greatly enhanced to provide additional support for the book.

CHAPTER STRUCTURE

Each chapter is organized as follows:

Overview
Section
Section Review
Section Exercises
Section

Section Review
Section Exercises
⋮
Notes
Chapter Review
Chapter Self-Test
Computer Exercises

Section reviews consist of exercises, with answers in the back of the book, that review the key concepts of the section. Notes contain suggestions for further reading. Chapter reviews provide reference lists of the key concepts of the chapters. Chapter self-tests contain four exercises per section, with answers in the back of the book. Computer exercises request implementation of some of the algorithms, projects, and other programming related activities. In addition, most chapters have Problem-Solving Corners.

EXERCISES

The book contains over 3500 exercises, 135 of which are computer exercises. Exercises felt to be more challenging than average are indicated with a star, ★. Exercise numbers in color (approximately one-third of the exercises) indicate that the exercise has a hint or solution in the back of the book. The solutions to the remaining exercises may be found in the *Instructor's Guide*. A handful of exercises are clearly identified as requiring calculus. No calculus concepts are used in the main body of the book and, except for these marked exercises, no calculus is needed to solve the exercises.

EXAMPLES

The book contains over 500 worked examples. These examples show students how to tackle problems in discrete mathematics, demonstrate applications of the theory, clarify proofs, and help motivate the material. Ends of examples are marked with the symbol ■.

PROBLEM-SOLVING CORNERS

The Problem-Solving Corner sections help students attack and solve problems and show them how to do proofs. Written in an informal style, each is a self-contained section following the discussion of the subject of the problem. Rather than simply presenting a proof or a solution to a problem, in these sections the intent is to show alternative ways of attacking a problem, to discuss what to look for in trying to obtain a solution to a problem, and to present problem-solving and proof techniques.

Each Problem-Solving Corner begins with a statement of a problem. After stating the problem, ways to attack the problem are discussed. This discussion is followed by techniques for finding a solution. After a solution is found, a formal solution is given to show how to correctly write up a formal solution. Finally, the problem-solving techniques used in the section are summarized. In addition, some of these sections include a Comments subsection, which discusses connections with other topics in mathematics and computer science, provides motivation for the problem, and lists references for further reading about the problem. Exercises conclude some Problem-Solving Corners.

INSTRUCTOR SUPPLEMENT

An *Instructor's Guide* is available at no cost from the publisher to instructors who adopt or sample this book. The *Instructor's Guide* contains solutions to the exercises not included in the book, tips for teaching the course, and transparency masters.

WORLD WIDE WEB SITE

A World Wide Web site

`www.prenhall.com/johnsonbaugh`

contains

- On-line true/false tests
- Expanded explanations of difficult material. The icon shown indicates that additional explanation is available.
- Links to other sites for additional information about discrete mathematics topics. The icon shown signals such a link.
- PowerPoint slides
- Supplementary material
- Computer programs
- Transparencies
- An errata list, also available at `www.depaul.edu/~rjohnson/dm5th/errata.txt`

Both instructors and students will find the PowerPoint slides useful. The supplementary material includes the section on Petri nets from the fourth edition.

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R.J.

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† This section can be omitted without loss of continuity

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† This section can be omitted without loss of continuity

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1

LOGIC AND PROOFS

Logic, logic, logic. Logic is the beginning of wisdom, Valeris, not the end.

FROM *Star Trek VI: The Undiscovered Country*

Logic is the study of reasoning; it is specifically concerned with whether reasoning is correct. Logic focuses on the relationship among statements as opposed to the content of any particular statement. Consider, for example, the following argument:

All mathematicians wear sandals.
Anyone who wears sandals is an algebraist.
Therefore, all mathematicians are algebraists.

Technically, logic is of no help in determining whether any of these statements is true; however, if the first two statements are true, logic assures us that the statement

All mathematicians are algebraists.

is also true.

Logical methods are used in mathematics to prove theorems and in computer science to prove that programs do what they are alleged to do. In the latter part of the chapter, we discuss some general methods of proof, one of which, mathematical induction, is used throughout mathematics and computer science. Mathematical induction is especially useful in discrete mathematics.

† This section can be omitted without loss of continuity.