World Scientific Lecture Notes in Physics 57

FOUNDATIONS OF QUANTUM CHROMODYNAMICS

Second Edition

T Muta

量子色动力学基础　第2版

World Scientific
世界图书出版公司

ISBN 7-5062-5636-3

9 787506 256360 >

ISBN 7-5062-5636-3/O · 336
WB5636 定价：63.00元

World Scientific Lecture Notes in Physics – Vol. 57

FOUNDATIONS OF QUANTUM CHROMODYNAMICS

Second Edition

An Introduction to Perturbative Methods in Gauge Theories

T Muta
Hiroshima University

World Scientific
Singapore • New Jersey • London • Hong Kong

Published by

World Scientific Publishing Co. Pte. Ltd.

P O Box 128, Farrer Road, Singapore 912805

USA office: Suite 1B, 1060 Main Street, River Edge, NJ 07661

UK office: 57 Shelton Street, Covent Garden, London WC2H 9HE

British Library Cataloguing-in-Publication Data
A catalogue record for this book is available from the British Library.

Cover picture reproduced by courtesy of the ALEPH experiment group.

First published 1998
Reprinted 2000

FOUNDATIONS OF QUANTUM CHROMODYNAMCIS (2ND EDITION)
An Introduction to Perturbative Methods in Gauge Theories

ISBN 981-02-2674-8
ISBN 981-02-4229-8 (pbk)

本书由世界科技出版公司授权重印出版，限于中国大陆地区发行.

PREFACE TO THE SECOND EDITION

It is to my surprise that the basis of quantum chromodynamics (QCD) has been made firmer and firmer in its applicability in the perturbative as well as nonperturbative regime since the first edition of this book was prepared in 1986. While much theoretical progress has been made in perturbative QCD, the basic structure of its formulation has remained unchanged and so the book is still of value for researchers who wish to learn fundamental items of perturbative QCD.

In preparing the second edition, I thoroughly inspected the book to make corrections of misprints as many as possible. Two graduate students, Kazutaka Kawakami and Norikazu Yamada, were of great help in this tiring and time-consuming work. I would like to thank them for their beautiful work. In the second edition, some new sentences are added corresponding to the new developments in theoretical and experimental studies of QCD. Advices given to the author by Takashi Ohsugi, Yoshio Sumi and Reisaburo Tanaka have been very useful in the process of renewing the book. The appendix D is added for the convenience of the readers who are not used to employing the Feynman rule in our book but accustomed to the traditional Feynman rule which appears in many books.

T. Muta

Hiroshima, Japan
March 1997

PREFACE TO THE FIRST EDITION

The strong interactions of fundamental particles have been successfully described by a non-Abelian gauge field theory called quantum chromodynamics (QCD). Because of its outstanding property of asymptotic freedom, a perturbative treatment makes sense for short-distance phenomena and predictions of the theory have been tested in a variety of elementary particle reactions. On the other hand, it is most important to control the nonperturbative regime of QCD in order to fully understand the strong interactions of hadrons. Unfortunately, however, no established method has surfaced to deal with the nonperturbative regime of QCD although some promising attempts are available.

In this book the techniques developed for the perturbative regime of QCD are explained in detail. To do this I have attempted to start at a level as low as that of a first-year graduate student's and continue from there to discuss the subject in a self-contained manner as far as possible. As a consequence the part of the book on background material for QCD became rather bulky. I think this part may also serve as a comprehensive introduction to gauge field theories. In the rest of the book the renormalization group method, the operator-product expansion and other technical developments are elucidated, and QCD predictions based on these techniques are derived for short-distance reactions. In the final chapter the infrared divergence is discussed in some detail since it is of relevance to the argument on the validity of the perturbative treatment.

The book grew out of lectures given in many places. I would like to thank those people who participated in the lectures and who made valuable criticisms.

During the course of writing the manuscript I have benefited from discussions and correspondences with many people as well as their assistance. I am happy to express my thanks to my friends, Bill Bardeen, Andrzej Buras and Dennis Duke for many discussions and their encouragement. Discussions with Kazuo Fujikawa, Yoichi Kazama, Noboru Nakanishi and Akio

Sugamoto have been very useful and I wish to express my gratitude to them. I am indebted to Tony Zee for valuable correspondences and to Hisao Nakkagawa and Akira Niégawa for critical comments. I would like to express my gratitude to Kiyoshi Kato, Tomoo Munehisa, Tatsumasa Tsurugai, and Sakue Yamada for providing me with experimental data and Monte Carlo results used in the book. The criticisms and comments by my colleagues and students have been undoubtedly beneficial. My hearty thanks are due to Jiro Kodaira, Toshinobu Maehara, Junji Okada, Juichi Saito, Ryuichi Najima, Masato Inoue, Kenji Hamada and other members of our laboratory.

The idea of turning my lecture notes into a book was first suggested to me four years ago by Prof. K.K. Phua. Since then he has waited for the completion of the book with great patience and kindness for which I would like to express my sincere gratitude. I would also like to thank Yasuo Hara for his concern about the completion of my book and his constant encouragement.

My friend and colleague, Yoshio Sumi, deserves many thanks for his kind help in drawing all the figures in the book in spite of his busy teaching and research schedule. It is my pleasure to thank my colleague, Masayoshi Kikugawa, for his help in computerizing the typing procedure, and Minoru Yonezawa and Seiichi Wakaizumi for their encouragement and support while I was writing the manuscript. A note of gratitude is due also to Hiroko Nishiura for her invaluable help in typing and correcting the whole manuscript. Without her persevering assistance the present book would not have materialized. I would also like to thank Emi Nakamoto for her cheerful assistance in typing the manuscript. Finally, I wish to thank my mother for her constant support of my research activities and my family for their patience during the years of preparing the manuscript.

Hiroshima, Japan T. Muta
December 1986

CONTENTS

CHAPTER

ONE

INTRODUCTION

1.1. GENERAL SURVEY

In the late fifties and in the sixties it was generally believed that the strong interactions of hadrons may not be described in any sense by the perturbative method of quantum field theory. In fact a naive application of the pertubative method to meson theory was recognized to fail although such a description of meson theory might have been prompted by the success of quantum electrodynamics. Accordingly, formulations based on the perturbative method were discarded in the theory of strong interactions and a suitable formulation of the theory independent of the perturbative approach was sought after. Extensive studies in this direction have brought about, among other things, the reduction technique for S-matrix elements, dispersion relations based on the analyticity of hadronic scattering amplitudes, Regge-pole theory and the dual resonance model. Although these developments were successful in many respects to describe hadronic reactions phenomenologically, they were not quite satisfactory since they did not stem from first principles.

The principal breakthrough which eventually led us to quantum chromodynamics (QCD) was put forward in 1973 when the property of asymptotic freedom of non-Abelian gauge field theories was discovered [tHo 72a, Gro 73, Pol 73]. The property of asymptotic freedom now allows perturbative treatment of strong interactions at short distances. We shall describe in the following the developments related to the discovery of asymptotic freedom.

In the late sixties studies on the classification of hadrons, hadron mass spectra and hadronic interactions strongly suggested that hadrons were made of quarks, the fundamental building blocks [Gel 64, Zwe 64].[1] It was then natural to look for the dynamics obeyed by quark systems which is responsible for the composition of hadrons as well as hadronic reactions.

[1] The term "quark" was adopted by Gell-Mann from the sentence, "Three quarks for Muster Mark," in James Joyce's *Finnegans Wake*.

In order to obtain experimental information on quark dynamics it seems to be the most sensible to probe the inside of hadrons (e.g., protons) by applying a beam of structureless particles (i.e., leptons). This method of studying the structure of target particles is essentially the same as the one utilized a long time ago by Geiger, Marsden and Rutherford in clarifying the structure of atoms. For the study of the hadronic structure we need much higher energies and larger momentum transfers to obtain the higher resolution. The first series of such experiments to probe the structure of the proton was initiated in the 1960's at SLAC (Stanford Linear Accelerator Center) and the process was called deep inelastic electron-proton scattering.

In 1969 Bjorken [Bj 69] reported that the scaling property of structure functions in electron-nucleon scatterings[2] was expected in the deep inelastic region where momentum transfer squared q^2 and energy transfer ν of electrons are very large with the ratio q^2/ν kept fixed. This scaling is called Bjorken scaling for which it is claimed that structure functions in the deep inelastic region depend only on the ratio q^2/ν rather than on two independent variables q^2 and ν. Immediately after Bjorken's proposal, experimental confirmation was found for it [Pan 68].

One of the easiest ways to understand Bjorken scaling was to assume that the projectile electrons scatter off almost-free pointlike constituents [Bj 69a] inside nucleons which were called partons[3] [Fey 69, 69a]. For deep inelastic electron-nucleon scatterings, the momentum transfer squared q^2 is large so that the spatial resolution for observing the target nucleon by projectile electrons is high. Thus Bjorken scaling implies that the constituents of the nucleon look almost free and point-like when observed with high spatial resolution. Hence, if one accepts the parton idea, the dynamics governing the parton system should have the property that the interaction between partons becomes weaker at shorter distances. The partons were later identified with quarks since experimentally it was suggested that their quantum numbers such as charges and spins were practically the same as those of quarks.

Searches for quark dynamics were initiated right after the foundation of the parton model. All the known quantum field theories at that time were surveyed as possible candidates for quark dynamics and were shown not to enjoy the above-mentioned property that the interaction between quarks gets weaker at short distances [Zee 73]. It was the non-Abelian gauge field theory[4] which was left untouched in this analysis.[5] This theory is a gauge theory

[2] For details, see Secs. 1.3 and 4.1.2.

[3] Coined by R.P. Feynman.

[4] See Sec. 2.1 for an introduction to non-Abelian gauge field theories.

[5] Symanzik suggested that $\lambda\phi^4$ theory with negative λ had the required property [Sym 73]. The theory, however, is unphysical since it has no lower bound for energy.

similar to quantum electrodynamics though differing from it in that the corresponding gauge symmetry is not Abelian, i.e., generators of the symmetry group are noncommutative. Such a theory was originally introduced by Yang and Mills [Yan 54]. 't Hooft [tHo 72a], Gross and Wilczek [Gro 73] and Politzer [Pol 73] examined non-Abelian gauge field theories by the use of the renormalization group method and found that they satisfied the desired property which is now called asymptotic freedom. Soon after it was shown that only the non-Abelian gauge field theory exhibited the property of asymptotic freedom among the known theories in four-dimensional space-time [Col 73].[6] It was quite fortunate that, by that time, the quantization of non-Abelian gauge field theories had been achieved [Fad 67] and their renormalizability had already been proven [tHo 71].

The dynamics governing quark systems therefore is to be found in non-Abelian gauge field theories. As mentioned above non-Abelian gauge field theories are generated by symmetries described by a noncommutative algebra. This means that quark systems are required to have an extra symmetry associated with the non-Abelian gauge field theory describing the quark dynamics. What is this extra symmetry among quarks?

In the meantime it had been frequently suggested that quarks must have a new quantum number called color and exhibit the color symmetry[7] in order to resolve several difficulties in the quark model. The difficulties may be summarized as follows[8]: (1) the problem of constructing baryon wave functions, (2) nonobservation of isolated quarks, (3) discrepancy between the prediction and experimental data on total cross sections of $e^+e^- \to$ hadrons and decay rates for $\pi^0 \to 2\gamma$.

Fritzsch and Gell-Mann [Fri 72, 73] proposed that the extra symmetry of the non-Abelian gauge field theory be identified with color symmetry. By this identification most of the difficulties in quark models by that time could be resolved in a natural way and thus the theory of quark dynamics was finally established. The theory was named quantum chromodynamics (QCD).[9] The term "chromo" refers to the "color" symmetry of quark systems.

[6] For the space-time with dimension different from four, there are some other possibilities of asymptotically free theories: the ϕ^3 theory in six dimensions [Mac 74], the four-fermion theory in two dimensions [Gro 74a] and the Wentzel model in three dimensions [Mut 77].

[7] The idea of the color quantum number may be traced back to the paper by Han and Nambu [Han 65]. Its present form was given by Gell-Mann [Gel 72] who coined the term "color." The scheme equivalent to color but based on para-fermi statistics was proposed by Greenberg [Gre 64] and the equivalence between the color and para-fermi schemes was explicitly shown by Ohnuki and Kamefuchi [Ohn 73].

[8] For detailed discussions, see Sec. 1.2.

[9] By M. Gell-Mann.

Just like the photon which is an Abelian gauge field mediating electromagnetic interactions between charged particles in quantum electrodynamics (QED), the non-Abelian gauge field in QCD mediates color interactions between quarks. This non-Abelian gauge field in QCD is called the gluon[10] as it is responsible for binding the quarks together. While photons have no electric charge (i.e., they are neutral), gluons carry color charges and hence interact with each other even in the absence of quarks. This property of gluons is an essential ingredient for having asymptotic freedom.[11]

In the quark model with color symmetries hadrons appear as colorless states while quarks carry color quantum numbers. It is assumed that only colorless states are physically realized and hence quarks cannot be observed in isolated states. There is a possibility of explaining this assumption as a dynamical effect in QCD. In fact serious infrared divergences due to the massless gluons (which are more serious than those in QED because of the self-couplings of gluons) may be responsible for confining quarks at long distances [Wei 73, Gro 73a]. Thus QCD has a desirable property that it enjoys the asymptotic freedom at short distances while it has a possibility of quark confinement at long distances.[12]

According to the property of asymptotic freedom of QCD, one may now safely use perturbation theory to discuss short-distance reactions. This approach in QCD of using perturbation theory is often referred to as *perturbative QCD*. The earliest application of QCD was made to deep inelastic electron-nucleon scattering [Gro 73a, 74, Geo 74] in order to see how Bjorken scaling could be recovered although it was eventually found to be logarithmically violated by the QCD corrections. In this application it is necessary to extract the purely short-distance part out of the deep inelastic cross section in order to guarantee the safe application of perturbative calculations. The operator product expansion (OPE) [Wil 69, Zim 71, 73] was, in this context, known to be the most powerful tool. Using OPE one can uniquely extract the short-distance part to which perturbative calculation is safely applied. Apparently one finds that the lowest-order results in the perturbative calculations reproduce Bjorken scaling and so the parton picture is recovered. It can, however, be easily recognized that large logs associated with the large mass scale q^2 (momentum-transfer squared) spoil the perturbative treatment and some improvement is necessary. The

[10] The term "gluon" was originally introduced within the framework of neutral vector theories without recourse to the color symmetry [Gel 62].

[11] For details, see Sec. 3.4.

[12] In this book we restrict our discussions to short-distance phenomena and will not consider the quark-confinement problem any further. For a review see, e.g., [Mar 78, Ban 81].

improvement may be attained by summing the large logs to all orders and this summation is most efficiently performed by the use of the renormalization group equations (RGE) [Chr 72]. After this resummation it was found already in the first application of QCD to deep inelastic scatterings that the Bjorken scaling was violated logarithmically [Gro 73a, 74, Geo 74]. Thus in QCD Bjorken scaling holds only in an approximate sense. The slight deviation from the Bjorken scaling in structure functions was later confirmed experimentally in deep inelastic muon-nucleon scatterings [Wat 75, Cha 75]. This observation, together with the later analyses of further experimental data, gave strong support to the foundation of QCD.

Another immediate application of QCD was made to electron-positron (e^+e^-) annihilation processes which in themselves are purely of short-distance nature. As this process is free from long-distance effects which may destroy the perturbative argument, one may directly use perturbation theory in this application. Perturbative calculations of the total cross section of e^+e^- annihilations were performed with the help of RGE immediately after the discovery of asymptotic freedom in QCD [App 73, Zee 73a]. Unfortunately the QCD effect is very small and has not yet been given full support by experimental data.

Some more applications of QCD were also made to decays of heavy quarkonia such as J/ψ's (charmonium) and Υ's (bottomium) and to nonleptonic decays of hadrons. The first series, J/ψ's, of heavy quarkonia was found right after the foundation of QCD and the first attempt to apply QCD was made to decays of these heavy quarkonia [App 75, 75b]. Here the decay width of heavy quarkonium may be decomposed into two parts: the square of the quarkonium wave function at the origin and the quark-antiquark annihilation cross section. The latter part is purely of the short-distance nature and is subject to perturbative treatment. The possibility of explaining the $\Delta I = 1/2$ rule in nonleptonic weak decays of hadrons was argued by applying OPE to the weak effective Hamiltonian [Gai 74, Alt 74]. This argument seems to be in the right track at least qualitatively but according to subsequent detailed studies its predictive power was found to be limited by the presence of the nonperturbative effects. The photon-photon scattering obtained as a subprocess of $e^+e^- \to e^+e^-X$ (X represents unobserved hadrons) is yet another example of an early application of pertubative QCD [Ahm 75, Wit 77]. This is one of the cleanest short-distance processes and has received much attention.

A formulation equivalent to the one based on the operator-product expansion but entirely phrased in parton language was proposed by Altarelli and Parisi [Par 76, Alt 77] and is called the Altarelli-Parisi formalism. This method is much more transparent in its physical meaning since it directly deals with the parton-distribution function [Bau 78, Kod 78] inside hadrons and the parton-fragmentation function [Owe 78, Uem 78], which are basic quantities for describing short-distance reactions.

In 1977 the second phase was opened in the development of perturbative QCD. There were three major ingredients which initiated the second phase: the completion of the calculation of the next-to-leading-order effect in deep inelastic scatterings [Flo 77, 78, Bar 78], the introduction of a new kind of purely short-distance processes, i.e., jets from quarks and gluons [Ste 77, Shi 78, Ein 78] and the generalization of OPE to accommodate a wider class of short-distance processes [Ell 78, 79, Ama 78, 78a, Lib 78, Mue 78].

The successful generalization of OPE as well as the discovery of a new type of short-distance processes, jets, eventually enlarged the region of applicability of perturbative QCD. In fact, since 1978 a variety of processes such as the Drell-Yan process $NN \to l^+l^-X$ (N: nucleon, $l^\pm$: charged leptons, X: unobserved hadrons), inclusive e^+e^- annihilation $e^+e^- \to$ hadron + X, large-p_T hadron reactions and multi-jets in e^+e^- annihilations have come under our control. It has, however, been argued that in some processes such as the Drell-Yan process, care must be taken of the infrared effects arising from soft gluons.

The computations of higher-order effects in perturbative QCD have been performed in many processes such as e^+e^- annihilations [Che 79, 80, Din 79, Cel 80, 80a], quarkonium decays [Bar 79a, Mac 81], photon-photon scattering [Bar 79], Drell-Yan processes [Alt 78, Kub 79, Con 79, Har 79, Hum 79], inclusive e^+e^- annihilations [Cur 80, Fur 80, Fol 81, Oka 81] and jets in e^+e^- annihilations [Ell 80, 81, Fab 80, 82, Ver 81]. In dealing with higher-order effects one immediately finds [Bar 78] that the calculated results crucially depend on the way one renormalizes divergent integrals appearing in the calculation. This is usually called the renormalization-scheme dependence of perturbative predictions and is a rather general phenomenon in the perturbative treatment of renormalizable quantum field theories [Cel 79, 79a].

The renormalization-scheme dependence of perturbative predictions can be seen even in quantum electrodynamics although in QED its observable effect is negligible as the coupling constant, the expansion parameter in perturbation series, is very small ($\alpha = e^2/4\pi \sim 1/137$). In QCD the effective coupling constant at the energy scale available presently is not very small, $\alpha_S (= g^2/4\pi) \sim 1/10$, and hence nonnegligible effects may be brought about by a different choice of the renormalization scheme. This effect may obscure the meaningful tests of our perturbative predictions in the phenomenological application of QCD. Nevertheless, with some limitations, the successful comparison of the predictions with the data has been made in a variety of processes.