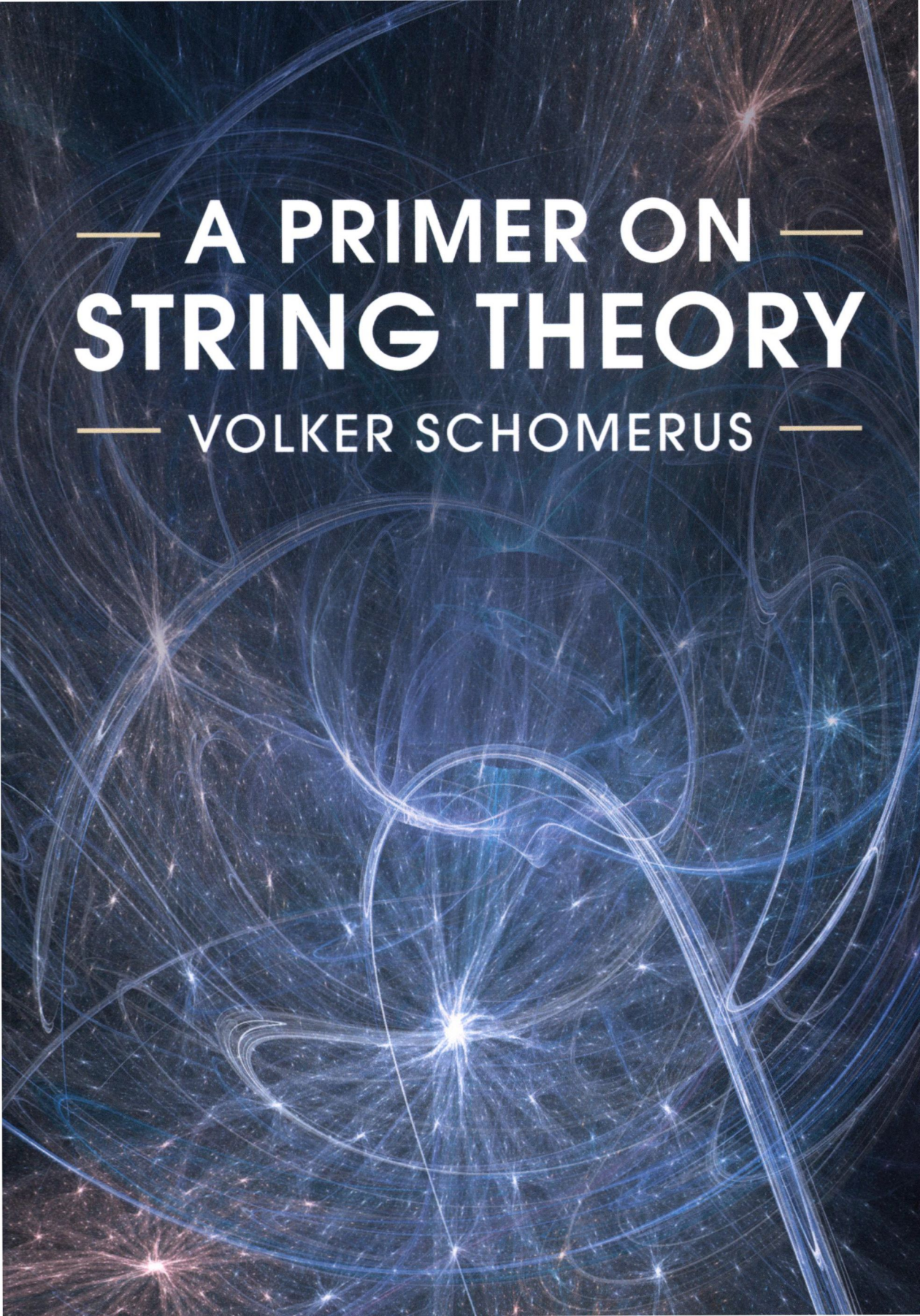
A PRIMER ON STRING THEORY

VOLKER SCHOMERUS



A systematic introduction to string theory that brings the reader to the forefront of contemporary developments in the field, written from an entirely modern perspective. The book covers an amazingly large number of topics with due attention to detail and logical rigour. Perfect book for first reading in string theory that can also be used as a reference by its practitioners.
Konstantin Zarembo, Nordic Institute for Theoretical Physics

This book successfully captures the essence of string theory that every theoretical physicist should know. I would like to use it in my next course on string theory at Caltech!
Hirosi Oguri, California Institute of Technology

Since its conception in the 1960s, string theory has been hailed as one of the most promising routes we have to unify quantum mechanics and general relativity. This book provides a concise introduction to string theory, explaining central concepts and mathematical tools and covering recent developments in physics, including compactifications and gauge/string dualities. With string theory being a multidisciplinary field interfacing with high-energy physics, mathematics, and quantum field theory, this book is ideal for both students with no previous knowledge of the field and scholars from other disciplines who are looking for an introduction to basic concepts.

Volker Schomerus holds a joint position as Senior Researcher in the DESY Theory Group and Professor of Mathematical Physics at Universität Hamburg. His work focuses on string theory at the interface of high-energy physics, mathematics, and statistical physics, for which he has received several distinctions, including the Guy-Lussac-Humboldt award in 2010. He is also co-author of the monograph *Boundary Conformal Field Theory and the World-Sheet Approach to D-Branes* (Cambridge, 2013).

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Preface

About half a century has passed since string theory was born, initially to explain the observed Regge trajectories of hadronic resonances. During these past decades, it has gone through several phases of dismissal and new appreciation, it has become a promising candidate for a quantum theory of gravity, and occasionally it has been popularized as a theory of everything. What most of these periods have in common are profound and fruitful interactions with rather different areas of theoretical physics, e.g., with quantum field theory, high-energy physics, gravity, mathematics, and even statistical and condensed matter physics. Indeed, there is a long list of string-inspired developments, a few of which we will have a chance to touch upon below. Among them are, e.g., new unified extensions of the standard model that are being tested in collider experiments, insights into black holes and their entropy, contributions to knot invariants or the theory of modular forms, and our understanding in particular of 2-dimensional critical systems, or the impressive recent progress in accessing both perturbative as well as non-perturbative effects in supersymmetric gauge theories. This interdisciplinary nature of string theory is its real strength, and it can serve as a good motivation to enter or explore the field.

While this textbook addresses primarily masters' or early PhD students, it may also be useful for scholars from neighboring fields who are looking for a short exposition of basic string theory. Compared to most other textbooks in the field, the scope here is fairly modest. Rather than attempting to cover string theory from its roots to all the fascinating modern developments, the intention is merely to provide a set of concepts and tools that are common to a wide range of recent research directions. By now many books have been written on string theory, which include various advanced and topical directions, such as [27, 28, 55, 56, 6, 39, 74, 41, 1] to list just a few that can be turned to for further reading. In addition, let me also mention a few shorter introductions such as [65, 68, 2]. Finally, a beautiful book by Barton Zwiebach [80] covers much less material but addresses undergraduates.

Some students might find it useful to consult this text, in particular to fill in occasional gaps in their undergraduate education.

The restrictions imposed on the choice of topics make it possible to cover the material of these Chapters in a one or two semester introductory course. Very few prerequisites are assumed, e.g., from quantum field theory, general relativity, or supersymmetry, so that master's students should have no problem following the exposition. The book consists of two parts. Part I describes strings in flat backgrounds. It introduces most of the basic constructions and results in string theory. This part of the material can be covered in a 10 to 12 lecture crash course, including Chapters 1–4, 6–8, and 10, along with Chapters 18 and 20 from Part II, for an outlook on modern developments. In this short form, the chapters assume some basic knowledge of supersymmetry. The minimal background is collected in Chapter 9. The remaining chapters from Part I deal with light-cone quantization, D-branes, and heterotic strings. These may be considered as additional material that may be included if time permits.

Part II contains somewhat more advanced topics on strings in curved backgrounds and string compactifications. It addresses three distinct subtopics. The first one is *conformal field theory*. As an example of the free field some very basic strategies and notions are presented in Chapter 13. The concept of modular invariance is the central theme of Chapter 14. It is explored with the example of the so-called orbifold construction. The third chapter on conformal field theory techniques deals with the SU(2) WESS-ZUMINO-NOVIKOV-WITTEN model, a theory that can be used to describe strings on a space of constant curvature. It allows us to illustrate most of the key techniques and concepts of conformal field theory.

The second subtopic treats CALABI-YAU spaces and the associated string *compactifications*. Chapter 16 introduces some background material on geometric concepts starting from complex manifolds and ending with some basic examples of complete intersections of CALABI-YAU spaces such as the famous quintic. This background material is then applied to discuss basic properties of string compactifications on CALABI-YAU spaces. The focus is on a space-time analysis, but a short discussion of the relevant world-sheet models is also provided, thereby interacting with the chapters on conformal field theory.

The book concludes with three chapters on *string dualities*. These possess the character of an overview rather than a detailed exposition. Chapter 18 starts with a brief discussion of T-duality, proceeds to the self-duality of type IIB superstring theory, and finishes with strongly coupled IIA models and some evidence for the existence of M-theory. Chapters 19 and 20 finally deal with the dualities between gauge theory and string theory that appear in the context of the *AdS/CFT correspondence*. These include a short discussion of large N_c limits before stating Maldacena's duality between the maximally (N=4) supersymmetric YANG-MILLS

(SYM) theory in four dimensions and strings in an $AdS_5 \times S^5$ background. Some basic entries of the dictionary between string and gauge theory are also introduced. The final chapter carries Maldacena's duality a bit deeper by providing more detail on N=4 SYM theory and on semiclassical strings in $AdS_5 \times S^5$.

I gave the lectures that make up this book or some of them on various occasions, in particular during the "String Steilkurs" at DESY, as one or two semester courses in Hamburg University, and at the APCTP Winter School. Many of the students who attended these courses have contributed through their questions and comments. I wish to thank all of them and in particular those who spent their time to compile seemingly never ending lists of misprints. I am particularly endebted to Philipp Höffer v. Loewenfeld and Johannes Oberreuter, who initiated these notes and typed the first 10 chapters. Some of the more advanced chapters were written with the assistence of my former students Vladimir Mitev and Maike Tormählen. Finally, I am grateful to Simon Capelin for his continued encouragement to complete the notes.

I dedicate this book to my wife Elena, to our sons Jonathan and Benedict, and to Leon.

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1

Historical Introduction and Overview

In this first chapter, I shall review some of the history of string theory, starting with early attempts to apply strings to hadronic physics. Then, I discuss how string theory emerged as a theory of gravitational physics and sketch the picture of modern string theory that was developed mostly during the second half of the 1990s. The discussion is based on images. It is the main goal of this book to introduce all the background that is required to give precise meaning to the cartoon you are about to see (Figures 1.2–1.6). In a first approach, the present chapter just serves for some rough and qualitative orientation.

1.1 String Theory and Strong Interactions

Most insight into the physics of particles comes from the study of scattering amplitudes. To take the simplest case, let us consider an elastic scattering process of two particles. After the scattering event, two particles emerge from the scattering event.

Such a $2 \to 2$ scattering process possesses many symmetries. The amplitude can depend only on kinematic invariants of the process. A widely used choice for these invariants is given by the so-called MANDELSTAM variables s, t, u, which are defined by

$$s = -(p_1 + p_2)^2, \tag{1.1}$$

$$t = -(p_1 + p_3)^2, \tag{1.2}$$

$$u = -(p_1 + p_4)^2. \tag{1.3}$$

Here, p_1 and p_2 denote the momenta of the incoming particles, and $-p_3, -p_4$ are momenta of the outgoing ones, see Figure 1.2. Throughout this book we shall work with a MINKOWSKI signature in which

$$q^2 = -q_0^2 + q_1^2 + \cdots + q_{D-1}^2.$$

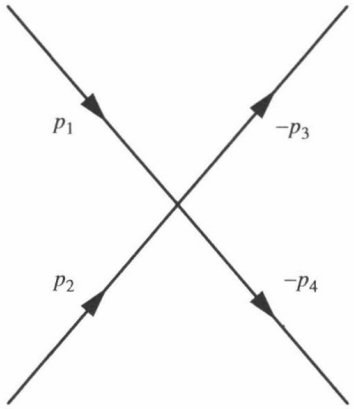


Figure 1.1 Kinematics of an elastic $2 \to 2$-scattering process of particles with momenta p_i.

The sum of the three MANDELSTAM variables is determined by the masses m_i of the particles through $s+t+u = \sum_i m_i^2$. From now on we shall consider amplitudes as functions of s, t.

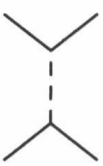
In the center of mass system, we can reexpress s and t through the energy $E = E_1 = E_2$ of the incoming particles and the center of mass scattering angle θ_s:

$$s = 4E_1^2 \,, \tag{1.4}$$

$$t = -\frac{s}{2}(1 - \cos\theta_s) \,. \tag{1.5}$$

In the late 1960s, physicists analyzed in great detail the scattering amplitudes for hadronic particles. What they found in the laboratory was an enormous number of new resonances. The longer the searches were pursued, the higher became the spins J and masses M of the observed particles. In addition, a curious linear relation $J = \alpha_0 + \alpha' M^2$ emerged that could be characterized by the so-called REGGE slope α' and intercept α_0; see, e.g. [37, 16]. Since at high energies s the t-channel exchange of a spin J resonance with mass M_J contributes a term $g_J^2(-s)^J/(t-M_J^2)$ to the total amplitude,[1] the overall contribution of the hadronic resonances takes the form

$$\mathcal{A}(s,t) \;=\; \sum_J$$



$$\sim -\sum_J g_J^2 \frac{(-s)^J}{t - M_J^2} \,, \tag{1.6}$$

where the sum is taken over all possible intermediate resonances that can be exchanged in the t channel. If there was only a finite number of such terms, i.e., if the sum stopped at some finite value of J, then the high-energy behavior

[1] The expression we work with here is valid only for large values of s. It receives lower order corrections for finite s that are well known but not relevant for our discussion.

would be dominated by the resonance with highest spin J, and the whole amplitude would diverge as we send s to infinity. If the sum is infinite, however, the outcome may be very different. As an example, let us consider the exponential function

$$\exp(-x) = 1 - x + \frac{1}{2}x^2 - \frac{1}{3!}x^3 + \dots .$$

Suppose now that we truncate this sum after the nth term. Then, the finite sum diverges for $x \to \infty$. On the other hand, the entire function $\exp(-x)$ is certainly finite. In fact, it even vanishes in the limit $x \to \infty$. Could something similar happen for hadronic scattering amplitudes, i.e., could there be some analytic expression $\mathcal{A}$ that reproduces the expansion (1.6) for small s but that stays finite at large s? The answer is positive, and one such expression is given by the so-called VENEZIANO amplitude [71]

$$\mathcal{A}_{\text{Ven}}(s,t) = \frac{\Gamma(-\alpha_0 - \alpha' s)\Gamma(-\alpha_0 - \alpha' t)}{\Gamma(-2\alpha_0 - \alpha'(s+t))} . \tag{1.7}$$

From the pole structure and shift properties of Γ functions it is easy to deduce the following expansion of $\mathcal{A}$ at small s:

$$\mathcal{A}_{\text{Ven}}(s,t) \sim -\sum_J \frac{P^J(s)}{\alpha' t - J + \alpha_0} . \tag{1.8}$$

Here, P^J is a polynomial of degree J. Hence, $\mathcal{A}$ does indeed encode the exchange of resonances that lie on a REGGE trajectory $M^2 = (J - \alpha_0)/\alpha'$. In 1970, string theory seemed to provide an exciting perspective on these findings. Namely, it was shown that simple (open) string theories in flat space [47, 50, 64] naturally reproduce the VENEZIANO amplitude with $\alpha_0 = 1$. This success of string theory is not too difficult to understand. String modes in flat space are harmonic oscillators, and it is well known from basic quantum mechanics that these possess a linear spectrum with a distance between the spectral lines that is determined by the tension T_s of the string. If we choose the latter to be $T_s \sim 1/\alpha'$, then we may identify hadronic resonances with vibrational modes of a string (provided we are willing to close an eye to the first resonance with $J = 0$, which is tachyonic because $\alpha_0 = 1$).

Obviously, the formula (1.7) must not be restricted to small center of mass energies. It can also be evaluated, e.g., for fixed angle scattering at large s. Using once more some simple properties of the Γ function one can derive

$$\mathcal{A}_{\text{Ven}}(s,t) \sim f(\theta_s)^{-1-\alpha' s} \quad \text{(large s)} , \tag{1.9}$$

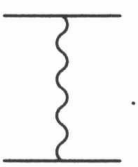
where f is some function of the center of mass scattering angle θ_s whose precise form is not relevant for us. The result shows that fixed angle scattering amplitudes predicted by flat space string theory fall off exponentially with the energy $\sqrt{s}$. Unfortunately for early string theory, this is not at all what is found in experiments,

which display much harder high-energy cross sections. The failure of string theory to produce the correct high-energy features of scattering experiments is once more easy to understand: strings are extended objects, and as such they do not interact in a single point but rather in an extended region of space-time. Consequently, their scattering amplitudes are rather soft at high energies (small distances), at least compared to point particles. In this sense, experiments clearly favored a point particle description of strongly interacting physics over fundamental GeV scale strings.

As we all know, a highly successful point particle model for strong interactions, known as Quantumchromodynamics (QCD), was established only a few years later. It belongs to the class of gauge theories that have ruled our description of nature for several decades now. Due to its asymptotic freedom, high-energy QCD is amenable to a perturbative treatment. On the other hand, low-energy (large distance) physics is strongly coupled and therefore remains difficult to address. Even though the problem to understand, e.g., confinement remains unsolved, QCD has at least never made any predictions that could be clearly falsified in a simple laboratory experiment, in contrast to what we have reviewed about early string theory. So, in spite of its intriguing success with hadronic resonances, string theory retreated from the area of strong interactions, and it even disappeared from physics before reemerging as a quantum theory of gravity.

1.2 Closed Strings and Supergravity

Before we can fully appreciate the role of closed string theory as a natural host for gravity, we would like to briefly recall the basic problem one faces when applying perturbation theory to quantize EINSTEIN's theory of gravity. In order to see the issue, let us compare the following two amplitudes:

$\mathcal{A}_0 =$, $\mathcal{A}_1 =$  . (1.10)

The second amplitude comes weighted with NEWTON's gravitational constant G_N. Therefore, by a simple dimensional analysis, we conclude that the dimensionless quotient of $\mathcal{A}_1/\mathcal{A}_0$ must be given by

$$\frac{\mathcal{A}_1}{\mathcal{A}_0} \sim \frac{G_N E^2}{\hbar c^5} , \tag{1.11}$$

i.e., it is proportional to E^2, a behavior that also follows directly from the fact that the gravitational interaction is mediated by a massless particle of spin $J = 2$ ($s^2/t \sim E^2$). From this formula it is clear that quantum corrections are divergent for

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