Advances in Mathematics and Its Applications

Yanyan Li Chi-Wang Shu
Rugang Ye Kang Zuo

中国科学技术大学出版社

当代科学技术基础理论与前沿问题研究丛书

Advances in Mathematics and Its Applications

中国科学技术大学出版社

内 容 简 介

本书是由一些综述性或原始研究论文组成的，涉及了数学的各个领域. 包括代数几何、应用数学、几何分析、图论、数学规划、数值分析和科学计算、运筹学和数学经济、算子代数、常微分和偏微分方程以及黎曼几何. 并且，每篇论文的作者至少有一位是中国科学技术大学数学系 1977 级学生.

本书适合数学研究者和研究生使用.

Advances in Mathematics and Its Applications
Yanyan Li, Chi-Wang Shu, Rugang Ye, Kang Zuo

Published by University of Science and Technology of China Press
96 Jinzhai Road, Hefei 230026, P. R. China

图书在版编目(CIP)数据

数学及其应用的进展＝Advances in Mathematics and Its Applications：英文/李岩岩等主编. —合肥：中国科学技术大学出版社，2009.1
（当代科学技术基础理论与前沿问题研究丛书：中国科学技术大学校友文库）
“十一五”国家重点图书
ISBN 978-7-312-02225-8

Ⅰ. 数… Ⅱ. 李… Ⅲ. 数学—文集—英文 Ⅳ. O1-53

中国版本图书馆 CIP 数据核字(2008)第 122452 号

出版发行 中国科学技术大学出版社
地址 安徽省合肥市金寨路 96 号，邮编：230026
印 刷 合肥晓星印刷有限责任公司
经 销 全国新华书店
开 本 710mm×1000mm 1/16
印 张 21
字 数 310 千
版 次 2009 年 1 月第 1 版
印 次 2009 年 1 月第 1 次印刷
定 价 58.00 元

总　　序

侯建国
(中国科学技术大学校长、中国科学院院士、第三世界科学院院士)

大学最重要的功能是向社会输送人才. 大学对于一个国家、民族乃至世界的重要性和贡献度，很大程度上是通过毕业生在社会各领域所取得的成就来体现的.

中国科学技术大学建校只有短短的五十年，之所以迅速成为享有较高国际声誉的著名大学之一，主要就是因为她培养出了一大批德才兼备的优秀毕业生. 他们志向高远、基础扎实、综合素质高、创新能力强，在国内外科技、经济、教育等领域做出了杰出的贡献，为中国科大赢得了“科技英才的摇篮”的美誉.

2008 年 9 月，胡锦涛总书记为中国科大建校五十周年发来贺信，信中称赞说：半个世纪以来，中国科学技术大学依托中国科学院，按照全院办校、所系结合的方针，弘扬红专并进、理实交融的校风，努力推进教学和科研工作的改革创新，为党和国家培养了一大批科技人才，取得了一系列具有世界先进水平的原创性科技成果，为推动我国科教事业发展和社会主义现代化建设做出了重要贡献.

据统计，中国科大迄今已毕业的 5 万人中，已有 42 人当选中国科学院和中国工程院院士，是同期（自 1963 年以来）毕业生中当选院士数最多的高校之一. 其中，本科毕业生中平均每 1000 人就产生 1 名院士和七百多名硕士、博士，比例位居全国高校之首. 还有众多的中青年才俊成为我国科技、企业、教育等领域的领军人物和骨干. 在历年评选的“中国青年五四奖章”获得者中，作为科技界、科技创新型企业界青年才俊代表，科大毕业生已连续多年榜上有名，获奖总人数位居全国高校前列. 鲜为人知的是，有数千名优秀毕业生踏上国防战线，为科技强军做出了重要贡献，涌现出二十多名科技将军和一大批国防科技中坚.

为反映中国科大五十年来人才培养成果，展示毕业生在科学研究中的最新进展，学校决定在建校五十周年之际，编辑出版《中国科学技术大学校友文库》，于2008 年 9 月起陆续出书，校庆年内集中出版 50 种. 该《文库》选题经过多轮严格的评审和论证，入选书稿学术水平高，已列为“十一五”国家重点图书出版规划.

入选作者中，有北京初创时期的毕业生，也有意气风发的少年班毕业生；有“两院”院士，也有 IEEE Fellow；有海内外科研院所、大专院校的教授，也有金融、IT 行业的英才；有默默奉献、矢志报国的科技将军，也有在国际前沿奋力拼搏的科研将才；有“文革”后留美学者中第一位担任美国大学系主任的青年教授，也有首批获得新中国博士学位的中年学者 …… 在母校五十周年华诞之际，他们通过著书立说的独特方式，向母校献礼，其深情厚意，令人感佩!

近年来，学校组织了一系列关于中国科大办学成就、经验、理念和优良传统的总结与讨论. 通过总结与讨论，我们更清醒地认识到，中国科大这所新中国亲手创办的新型理工科大学所肩负的历史使命和责任. 我想，中国科大的创办与发展，首要的目标就是围绕国家战略需求，培养造就世界一流科学家和科技领军人才. 五十年来，我们一直遵循这一目标定位，有效地探索了科教紧密结合、培养创新人才的成功之路，取得了令人瞩目的成就，也受到社会各界的广泛赞誉.

成绩属于过去，辉煌须待开创. 在未来的发展中，我们依然要牢牢把握“育人是大学第一要务”的宗旨，在坚守优良传统的基础上，不断改革创新，提高教育教学质量，早日实现胡锦涛总书记对中国科大的期待：瞄准世界科技前沿，服务国家发展战略，创造性地做好教学和科研工作，努力办成世界一流的研究型大学，培养造就更多更好的创新人才，为夺取全面建设小康社会新胜利、开创中国特色社会主义事业新局面贡献更大力量.

是为序.

2008 年 9 月

Preface I

I am very honored to write a short introduction for this volume, at the request of the four editors.

Thirty years ago, as China began its far-reaching reform and opening to the outside world, the entrance exam for study at colleges was re-implemented across the country. All teachers at the Mathematics Department of the University of Science and Technology of China (USTC), myself included, were very excited about this new development. We eagerly awaited a new generation of young students, when the entrance exam took place. In the spring of 1978, the new students arrived on the campus of USTC, located in Hefei, a city with more than 1700 years of history. Sixty-three students were enrolled in the Mathematics Department, who formed Class 771. They were very talented and studied very hard. Their progress at USTC was amazing, and their achievements later on in their careers are even more remarkable.

Beginning in the summer of 1978, at the suggestion and recommendation of Prof. S. S. Chern and Prof. W. T. Wu, I spent two years in the United States to visit University of California at Berkeley and the Institute for Advanced Study at Princeton. On my way back to China, I visited Bonn University in Germany. In 1980, four divisions of specialization were introduced for Class 771, which were Algebra, Analysis, Global Analysis and Differential Geometry, and Numerical Analysis and Applied Mathematics. I was in charge of Global Analysis and Differential Geometry. The professors at Bonn University were very interested in having students from my division, hence I sent three students there for pursuing graduate study, including two editors of this volume, Rugang Ye and Kang Zuo. Several other authors of this volume were also in my division. Other students in Class 771 went to Chinese Academy of Sciences and other institutions in China, as well as universities in USA and Europe to pursue graduate study.

An impressive number of people from Class 771 are continuing with doing mathematical research at high levels today. Moreover, they cover a number of

diverse fields, as the collection of papers in this volume manifest. As leading experts in their fields, they take the opportunity of the 50th anniversary of the founding of USTC and the 30th birthday of Class 771 to present surveys of various branches of mathematical research. I am very impressed by the high quality, in particular the depth and breadth, and nice styles of these papers. I believe that this volume is an excellent contribution to the mathematical community and highly recommend it to anyone who is interested in learning about recent advances and current trends in mathematics and its applications.

Chia-Kuei Peng (Jiagui Peng)
School of Mathematical Sciences
Graduate University of Chinese Academy of Sciences
Beijing, China

December 2007

Preface II

It is my great pleasure to write an introduction for this collection of papers.

The University of Science and Technology of China (USTC) was founded by the Chinese Academy of Sciences (CAS) in Beijing in 1958 in an effort to meet the needs of scientific and technological developments in China at that time. Based on the principle of an integrated structure of institutes and departments, Prof. Luo-Geng Hua was named the chair of the Mathematics Department, while Prof. Kang Feng was the vice chair of the department as well as the director of the computational mathematics division. I was assigned to a teaching position with the USTC by the Institute of Computing Technology, CAS in 1961 and was involved in the establishment of the specialty study in computational mathematics at the time. I formally joined the faculty of USTC in 1965 and was appointed as the director of the Computing Center, CAS in 1986.

During the twenty-five years of my teaching career in the USTC Mathematics Department from 1961 to 1986, I experienced the historical periods of USTC spanning its establishment, growth, stagnation and, finally, the revival since the beginning of the reform of China. I witnessed generations of young students who were trained and brought to the society by the USTC Mathematics Department. The young generations have found places in our country and worldwide and evolved into a driving force of the national as well as international leaders in the field of mathematics. Their accomplishments have brought great gratification to every one of us at USTC.

Students of Class 1977 were the first recruitment upon the re-establishment of the College Entrance Examination, a major development since the relocation of USTC to Hefei in 1970. The college graduates of this new generation inherited and further developed the USTC tradition. Their hard work and dedication have led to outstanding achievements. The fourteen papers included in this collection serve as a partial representation of the major scientific research accomplishments achieved by the Class 1977 graduates over the years. They present magnificent results and directions of future

development in the contemporary international mathematical researches, covering a wide range of fields ranging from pure mathematics to applied and computational mathematics. The depth and breadth of thinking shown in this collection fully demonstrate the success of USTC education which is characterized by solid foundation, wide knowledge base and strong developmental potential. Dedicated to the fifty-year anniversary of the founding of USTC (1958-2008), this collection carries special significance and I am very happy to see it appearing.

Zhong-ci Shi
Academy of Mathematics and Systems Science
Chinese Academy of Sciences
Beijing, China

February 2008

Preface III

It gives me great pleasure to write a preface for this volume, which brings me into deep thoughts and back to the year 1978.

In the spring of 1978, a large number of outstanding students were admitted to the University of Science and Technology of China after a national college entrance examination, the first one after the ten-year cultural revolution. Among them, 63 top quality students came to study, with great interest and passion in mathematics, in the Department of Mathematics, constituting Class 771. To train them to be promising mathematicians, the Department offered various basic courses as well as more advanced courses in modern mathematics during their four and half years of study. To ensure the success of the program, a number of teachers of high caliber were selected to teach the courses.

It was my pleasure to be appointed as the teacher of the first basic course Mathematical Analysis for Class 771. This was the most important basic course, the basics of the basic courses and a course of vital importance to the academic success of the students. The appointment let me enjoy the trust of the Department. I was fully aware of the importance and difficulty of the mission, and determined to train the students completely and thoroughly.

To start, I sounded them out about their mathematical knowledge and potential by holding two exams. Following a whole set of teaching methods created by my teacher, the internationally renowned Professor W. T. Wu, I began to train the students rigorously at increasing levels of materials and increasing degrees of difficulty in accordance with their aptitudes, in combination with the assignments of a large number of difficult extracurricular exercises. Then I let them share their individual solutions via wall postings and seminars. For example, 23 wall postings were put up and 13 seminar sessions took place during the first semester of their first college year. After the training in the first semester the students greatly broadened their scopes of thoughts and showed even more interest and passion in mathematics. During the process, top students were emerging, while the entire class was progressing rapidly. Everybody studied hard and diligently, and acquired the ability of

analyzing and solving problems on his/her own, which undoubtedly laid a firm foundation for future study and research. Professor W. T. Wu gave a highly positive assessment of this process of training. I myself was already deeply convinced at the time that the students of Class 771 would make outstanding contributions to mathematics later on by bringing into full play their intelligence, diligence and solid foundation.

The second course I taught was Real Analysis. It is traditionally a very difficult basic course in the Department of Mathematics. The difficulty lies in the complex concepts and techniques, and how to help students to acquire the ability of solving problems. I also taught the course Differentiable Manifolds. It was an obligatory course of modern mathematics in the twentieth century, and provided a bridge for the students to enter modern mathematical research.

Today, thirty years later, we see 27 people from Class 771 being so active in the mathematical community. They are well-known professors at famous universities and research institutes in China, France, Germany, the United States, and other countries. They have published a large number of papers in important mathematical journals, including a large number of papers in top journals. The present volume presents only a small part of their works, from which one can have a glance at their outstanding achievements.

Let me take as an example Mr. Yanyan Li who showed his talent as early as his college time. He has published more than 100 papers in first-rate mathematical journals and gave a 45-minute invited speech at the International Congress of Mathematicians. He is currently a professor at Rutgers University and a lectureship professor of Yangtze River Scholarship Project at Beijing Normal University. He is recognized by Thomason ISI as a "highly cited researcher". Another example is Mr. Chi-Wang Shu who was among the students of the best academic achievements at his college time. He has published more than 140 papers in first-rate mathematical journals. He was the chair of the Division of Applied Mathematics at Brown University and is currently a professor at Brown University and a lectureship professor of Yangtze River Scholarship Project at the University of Science and Technology of China. He has won the NASA achievement award in Computational Fluid Dynamics, the SIAM/ACM Prize in Computational Science and Engineering in the United States, and Feng Kang Prize of Scientific Computing at Chinese Academy of Sciences. He is also recognized by Thomason ISI as a "highly cited researcher". Among the Thomason ISI highly cited researchers in mathematics, a total of 10 received their Bachelor degrees from the mainland

China. As we see, two are from Class 771.

It is rare that so many mathematical talents who have published so large a number of academic papers of so high caliber have come from a single class at a university. We, teachers and students of the University of Science and Technology of China, are proud of and elated at the achievements of the people from Class 771.

Senlin Xu, Professor Emeritus
Department of Mathematics
University of Science and Technology of China
Hefei, China

January 2008

Preface IV

In 1977, after a ten-year period of interruption, the entrance exam for study at colleges was reintroduced in China, opening the door to college education to all qualified young people and ensuring the supply of students of high quality and motivation to colleges. Close to 6 million people took the entrance exam in December 1977. Over two hundred thousand people passed the exam and went on to study at colleges across China. They have often been referred to as students of the year 77, though they entered colleges in the spring of 1978. Since they graduated in 1982, they are also referred to as the graduates of the year 82. This major event ushered in a new era of higher education and developments of science and technology in China.

Sixty-three students of the year 77 were enrolled in the Mathematics Department of the University of Science and Technology of China (USTC), a renowned institution of higher education in China, and formed the so-called Class 771 of USTC. Obviously, the number 77 refers to the year 1977, while the number 1 is the numerical order of the Mathematics Department among the academic departments of USTC. The present volume is dedicated to the 50th anniversary of the founding of USTC, as well as to the 30th anniversary of the starting of college education at USTC of this group of people and the birth of Class 771. A number of people from Class 771 remain at the forefront of mathematical research today. In this volume, they take the opportunity of the anniversary to present in-depth surveys or original research of various branches of mathematics, which include algebraic geometry, applied mathematics, geometric analysis, graph theory, mathematical programming, numerical analysis and scientific computing, operations research and mathematical economics, operator algebra, ordinary and partial differential equations, and Riemannian geometry. Some additional contributions in other fields such as probability theory, topology and mathematical physics are not included due to time reason. We hope to add those contributions in a revised edition of this volume in the future.

Seven years into the new millennium, it is more clear than before in which directions mathematical research is and should be developing. First, interactions between and integration of different areas of mathematics play an increasingly prominent role. This is amply demonstrated by, e.g. Perelman's work on geometric deformations of 3-manifolds in terms of the Ricci flow, which covers the famous Poincaré conjecture and Thurston's geometrization conjecture. Here, an important branch of geometric analysis – the Ricci flow theory, invented by R. Hamilton, found a culminating application in a central topological problem. Various tools from Riemannian geometry and partial differential equations are integrated here for the purpose of analyzing the structures of the Ricci flow, in particular the structures of blow-up singularities of the Ricci flow. Secondly, interactions of mathematical research with natural sciences and other branches of sciences are becoming more and more important, and are creating new frontiers of research in mathematics. These include, among other things, interactions with high energy physics and physics of condensed matter, and new methods of mathematical computations. Thirdly, fundamental themes and problems of various branches of mathematics are being investigated on deeper levels and in broader scopes, based on new insights and newly developed tools. As examples, we would like to mention the Ricci flow, the Poincaré conjecture, the minimal model problem for algebraic varieties, and many deep problems in analysis and partial differential equations.

As editors of this volume, we are very happy to witness the ongoing successful research activities of our former classmates, and we are very happy to be able to put together in this volume the excellent surveys and research papers contributed by them. We believe that these papers reflect to various degrees the three major trends of current mathematical research mentioned above. The surveys are written both for the experts and the general audience. For the experts, they offer new perspectives and integrated pictures of the covered research areas. For the general audience, these surveys provide gentle introductions to a number of current research topics, including motivational backgrounds, central concepts, main results, and applications. In particular, we hope that these surveys can provide helpful hints for doing mathematical research for students. All the survey and research papers in this volume went through the standard process of multiple peer reviews.

When we look back at our four year college study at USTC, it is so clear how much we are indebted to USTC and our teachers at USTC. We would

like to take this opportunity to express our deep gratitude. During the period of preparing for this volume, a number of our former classmates shared their sweet memories of the four year college life at USTC. These are mostly different in nature from mathematical research and include many anecdotes of great fun. We hope that these memories can be put together in another book, so that they can be kept and shared for a very long time.

Yanyan Li, Rutgers University
Chi-Wang Shu, Brown University
Rugang Ye, University of California at Santa Barbara
Kang Zuo, Universität Mainz

December 2007

Contents