



# Flight Vehicle Aerodynamics

Mark Drela

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# Preface

## Objective

This book is intended as a general reference for the physics, concepts, theories, and models underlying the discipline of aerodynamics. An overarching theme is the technique of velocity field representation and modeling via source and vorticity fields, and via their sheet, filament, or point-singularity idealizations. These models provide an intuitive feel for aerodynamic flow behavior, and are also the basis of aerodynamic force analysis, drag decomposition, flow interference estimation, wind tunnel corrections, computational methods, and many other important applications.

This book covers some topics in depth, while offering introductions or summaries of others. In particular, Chapters 3,4 on Boundary Layers, Chapter 7 on Unsteady Aerodynamics, and Chapter 9 on Flight Dynamics are intended as introductions and overviews of those topics, which deserve to be properly treated in separate dedicated texts. Similarly, there are only glancing mentions of the related topic of Propulsion, which is its own discipline.

Computational Fluid Dynamics (CFD) and computational methods in general are indispensable for today's practicing aerodynamicist. Hence a few computational methods are described here, primarily the vortex lattice and panel methods which are based on the source and vorticity flow-field representation. The main goal is to provide improved understanding of the concepts and physical models which underlie such methods.

Most of this book is based on the lecture notes, handouts, and reference materials which have been developed for the course *Flight Vehicle Aerodynamics* (course number 16.110) taught by the author at MIT's Department of Aeronautics and Astronautics. This course is intended for first-year graduate students, but has also attracted a significant number of advanced undergraduates.

## Preparation

This book assumes that the reader is well versed in basic physics and vector calculus, and already has had exposure to basic fluid mechanics and aerodynamics. Hence, little or no space is devoted to introduction or discussion of basic concepts such as fluid velocity, density, pressure, viscosity, stress, etc. Chapter 1 on the Physics of Aerodynamics Flows is intentionally concise, since it is intended primarily as a reference for the underlying physical principles and governing equations of fluid flows rather than as a first introduction to these topics. The author's course at MIT begins with Chapter 2.

Some familiarity with aerodynamics and aeronautics terminology is assumed on the part of the reader. However, a summary of advanced vector calculus notation is given in Appendix A, since this is not commonly seen in basic vector calculus texts.

## Acknowledgments

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# Nomenclature

The page numbers indicate where each symbol is first used.

## Roman letters

$a$	Speed of sound, p. 1
$\alpha$	Boundary layer edge velocity power-law exponent, p. 71
$A$	Cross-sectional area, p. 45
$AR$	Aspect ratio, p. 113
$A_{ij}$	Aerodynamic influence coefficient (AIC) matrix, p. 114
$\mathcal{A}$	G-beta constant for equilibrium turbulent flow, p. 73
$\mathcal{A}_n$	Fourier coefficients of spanwise circulation distribution, p. 113
$\mathcal{A}_n$	Fourier coefficients of airfoil's vortex sheet strength distribution, p. 254
$\bar{\mathbf{A}}$	Flight-dynamics system matrix, p. 207
$b$	Boundary layer lateral width parameter, p. 75
$b$	Wingspan, p. 104
$b$	Reference span, p. 209
$B$	Wind tunnel cross-section aspect ratio, p. 233
$B_{ij}$	Source-influence matrix for the potential, p. 151
$\mathcal{B}$	G-beta constant for equilibrium turbulent flow, p. 73
$\mathcal{B}_n$	Fourier coefficients airfoil thickness distribution, p. 257
$\bar{\mathbf{B}}$	Flight-dynamics control matrix, p. 207
$c$	Airfoil chord, p. 9
$c$	Reference chord, p. 209
$c_d$	2D section drag coefficient, p. 40
$C_D$	Drag coefficient, p. 125
$c_{\mathcal{D}}$	Dissipation coefficient (locally normalized), p. 66
$c_f$	Skin friction coefficient (locally normalized), p. 65
$C_f$	Skin friction coefficient, p. 12
$\bar{C}_f$	Average skin friction coefficient, p. 88
$c_{\ell}$	2D section lift coefficient, p. 40
$C_L$	Lift coefficient, p. 125
$C_{\ell}$	Rolling moment coefficient, p. 125
$C_m$	Pitching moment coefficient, p. 125
$C_n$	Yawing moment coefficient, p. 125

$C_p$	Pressure coefficient, p. 12
$c_p$	Specific heat at constant pressure, p. 2
$c_v$	Specific heat at constant volume, p. 2
$C_Y$	Sideforce coefficient, p. 125
$C$	Theodorsen function, p. 156
$d$	Body diameter, p. 115
$\mathcal{D}$	Dissipation integral, p. 66
$D$	Drag, p. 100
$D'$	Drag per unit span (in 2D), p. 40
$D_i$	Induced drag, p. 105
$D_p$	Profile drag, p. 105
$D_w$	Wave drag, p. 188
$e$	Specific energy, p. 2
$e$	Span efficiency, p. 113
$\dot{E}$	Energy rate (power), p. 59
$f$	Body force per unit mass field, p. 4
$F$	Force, p. 59
$\mathbf{F}$	Force vector, p. 124
$\mathcal{F}_\theta$	Momentum thickness growth function, p. 81
$g$	Gravitational acceleration magnitude, p. 19
$\mathbf{g}$	Gravitational acceleration vector, p. 4
$g$	Gain of control variable on surface deflection, p. 132
$G$	Clauser shape parameter, p. 73
$h$	Specific enthalpy, p. 2
$h$	Unsteady airfoil heave displacement, p. 152
$h$	Hyperbolic radius, p. 185
$h$	Wind tunnel cross-section height, p. 239
$\hat{\mathbf{h}}$	Hinge-axis unit vector, p. 132
$\mathbf{h}$	Angular momentum of onboard rotors, p. 205
$H$	Boundary layer shape parameter, p. 65
$H^*$	Kinetic energy shape parameter, p. 66
$H^{**}$	Density flux thickness shape parameter, p. 66
$\mathbf{H}$	Total aircraft angular momentum, p. 205
$\bar{\mathbf{I}}$	Unit tensor, identity matrix, p. 6
$\bar{\mathbf{I}}$	Mass moment of inertia tensor, p. 205
$I$	Moment of inertia component, p. 206
$k$	Heat conductivity, p. 1
$k$	Reduced frequency, p. 156
$K$	Kernel function for scalar field, p. 32
$\mathbf{K}$	Kernel function for vector field, p. 25

$K$	Boundary layer kinetic energy defect, p. 59
$K_f$	Profile drag form factor, p. 89
$Kn$	Knudsen number, p. 1
$\ell$	Body length, p. 1
$\ell$	Coordinate on surface (of $sln$ system), p. 26
$l$	Circuit or control volume integration coordinate, p. 41
$L$	Lift, p. 100
$L'$	Lift per unit span (in 2D), p. 40
$\mathcal{L}$	Rolling moment, p. 124
$\mathcal{L}$	Lagrangian function, p. 118
$m$	Boundary layer mass defect, p. 49
$m$	Aircraft mass, p. 205
$\dot{m}$	Mass flow, p. 49
$M$	Mach number, p. 10
$\mathbf{M}$	Moment vector, p. 124
$\mathcal{M}$	Pitching moment, p. 124
$n$	Coordinate normal to surface (of $sln$ system), p. 26
$\hat{\mathbf{n}}$	Unit normal vector, p. 4
$\tilde{N}$	Laminar instability amplitude exponent, p. 93
$\mathcal{N}$	Yawing moment, p. 124
$p$	Pressure, p. 1
$p'$	Dynamic part of pressure field, p. 20
$\bar{p}$	Roll rate, p. 124
$\bar{p}$	Dimensionless roll rate ( $= pb_{\text{ref}}/2V_{\infty}$ ), p. 125
$Pr$	Prandtl number, p. 10
$P$	Boundary layer momentum defect, p. 59
$q$	Pitch rate, p. 124
$\bar{q}$	Dimensionless pitch rate ( $= qc_{\text{ref}}/2V_{\infty}$ ), p. 125
$\dot{q}_s$	Heating rate per unit area, p. 6
$\dot{q}_v$	Heating rate per unit volume, p. 5
$\dot{\mathbf{q}}$	Heat flux vector, p. 6
$q_{\infty}$	Freestream dynamic pressure ( $= \frac{1}{2}\rho_{\infty}V_{\infty}^2$ ), p. 12
$Q$	Freestream dynamic pressure, for flight dynamics, p. 209
$Q$	Coefficient in PP2 equation, p. 170
$\mathbf{r}$	Cartesian position vector ( $= x \hat{\mathbf{x}} + y \hat{\mathbf{y}} + z \hat{\mathbf{z}}$ ), p. 3
$r$	Magnitude of Cartesian position vector, ( $= \sqrt{x^2 + y^2 + z^2}$ ), p. 3
$r$	Distance to $x$ axis in axisymmetric cases ( $= \sqrt{y^2 + z^2}$ ), p. 187
$r$	Yaw rate, p. 124
$\bar{r}$	Dimensionless yaw rate ( $= rb_{\text{ref}}/2V_{\infty}$ ), p. 125
$R$	Specific gas constant, p. 2

$R$	Radius kernel function, p. 38
$R$	Radius of axisymmetric body, p. 139
$Re$	Reynolds number, p. 10
$Re_\theta$	Momentum thickness Reynolds number, p. 81
$\mathbf{R}$	Position vector in Earth frame, p. 145
$\mathbf{R}_o$	Position vector of body-axes origin in Earth frame, p. 149
$s$	Specific entropy, p. 13
$s$	Coordinate on surface (of $sln$ system), p. 26
$S$	Reference area, p. 209
$St$	Strouhal number, p. 11
$\mathcal{S}$	Surface area, p. 7
$\mathcal{S}$	Sonic discriminator ( $= 1 - M^2$ ), p. 196
$\bar{\mathbf{S}}$	Stability-axes to body-axes $xz$ rotation matrix, p. 210
$t$	Time, p. 7
$t$	Airfoil thickness, p. 41
$T$	Temperature, p. 1
$\mathcal{T}$	Normalized skin friction function, p. 81
$T_s$	Sutherland's constant for air ( $= 110\text{ K}$ ), p. 1
$\bar{\mathbf{T}}$	Rotation matrix (direction cosine matrix), p. 124
$u$	Cartesian $x$ -velocity component, p. 3
$u_1$	Streamwise velocity component of 3D boundary layer, p. 76
$u_2$	Crossflow velocity component of 3D boundary layer, p. 76
$U$	Normalized boundary layer velocity ( $= u/u_e$ ), p. 60
$\mathbf{U}$	Body velocity vector, p. 4
$v$	Cartesian $y$ -velocity component, p. 3
$V$	Fluid speed, p. 4
$\mathbf{V}$	Fluid velocity, p. 4
$V_h$	Horizontal-tail volume coefficient, p. 218
$V_v$	Vertical-tail volume coefficient, p. 219
$\mathbf{v}$	Flight-dynamics eigenvector, p. 207
$\dot{\mathcal{V}}$	Volume outflow rate, p. 31
$\mathcal{V}$	Volume, p. 7
$w$	Cartesian $z$ -velocity component, p. 3
$\dot{w}_s$	Work rate per unit area, p. 6
$\dot{w}_v$	Work rate per unit volume, p. 5
$x$	Cartesian coordinate, p. 3
$\hat{x}$	Cartesian $x$ unit vector, p. 3
$\mathbf{x}$	Flight-dynamics state vector, p. 206
$X$	Axial force, p. 206
$X$	Normalized wind tunnel coordinate, p. 225

$y$	Cartesian coordinate, p. 3
$\hat{y}$	Cartesian $y$ unit vector, p. 3
$Y$	Sideforce, p. 100
$Y$	Normalized wind tunnel coordinate, p. 225
$z$	Cartesian coordinate, p. 3
$\hat{z}$	Cartesian $z$ unit vector, p. 3
$Z$	Normal force, p. 206
$Z$	Slender body camberline shape, p. 139
$Z$	Thin-airfoil camberline shape, p. 152
$Z$	Normalized wind tunnel coordinate, p. 225
$Z'$	Airfoil surface slope, p. 174

## Greek letters

$\alpha$	Angle of attack, p. 124
$\beta$	Clauser pressure gradient parameter, p. 73
$\beta$	Sideslip angle, p. 124
$\beta$	Subsonic Prandtl-Glauert factor ( $= \sqrt{1 - M_\infty^2}$ ), p. 173
$\beta$	Supersonic Prandtl-Glauert factor ( $= \sqrt{M_\infty^2 - 1}$ ), p. 183
$\gamma$	Ratio of specific heats ( $= c_p/c_v$ ), p. 3
$\gamma$	Vortex sheet strength, p. 23
$\gamma$	Vortex sheet strength (vector, in 3D), p. 23
$\Gamma$	Vortex filament strength, p. 27
$\Gamma$	Vortex filament strength vector (in 3D), p. 27
$\tilde{\Gamma}$	Circulation about closed circuit, p. 31
$\delta$	Boundary layer normal length scale, p. 70
$\delta^*$	Displacement thickness, p. 49
$\delta_{FS}$	Falkner-Skan boundary layer normal length scale, p. 71
$\delta^{**}$	Density flux thickness, p. 66
$\Delta^*$	Displacement area, p. 75
$\delta$	Flight-dynamics control vector, p. 206
$\epsilon$	Small quantity, p. 169
$\varepsilon$	Maximum camber, p. 175
$\varepsilon$	Wing downwash angle at tail, p. 217
$\zeta$	Damping ratio, p. 212
$\eta$	Boundary layer normal coordinate ( $= n/\delta$ ), p. 70
$\vartheta$	Glauert angle coordinate, p. 112
$\theta$	Polar angle coordinate, p. 38
$\theta$	Aircraft pitch angle, p. 202
$\Theta$	Angle kernel function, p. 38
$\theta$	Momentum thickness, p. 59

$\theta^*$	Kinetic energy thickness, p. 59
$\Theta$	Momentum area, p. 75
$\Theta^*$	Kinetic energy area, p. 75
$\kappa$	2D doublet strength, p. 29
$\mathcal{K}$	3D doublet strength, p. 29
$\lambda$	Molecular mean free path, p. 1
$\lambda$	Source sheet strength, p. 23
$\lambda$	Thwaites pressure gradient parameter, p. 81
$\Lambda$	Scaled pressure gradient parameter, p. 84
$\lambda$	Flight-dynamics eigenvalue, p. 207
$\lambda$	Wing taper ratio, p. 216
$\Lambda$	Source filament strength, p. 27
$\Lambda$	Wing sweep angle, p. 77
$\Lambda$	Lagrange multiplier, p. 119
$\mu$	Doublet sheet strength, p. 29
$\mu$	Viscosity, p. 1
$\mu_t$	Eddy viscosity, p. 61
$\nu$	Kinematic viscosity ( $= \mu/\rho$ ), p. 17
$\xi$	Characteristic variable, p. 183
$\rho$	Density, p. 1
$\sigma$	Real part of flight-dynamics eigenvalue (time constant), p. 207
$\sigma$	Source density (dilatation rate $\nabla \cdot \mathbf{V}$ ), p. 6
$\Sigma$	Source point strength, p. 27
$\bar{\tau}$	Viscous stress tensor, p. 5
$\tau$	Viscous stress vector, p. 5
$\Upsilon$	Effective wing dihedral angle, p. 216
$\varphi$	Perturbation velocity potential, p. 38
$\phi$	Velocity potential, p. 19
$\phi$	Normalized perturbation potential, p. 170
$\phi$	Aircraft roll angle, p. 202
$\Phi$	Wagner function, p. 154
$\Phi$	Full velocity potential, p. 166
$\chi$	Flow curvature from wind tunnel images, p. 228
$\psi$	Aircraft heading angle, p. 202
$\Psi$	Küssner function, p. 154
$\omega$	Imaginary part of flight-dynamics eigenvalue (radian frequency), p. 207
$\omega$	Vorticity (in 2D), p. 28
$\omega$	Vorticity vector (in 3D), p. 17
$\Omega$	Body angular velocity vector, p. 4

## Operators

- $D(\cdot)/Dt$  Substantial derivative ( $= \frac{\partial(\cdot)}{\partial t} + \mathbf{V} \cdot \nabla(\cdot)$ ), p. 8  
 $\nabla(\cdot)$  Gradient ( $= \frac{\partial(\cdot)}{\partial x} \hat{\mathbf{x}} + \frac{\partial(\cdot)}{\partial y} \hat{\mathbf{y}} + \frac{\partial(\cdot)}{\partial z} \hat{\mathbf{z}} = \frac{\partial(\cdot)}{\partial s} \hat{\mathbf{s}} + \frac{\partial(\cdot)}{\partial \ell} \hat{\mathbf{\ell}} + \frac{\partial(\cdot)}{\partial n} \hat{\mathbf{n}}$ ), p. 6  
 $\tilde{\nabla}(\cdot)$  Surface gradient ( $= \frac{\partial(\cdot)}{\partial s} \hat{\mathbf{s}} + \frac{\partial(\cdot)}{\partial \ell} \hat{\mathbf{\ell}}$ ), p. 28  
 $(\dot{\cdot})$  Time derivative ( $= \frac{\partial(\cdot)}{\partial t}$ ), p. 146

## Subscripts

- $(\cdot)_0$  Trim-state quantity, p. 207  
 $(\cdot)_{\infty}$  Freestream, p. 9  
 $(\cdot)_A$  Apparent-mass force or moment, p. 154  
 $(\cdot)_b$  Related to boundary conditions, p. 25  
 $(\cdot)_e$  At edge of boundary layer, p. 47  
 $(\cdot)_{\text{eff}}$  Effective-freestream quantity (excludes near-field contributions), p. 104  
 $(\cdot)_i$  Equivalent Inviscid Flow, p. 47  
 $(\cdot)_i$  Related to induced drag or downwash, p. 104  
 $(\cdot)_i$  Control-point index in vortex lattice method, p. 132  
 $(\cdot)_j$  Horseshoe vortex index in vortex lattice method, p. 132  
 $(\cdot)_l$  On lower surface, p. 51  
 $(\cdot)_l$  Control variable index in vortex lattice method, p. 132  
 $(\cdot)_{LE}$  At leading edge, p. 37  
 $(\cdot)_o$  Stagnation (total) quantity, p. 4  
 $(\cdot)_Q$  Quasi-steady force or moment, p. 154  
 $(\cdot)_{\text{ref}}$  Reference value, p. 2  
 $(\cdot)_{\text{SL}}$  Sea-level Standard Atmosphere, p. 1  
 $(\cdot)_{TE}$  At trailing edge, p. 37  
 $(\cdot)_{\text{tr}}$  At transition location, p. 89  
 $(\cdot)_u$  On upper surface, p. 51  
 $(\cdot)_u$  Measured quantity uncorrected for tunnel boundary effects, p. 223  
 $(\cdot)_w$  At wall, p. 47  
 $(\cdot)_{\perp}$  Component or quantity perpendicular to wing, p. 77  
 $(\cdot)_{\parallel}$  Component or quantity parallel to wing, p. 77

## Superscripts

- $(\cdot)'$  Dummy variable of integration, p. 2  
 $(\cdot)^b$  Vector component in body axes, p. 265  
 $(\cdot)^e$  Vector component in Earth axes, p. 265  
 $(\cdot)^s$  Vector component in stability axes, p. 124  
 $(\cdot)^w$  Vector component in wind axes, p. 125  
 $(\bar{\cdot})$  Dimensionless quantity (in Chapters 1, 6), p. 10  
 $(\bar{\cdot})$  Quantity in Prandtl-Glauert space (in Chapter 8), p. 173

# Contents

Preface	xv
Nomenclature	xvii
<b>1 Physics of Aerodynamic Flows</b>	<b>1</b>
1.1 Atmospheric Properties . . . . .	1
1.2 Ideal-Gas Thermodynamic Relations . . . . .	2
1.3 Conservation Laws . . . . .	3
1.3.1 Mass, momentum, energy fluxes . . . . .	4
1.3.2 Volume forces, work rate, heating . . . . .	4
1.3.3 Surface forces, work rate, heating . . . . .	5
1.3.4 Integral conservation laws . . . . .	6
1.4 Differential Conservation Equations . . . . .	7
1.4.1 Divergence forms . . . . .	7
1.4.2 Convective forms . . . . .	8
1.4.3 Surface boundary conditions . . . . .	8
1.5 Units and Parameters . . . . .	9
1.5.1 Unit systems . . . . .	9
1.5.2 Non-dimensionalization . . . . .	10
1.5.3 Unsteady-flow parameters . . . . .	10
1.5.4 High Reynolds number flows . . . . .	11
1.5.5 Standard coefficients . . . . .	11
1.6 Adiabatic Flows . . . . .	12
1.7 Isentropic Flows . . . . .	13
1.7.1 Requirements for isentropy . . . . .	13
1.7.2 Isentropic relations . . . . .	14
1.7.3 Speed of sound . . . . .	15
1.7.4 Total pressure and density . . . . .	15
1.8 Low Speed and Incompressible Flows . . . . .	16

1.9	Vorticity Transport and Irrotationality . . . . .	17
1.9.1	Helmholtz vorticity transport equation . . . . .	17
1.9.2	Crocco relation . . . . .	19
1.9.3	Bernoulli equation . . . . .	19
1.10	Aerodynamic Flow Categories . . . . .	21
<b>2</b>	<b>Flow-Field Modeling</b>	<b>23</b>
2.1	Vector Field Representation Methods . . . . .	23
2.2	Velocity / Vorticity-Source Duality . . . . .	24
2.3	Aerodynamic Modeling – Vorticity and Source Lumping . . . . .	25
2.3.1	Sheets . . . . .	26
2.3.2	Lines . . . . .	26
2.3.3	Points . . . . .	27
2.3.4	2D forms . . . . .	27
2.4	3D Vortex Sheet Strength Divergence Constraint . . . . .	28
2.5	Equivalence of Vortex and Doublet Sheets . . . . .	29
2.6	Integral Velocity / Vorticity-Source Relations . . . . .	31
2.7	Velocity-Potential Integrals . . . . .	32
2.7.1	3D potentials . . . . .	32
2.7.2	2D potentials . . . . .	32
2.8	Physical Requirements . . . . .	33
2.8.1	Sources in incompressible flow . . . . .	34
2.8.2	Sources in compressible flow . . . . .	34
2.8.3	Vorticity in high Reynolds number flows . . . . .	35
2.9	Flow-Field Modeling with Source and Vortex Sheets . . . . .	36
2.9.1	Source sheet applications . . . . .	36
2.9.2	Vortex sheet applications . . . . .	37
2.10	Modeling Non-uniqueness . . . . .	37
2.11	2D Far-Field Approximations . . . . .	38
2.11.1	2D source and vortex distribution far-field . . . . .	38
2.11.2	Far-field effect of lift and drag . . . . .	40
2.11.3	Far-field effect of thickness . . . . .	41
2.11.4	Far-field effect of lift's pitching moment . . . . .	42
2.11.5	Doublet orientation . . . . .	43
2.11.6	2D far-field observations . . . . .	43
2.12	3D Far-Fields . . . . .	44
2.12.1	3D far-field effect of drag . . . . .	44
2.12.2	3D far-field effect of volume . . . . .	45

<b>3 Viscous Effects in Aerodynamic Flows</b>	<b>47</b>
3.1 Inviscid Flow Model . . . . .	47
3.2 Displacement Effect . . . . .	48
3.2.1 Normal mass flux matching . . . . .	48
3.2.2 Normal mass flux in real flow . . . . .	49
3.3 Improved Inviscid Flow Models . . . . .	49
3.3.1 Displacement Body model . . . . .	49
3.3.2 Wall Transpiration model . . . . .	50
3.3.3 Wake modeling . . . . .	51
3.3.4 Improved flow model advantages . . . . .	52
3.4 Viscous Decambering Stall Mechanism . . . . .	52
3.5 Considerations in Flow Model Selection . . . . .	53
<b>4 Boundary Layer Analysis</b>	<b>57</b>
4.1 Boundary Layer Flow Features and Overview . . . . .	57
4.2 Defect Integrals and Thicknesses . . . . .	58
4.2.1 Mass flow comparison . . . . .	58
4.2.2 Momentum and kinetic energy flow comparisons . . . . .	58
4.2.3 Other integral thickness interpretations . . . . .	60
4.3 Boundary Layer Governing Equations . . . . .	61
4.3.1 Thin Shear Layer approximations . . . . .	61
4.3.2 Boundary layer equations . . . . .	61
4.3.3 Characteristics of turbulent boundary layers . . . . .	62
4.4 Boundary Layer Response to Pressure and Shear Gradients . . . . .	63
4.5 Integral Boundary Layer Relations . . . . .	64
4.5.1 Integral momentum equation . . . . .	64
4.5.2 Integral kinetic energy equation . . . . .	65
4.5.3 Integral defect evolution . . . . .	66
4.5.4 Integral defect / profile drag relations . . . . .	68
4.6 Self-Similar Laminar Boundary Layers . . . . .	70
4.6.1 Wedge flows . . . . .	72
4.7 Self-Similar Turbulent Boundary Layers . . . . .	73
4.8 Axisymmetric Boundary Layers . . . . .	74
4.9 3D Boundary Layers . . . . .	76
4.9.1 Streamwise and crossflow profiles . . . . .	76
4.9.2 Infinite swept wing . . . . .	77
4.9.3 Crossflow gradient effects . . . . .	79

4.10	2D Boundary Layer Solution Methods – Overview	80
4.10.1	Classical boundary layer problem	80
4.10.2	Finite-difference solution methods	80
4.10.3	Integral solution methods	80
4.11	Integral Boundary Layer Solution	81
4.11.1	Thwaites method	81
4.11.2	White's equilibrium method	84
4.11.3	Two-equation methods	84
4.11.4	Viscous dissipation relations	85
4.12	Coupling of Potential Flow and Boundary Layers	87
4.12.1	Classical solution	87
4.12.2	Viscous/inviscid coupling	87
4.13	Profile Drag Prediction	88
4.13.1	Wetted-area methods	88
4.13.2	Local-friction and local-dissipation methods	90
4.13.3	Boundary layer calculation methods	90
4.14	Transition	92
4.14.1	Transition types	92
4.14.2	TS-wave natural transition prediction	93
4.14.3	Influence of shape parameter	93
4.14.4	Transitional separation bubbles	95
<b>5</b>	<b>Aerodynamic Force Analysis</b>	<b>99</b>
5.1	Near-Field Forces	99
5.1.1	Force definitions	99
5.1.2	Near-field force calculation	100
5.2	Far-Field Forces	101
5.3	Flow-Field Idealization	102
5.4	Wake Potential Jump	102
5.5	Lifting-Line Analysis	104
5.6	Idealized Far-Field Drag	106
5.6.1	Profile drag relations	107
5.6.2	Trefftz-plane velocities	108
5.6.3	Induced drag relations	109
5.7	Idealized Far-Field Lift and Sideforce	111
5.8	Trefftz Plane Integral Evaluation	112
5.8.1	Fourier series method for flat wake	112

5.8.2	Discrete panel method for a general wake . . . . .	114
5.9	Fuselage wake contraction effect . . . . .	115
5.10	Minimum Induced Drag . . . . .	116
5.10.1	Minimum induced drag problem statement . . . . .	116
5.10.2	Optimum normal sheet velocity . . . . .	117
5.10.3	Optimum potential jump calculation . . . . .	118
5.10.4	Additional constraints . . . . .	118
5.10.5	Example optimum load distributions . . . . .	119
<b>6</b>	<b>Aerodynamics of Aircraft in Maneuver</b>	<b>123</b>
6.1	Aircraft Motion Definition . . . . .	123
6.1.1	Aircraft velocity and rotation . . . . .	123
6.1.2	Body-point velocity . . . . .	124
6.2	Axis Systems . . . . .	124
6.2.1	Stability axes . . . . .	124
6.2.2	Wind axes . . . . .	125
6.3	Non-Dimensionalization and Parameterization . . . . .	125
6.3.1	Dimensionless variables . . . . .	125
6.3.2	Quasi-steady force and moment parameterization . . . . .	126
6.4	Lifting Surface Theory . . . . .	127
6.4.1	Vortex/doublet sheet geometry . . . . .	127
6.4.2	Lifting-surface problem formulation . . . . .	127
6.4.3	Near-field loads . . . . .	128
6.4.4	Trefftz-plane loads . . . . .	130
6.5	Vortex Lattice Method . . . . .	130
6.5.1	Vortex lattice discretization . . . . .	131
6.5.2	Velocity field representation . . . . .	131
6.5.3	Flow tangency condition . . . . .	132
6.5.4	Linear system setup and solution . . . . .	133
6.5.5	Near-field force and moment calculation . . . . .	134
6.5.6	Trefftz-plane force calculation . . . . .	135
6.5.7	Stability and control derivative calculation . . . . .	136
6.6	Slender Body Theory . . . . .	136
6.6.1	Slender body geometry . . . . .	136
6.6.2	Slender body flow-field . . . . .	137
6.6.3	2D unsteady flow interpretation . . . . .	138
6.6.4	Local 2D far-field . . . . .	138