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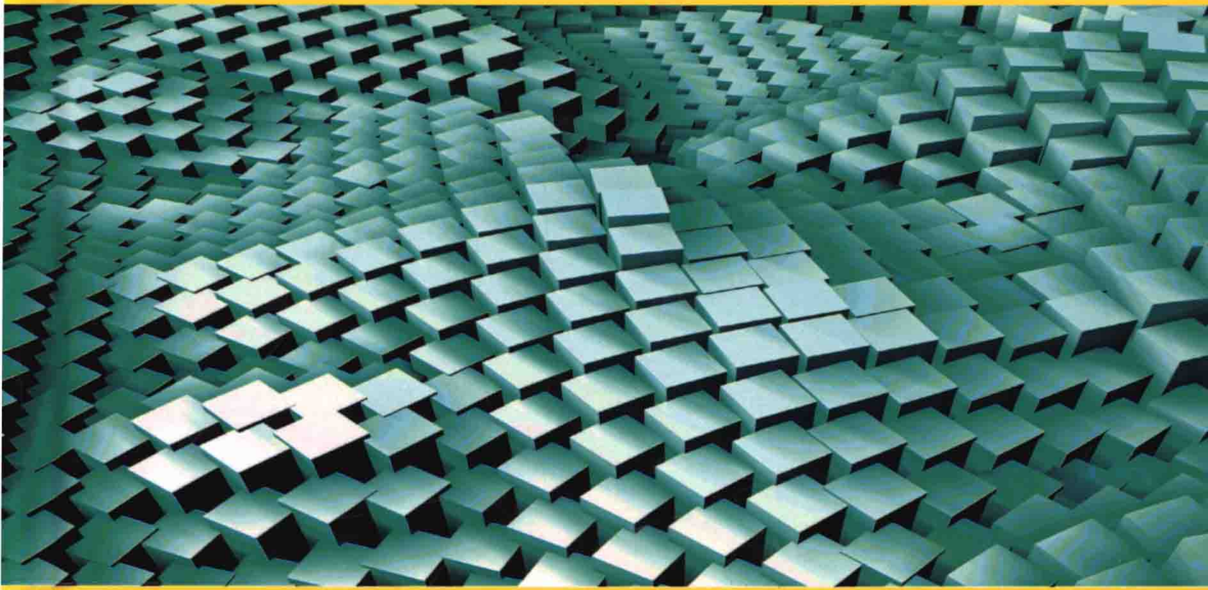

Volume 3

Movement Equations 3

Dynamics and Fundamental Principle

Michel Borel and Georges Vénizélos

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Coordinated by Abdelkhalak El Hami

This volume is the focal point of the work undertaken in the previous volumes of this set of books: the statement of the fundamental principle of the dynamics whose implementation, according to two paths whose choice depends on the problem to be treated, leads to equations of motion.

In order to achieve this, it is treated first of all in the context of solids in their environment, as a prerequisite for the formulation of the fundamental principle. Then, in addition to its use in some exercises, the approach is illustrated by three particular cases.

The first is an example where it is developed end-to-end and addresses the two approaches that lead to the equations of motion. The two other examples deal with two classical but important subjects, the movement of the Earth according to the hypotheses that can be stated about it, and Foucault's pendulum.

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Movement Equations 3

Introduction

With this third volume, the series on non-deformable solids reaches its acme; this is where we introduce and enlarge on the movement equations of non-deformable solids, which was always the initial goal.

The first volume of the series served to prepare the material necessary for writing these equations, that is how best to situate a solid in space to study its motion, how to describe its kinematics, the velocity and acceleration fields that drive it, how to characterize a solid through its inertial and kinetic configurations, and determine the energy statement of its motion.

But the development of this material, to arrive at the movement equations, requires various mathematical tools which the authors thought useful to remind rather than letting research them individually. This is the point of Volume 2.

With this third volume, readers are ready to touch on the core of the matter, the fundamental principle of dynamics and its application to cases where solids are free, or considered to be linked when there are bonds restricting their motion.

Chapter 1 of the book proposes a global vision of the fundamental principle and the conditions for its use, in particular the case where the observation frame of the motion of a solid is non-Galilean. The frame from which the motion of a solid is observed is crucial as it is this environment which exerts efforts upon it, affecting its progression.

The efforts, whether they are known or unknown (the links), have on the motion energetic consequences which we will evaluate by applying the fundamental principle. Chapter 2 places the solid in its environment, identifies the efforts and characterizes the power and energetic aspects they put into play throughout the motion.

The data for the problem are therefore acquired through the two first chapters, that means the following one, Chapter 3, is then in a position to begin applying the fundamental principle by presenting and enlarging on the scalar consequences that result from it and which produce the movement equations. Chapter 3 then ends with an example which serves to look through the different forms of these scalar consequences, knowing that the one which eventually is chosen depends essentially on the problem at hand.

Chapter 4 proposes two interesting cases for the application of the fundamental principle and shows how movement equations are used in various complex problems the solutions to which can only be obtained from hypotheses and simplifications without which the problem would not be treatable. These two cases are the motion of the Earth using inertial assumptions, and Foucault's pendulum according to the study by Michel Cazin in *Sciences* magazine in July 2000 where he bases himself on simplifying hypotheses to propose a credible explanation to the observed motion.

Chapter 5, which is the final chapter, plays a completely different role. Developing applications of the fundamental principle and establishing its scalar consequences require being familiar with the elements which contribute to its formulation, as they are presented in the first entry in the series. To grant readers with autonomy when using this book, a methodological formulary has been included, which recaps all essential points from Volume 1. This is the purpose of Chapter 5.

Arriving at this point, it is interesting to continue exploring certain individual cases through the ways they are used. This will be the subject of the fourth and fifth books in this series; the first among them will focus on the study of equilibrium situations for non-deformable solids and on small motions (or oscillations) that they experience around them; the final entry in the series will look at the motions of solid systems including cases of equilibrium and oscillations, with an introduction to robotics.

With this present volume and with the ones that preceded it and will follow it, the authors wished to explore the motion of non-deformable solids, and provide professional or student users with a structured mathematical approach. The lessons they have been giving at the CNAM since the 1970s has convinced them of the benefits of using such an approach and encouraged them to create this series.

Table of Notations

M	material point
t	time
m_H	mass of the sun $- 2.10^{30}\,kg$
m_T	mass of the Earth $- 6.10^{24}\,kg$
G_H	center of inertia of the Sun
G_T	center of inertia of the Earth
G_TG_H	distance between the Sun and the Earth $\sim 150.10^9\,m$
$\mathscr{G}$	Universal gravitational constant $6,67.10^{-11}\,m^3 kg^{-1} s^{-2}$
$m(S)$	mass of a solid (S)
δ_{ij}	Kronecker symbol
ε_{ijk}	three-index permutation symbol
$\vec{V}$	vector
$(\lambda) \equiv (\overrightarrow{x_\lambda}\ \overrightarrow{y_\lambda}\ \overrightarrow{z_\lambda})$	basis
$\langle\lambda\rangle \equiv \langle O_\lambda \| \overrightarrow{x_\lambda}\ \overrightarrow{y_\lambda}\ \overrightarrow{z_\lambda}\rangle$	frame
ψ, θ, φ	Euler angles, specifically the precession, nutation and spin angles in order
$\Pi(\vec{V}, \vec{W})$	plane of the two vectors $\vec{V}$ and $\vec{W}$
$\Pi(O \| \vec{V}, \vec{W})$	plane of the two vectors $\vec{V}$ and $\vec{W}$ passing through point O
$\overrightarrow{OM}$	bipoint vector

$\overrightarrow{O_\lambda O_S}$	situation bipoint or situation vector of point O_S in relation to the point O_λ of selected frame of reference $\langle\lambda\rangle$
$\widehat{\vec{V},\vec{W}}$	angle of two vectors oriented from $\vec{V}$ towards $\vec{W}$
$\lVert\vec{V}\rVert$	norm of vector $\vec{V}$
$\vec{V}\cdot\vec{W}$	scalar product of vectors $\vec{V}$ and $\vec{W}$
$\vec{V}\wedge\vec{W}$	vector product of vectors $\vec{V}$ and $\vec{W}$
$\vec{u}(\alpha)$	polar unit vector in cylindrical-polar coordinates
$\vec{k}(\alpha,\beta)$	polar unit vector in spherical coordinates
$\mathscr{R}_{\vec{u},\alpha}\equiv\left[\vec{u}\middle\vert\alpha\right]$	vector rotation of angle α around the axis defined by vector $\vec{u}$
$\Gamma_\lambda^{(t_i,t_f)}(\mathrm{M})$	trajectory, in the frame $\langle\lambda\rangle$, of material point M, during the time interval $\left[t_i,t_f\right]$
$\overrightarrow{v^{(\lambda)}}(\mathrm{M}\vert t)$ ou $\overrightarrow{v^{(\lambda)}}(\mathrm{M})$	velocity at time t of the material point M during its motion in the frame $\langle\lambda\rangle$
$\overrightarrow{J^{(\lambda)}}(\mathrm{M}\vert t)$ ou $\overrightarrow{J^{(\lambda)}}(\mathrm{M})$	acceleration at time t of the material point M throughout its motion in the frame $\langle\lambda\rangle$
$\overrightarrow{\omega_S^\lambda}$	rotation vector or rotation rate of the solid (S) in its motion in relation to frame $\langle\lambda\rangle$
$\vec{v}_e\left(\mathrm{M}\vert_\mu^\lambda\right)$	drive velocity of the material point M in the relative motion of the frame $\langle\mu\rangle$ in relation to the frame $\langle\lambda\rangle$
$\vec{J}_e\left(\mathrm{M}\vert_\mu^\lambda\right)$	drive acceleration of the material point M in the relative motion of the frame $\langle\mu\rangle$ in relation to the frame $\langle\lambda\rangle$
$\vec{J}_c\left(\mathrm{M}\vert_\mu^\lambda\right)$	Coriolis acceleration applied to the material point M during its relative motion of the frame $\langle\mu\rangle$ in relation to the frame $\langle\lambda\rangle$
$\dfrac{d^{(\lambda)}}{dt}\vec{V}$	derivative in relation to time of the vector $\vec{V}$ in the frame $\langle\lambda\rangle$

$\{\mathscr{T}\}_P = \left[\vec{s}\{\mathscr{T}\} \middle\vert \overrightarrow{\mathscr{M}_P}\{\mathscr{T}\}\right]$	torsor characterized by its two reduction elements at point P
$\vec{s}\{\mathscr{T}\}$	resultant of the torsor $\{\mathscr{T}\}$: 1st reduction element
$\overrightarrow{\mathscr{M}_P}\{\mathscr{T}\}$	moment at P of the torsor $\{\mathscr{T}\}$: 2nd reduction element $\overrightarrow{\mathscr{M}_Q}\{\mathscr{T}\} = \overrightarrow{\mathscr{M}_P}\{\mathscr{T}\} + \overrightarrow{QP} \wedge \vec{s}\{\mathscr{T}\}$
$\mathscr{I} = \vec{s}\{\mathscr{T}\} \cdot \overrightarrow{\mathscr{M}_P}\{\mathscr{T}\}$	scalar invariant of the torsor $\{\mathscr{T}\}$, independent of point P
$\{\mathscr{T}_1\} \otimes \{\mathscr{T}_2\}$	product of two torsors
$\{^{\lambda}_{S}\}_{P_S} = \left[\overrightarrow{\omega_S^{\lambda}} \middle\vert \overrightarrow{v^{(\lambda)}}(P_S)\right]$	velocity distributing torsor or kinematic torsor associated with the motion of the material point P_S of the solid (S)
$\{p_S^{\lambda}\}$	kinetic torsor associated with the motion of the solid (S) in the frame $\langle\lambda\rangle$
$\{\mathscr{A}_S^{\lambda}\}$	dynamic torsor associated with the motion of solid (S) in the frame $\langle\lambda\rangle$
$I_{O_S}(S\vert m)$	inertia operator of the solid (S) provided the measure of mass m
$\{\mathscr{A}_s^e \vert ^g_\lambda\}$	inertia drive torsor of the solid (S) in the relative motion of $\langle\lambda\rangle$ in relation to $\langle g\rangle$
$\{\mathscr{A}_s^c \vert ^g_\lambda\}$	inertia Coriolis torsor of solid (S) in the motion relative of $\langle\lambda\rangle$ in relation to $\langle g\rangle$
$\{\Delta\}$	torsor of known efforts
g	acceleration of Earth's gravity $\sim 9.80665\ ms^{-2}$ (9.81 on average) depending on the location and latitude of the body which is subject to
$\{\mathscr{L}\}$	torsor of unknown efforts
$\mathscr{L}\left(Q_1,\dots Q_6 \middle\vert Q_1',\dots Q_6' \middle\vert t\right)$	link acting upon a solid
$\{\mathscr{L} \to S\}$	torsor of link efforts applied to the solid (S)

$\mathscr{P}^{(g)}(\mathrm{F} \to S)$	power developed by the set of forces F acting upon the solid (S) throughout its motion
$\Pi_{\alpha}^{(g)}(\mathrm{F} \to S)$	partial power relative to the variable Q_α, developed by the set of forces F acting upon the solid (S) throughout its motion
$T^{(\lambda)}(S)$	kinetic energy of the solid (S) throughout its motion in relation to the frame $\langle\lambda\rangle$
(L_α)	Lagrange equation relative to the variable Q_α

When the situation of the solid (S) in the frame $\langle\lambda\rangle$ is represented by the parameters Q_α, we write : $\left\{ {}^{\lambda}_{S} \right\} = \left\{ {}^{\lambda}_{S,\alpha} \right\} Q_\alpha'$ where

$\left\{ {}^{\lambda}_{S,\alpha} \right\}_{\mathrm{O}_S} = \left[\overrightarrow{{}^{\alpha}\delta(\lambda,S)} \middle| \overrightarrow{{}^{\alpha}d_{\mathrm{O}_S}} \right]$	partial distributing torsor relative to the variable Q_α
$\overrightarrow{{}^{\alpha}\delta(\lambda,S)}$	partial rotation rate relative to the variable Q_α, component of the variable Q_α' of the rotation rate, such that $\overrightarrow{\omega_S^{\lambda}} = \overrightarrow{{}^{\alpha}\delta(\lambda,S)}\, Q_\alpha'$
$\overrightarrow{{}^{\alpha}d_{\mathrm{O}_S}}$	component of the variable Q_α' of the velocity vector of the point O_S, such that $\overrightarrow{v^{(\lambda)}}(\mathrm{O}_S) = \overrightarrow{{}^{\alpha}d_{\mathrm{O}_S}}\, Q_\alpha'$

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