



PROCEEDINGS OF THE CONFERENCE OF GLOBAL CHINESE SCHOLARS ON HYDRODYNAMICS

全球华人水动力学学术会议论文集

Editor-in-Chief: Zhu De-xiang

Zhou Lian-di

Yang Xian-cheng

SHANGHAI UNIVERSITY PRESS

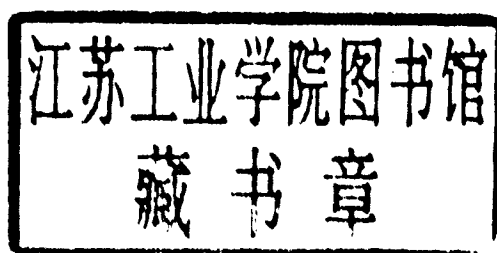
上海大学出版社

全球华人水动力学学术会议论文集

**PROCEEDINGS OF THE CONFERENCE
OF GLOBAL CHINESE SCHOLARS
ON HYDRODYNAMICS**

Editor-in-Chief

Zhu De-xiang
Zhou Lian-di
Yang Xian-cheng



上海大学出版社

Shanghai University Press

图书在版编目 (CIP) 数据

全球华人水动力学学术会议论文集/朱德祥, 周连第,
杨显成主编. —上海: 上海大学出版社, 2006. 7

ISBN 7-81058-978-4

I. 全... II. (1)朱... (2)周... (3)杨... III 水动力
学—学术会议—文集—英文 IV. TV131.2-53

中国版本图书馆 CIP 数据核字 (2006) 第 075177 号

Responsible Editors:

Wang Yue-sheng (王悦生)

Zhao Gang (赵 钢)

Cover Designer:

Ke Guo-fu (柯国富)

©Copyright 2006, Shanghai University Press

Published by Shanghai University Press, Shanghai, China

ISBN 7-81058-978-4/TV·001

Price: 150.00

Technical Advisory Committee

Chairman:

DONG Shitang (Wuxi, China)

Members:

BIAN Yingui	(Beijing, China)	SONG Charles C.S	(Minnisota, U.S.A.)
CAI Shutang	(Shanghai, China)	WU Deming	(Harbin, China)
CHEN Yaosong	(Beijing, China)	WU Theodore Y.	(Cambridge, U.S.A.)
CHIEN Weizang	(Shanghai, China)	WU Xiuheng	(Wuhan, China)
CHWANG Allent T.	(Hong Kong, China)	XU Zhenfang	(Beijing, China)
DAI Yishan	(Harbin, China)	XU Xieqing	(Beijing, China)
HE Yousheng	(Shanghai, china)	YAN Kai	(Nanjing, China)
HUI Waihow	(Hong Kong, China)	ZHAN Zhaoshun	(Beijing, China)
LIANG Zaichao	(Wuhan, China)	ZHAO Wenqian	(Chengdu, China)
LIN Chia-chiao	(Cambridge, U.S.A.)	ZHENG Qingcun	(Beijing, China)
LING Sung-ching	(Silver Spring, U.S.A.)	ZHOu Heng	(Tianjin, China)
LIU Gaolian	(Shanghai, China)	ZHUANG Lixian	(Hefei, China)
MEI Chiang C.	(Cambridge, U.S.A.)	OU Shan-hwei	(Tainan, China)
QIU Dahong	(Dalian, China)		

Organizing Committee

Chairman:

WU Yousheng (Wuxi, China)

Vice Chairmen:

DAI Shiqiang	(Shanghai, China)
DONG Shitang	(Wuxi, China)
MIAO Guoping	(Shanghai, China)
ZHU Dexiang	(Shanghai, China)

Members:

CAI Daming	(Wuxi, China)	DAI Huihui	(Hong Kong, China)
CHEN Hamnching	(Texas, U.S.A.)	DUAN Wenyang	(Harbin, China)
CHEN Xiaobo	(Paris, France)	HUANG Guangwei	(Tokyo, Japan)
CHENG Liang	(West Australia, Australia)	HUANG Shan	(Glasgow, U.K.)
CUI Weicheng	(Wuxi, China)	HUAI Wenxin	(Wuhan, China)
CUI Haiqing	(Daqing, China)	JIANG Tao	(Duisburg, Germany)

LI Ding	(West Lafayette, U.S.A.)	TAO Mingde	(Shanghai, China)
LI Jiachun	(Beijing, China)	WAN Decheng	(Dortmund, Germany)
LI Qiusheng	(Hong Kong, China)	WANG Daozeng	(Shanghai, China)
LI Yoksheung	(Hong Kong, China)	WANG Qianxi	(Singapore)
LI Yucheng	(Dalian, China)	WU Guoxiong	(London, U.K.)
LIN Bingyao	(Hangzhou, China)	WU Jianhua	(Nanjing, China)
LIN Jianzhong	(Hangzhou, China)	XU Kun	(Hong Kong, China)
LIN Pengzhi	(Singapore)	XU Li	(Beijing, China)
LIU Hua	(Shanghai, China)	YAN Kai	(Wuxi, China)
LIU Pengfai	(St. John's, Canada)	YANG Chi	(Fairfax, U.S.A.)
LIU Philip L.F.	(Ithaca, U.S.A.)	YIN Hongjun	(Daqing, China)
LIU Yingzhong	(Shanghai, China)	YU Xiping	(Beijing, China)
LIU Yulu	(Shanghai, China)	YUAN Yeli	(Qingdao, China)
LIU Zhaorong	(Shanghai, China)	ZHAN Jieman	(Guangzhou, China)
LU Chuangjing	(Shanghai, China)	ZHANG Hongwu	(Beijing, China)
LU Weizhen	(Hong Kong, China)	ZHANG Ruikai	(Nanjing, China)
LU Xiyun	(Hefei, China)	ZHAO Rong	(Trondheim, Norway)
MOK Kaimeng	(Macau, China)	ZHENG Xin	(London, U.K.)
NG Chiuon	(Hong Kong, China)	ZHU Songping	(Wollongong, Australia)
QIAN Yuehong	(Princeton, U.S.A.)	ZOU Zaojian	(Shanghai, China)
QIAO Fangli	(Qingdao, China)	ZU Qihua	(Nanjing, China)
SHEN Hongcui	(Wuxi, China)	XueNong Chen	(Karlsruhe, Germany)
TANG Hansong	(Seattle, U.S.A.)		

Secretary General:

ZHOU Liandi (Shanghai, China)

PREFACE

When we are entering the 21st century, along with the growth of global economy, especially in the variety of fields, such as naval architecture, ocean and coastal engineering, hydraulic and hydropower engineering, environment engineering, agriculture engineering, biochemical and biomedical engineering, the interest in the science and technology of hydrodynamics, one of the most important branches of theoretical and experimental fluid mechanics, grows faster than ever before. Great progress in the theoretical, computational and experimental hydrodynamics was achieved during the past decades by the efforts of the scientists and specialists working in the field all over the world, among which contributions made by Chinese scholars at home and abroad have attracted even more prominent attentions.

Chinese nation is of the long history of 5000 years, in which there were full of inspiring and prosperous cultural and scientific achievements. Dated back hundreds years ago, the famous Du-Jiang weir of Li Bing in Sichuan, the great freight of Zheng He sailing round the vast oceans were two of the brilliant examples of Chinese innovative achievements in the field of hydrodynamics. In the new era of a developing China, the Three Gorges Dam and Hydropower Station, the blooming shipbuilding industry reaching the third largest one in the world since 1996 etc., to just mention a few, are the examples of the achievements in research and applications of modern hydrodynamics.

To encourage the exchange of knowledge and to promote the levels of research, development and application of hydrodynamics in multidisciplinary fields for better fitting in the needs of the rapid economy growth, late Professor Gu Maoxiang and a group of specialists, representing all those in the country working in the field of hydrodynamics, initiated the idea, set up the editorial board and started the publication of "*Journal of Hydrodynamics*" in 1986. Since then for 19 years, organized by the Editorial Board of the Journal, the annual "*National Conference on Hydrodynamics*" has been held in series in the mainland of China, Hong Kong and Macao. The successful journal and the annual conferences have been widely and warmly responded and strongly supported by Chinese scholars at home and abroad. This makes one of the most vivid activities of scientific and technical exchange in China. At present over 30 famous over sea Chinese scholars are the members of the Editorial Board of the Journal. In 1994, the Editorial Board of the Journal also initiated the "*International Conference on Hydrodynamics*". It is now an international conference held worldwide every two years under the supervision of well-known scholars and organizations from different countries, regions and nations.

On the occasion of celebrating the 20th anniversary of the *Journal of Hydrodynamics*, the **Conference of Global Chinese Scholars on Hydrodynamics** is held to provide an opportunity for gathering together the Chinese scholars from different parts of the world to present their latest research achievements and to stimulate the discussions of important topics in the relevant areas of hydrodynamics.

This conference is jointly organized and sponsored by the **Editorial Board of *Journal of Hydrodynamics***, **Chinese Society of Theoretical and Applied Mechanics**, **Chinese Society of Naval Architects and Marine Engineers**, **Chinese Hydraulic Engineering Society**, **Chinese Society of Oceanography**, and also supported by China Ship Scientific Research Center, Shanghai Jiaotong University, Shanghai Institute of Applied Mathematics and Mechanics, the First Institute of Oceanography of the State Ocean Administration, Shanghai Society of Theoretical and Applied Mechanics, Daqing Petroleum Institute and National Laboratory of Hydrodynamics, China.

The present proceedings contain 87 papers being presented at the conference. You may find from the contents and authors of the papers that the conference will surely be creative and productive, and the meeting will definitely be a great event for Chinese scholars. On behalf of the Organizing Committee I would like to thank all the participants for their great contributions to the successful conference, and all the supporting organizations listed above.

Had been devoted for nearly half century to the research and development of hydrodynamics in China, Professor Gu Maoxiang left us 10 years ago. By holding this conference we also deeply cherish the memory of late Professor Gu for his great technical achievements and outstanding personality.

Wu You-sheng

Chairman of CSH'06 Organizing Committee

July 8, 2006, Wuxi, China

CONTENTS

Keynote Lectures

Long-period oscillations in a harbor excited by broad-banded random sea waves CHEN Mengyi, MEI Chiang-chung	3
Boundary layer flows under solitary wave LIU Philip L.-F., PARK, Yong Sung	9
Hydrodynamic problems associated with construction of sea-crossing bridges LIU Hua	13
Origin of vortical action as the fabric of the universe LING Sung-Ching, PAO Hsien-Ping	19
Recent progress in dynamics of boundary layer transition LEE Cun-biao LI Rui-qu	26
Lattice Boltzmann models for hydrodynamics Qian Yue-hong	31
Numerical simulation of aquatic eco-environment of Bo Hai Bay Tao Jian-hua	34

Session A1

Exact variational formulation of free-surface gravity flow around hydrofoil accounting for surface tension LIU Gao-lian	45
Nuclear Solitary Wave Xue-Nong Chen, Werner Maschek	49
Finding multiple solutions of nonlinear problems by means of the homotopy analysis method Shijun LIAO	54
Dispersion in open-channel flow subject to the processes of sorptive exchange on the bottom and air-water exchange on the free surface NG Chiu-on	57
The effect of temperature distribution on the mass species transport in the micro-channel driven by electroosmosis ZHANG Kai, LIN Jiang-zhong	65

Session B1

The gas-kinetic scheme for shallow water equations XU Kun	73
---	----

Numerical analysis of the effects of sinker weight on the hydrodynamics behaviour of gravity cage net in uniform flow 77

LI Yucheng, ZHAO Yunpeng, GUI Fukun, TENG Bin, GUAN Changtao

An efficient computational method for nonlinear three-dimensional wave-wave and wave-body Interaction 84

Hongmei Yan, Yu-ming Liu, Dick K.P. YUE

Boussinesq-type modeling in surf zone using mesh-less least-square-based finite difference method 89

WANG Ben-long, ZHU Yuan-qing, SONG Zhi-ping, LIU Hua

Modeling of liquid-solid flows with large number of moving particle by multigrid fictitious boundary method 93

Decheng Wan, Stefan Turck

Session A2

The Lie-group shooting method for boundary layer equations in fluid mechanics 103
CHANG Chih-wen, CHANG Jiang-ren, LIU Chein-shan

The coagulation and re-separation process of particles in a slow viscous flow 109
SUN Ren

A unified framework for incompressible and compressible fluid flows 113
Ding Li, Charles L. Merkle

Euler/Euler theory and application of fluid-particle two-phase jet 120
WANG Ming-bo, WANG Rui-he

Session B2

Simulations of interactions between nonlinear waves and multi or an array of cylinders 127
C. Z. Wang, G. X. Wu

The numerical simulation of local scour in front of a vertical-wall breakwater 134
CHEN Bing

A Finite element solution of wave forces on a horizontal circular cylinder close to the sea-bed 139
ZHAO Ming, CHENG Liang, AH Hongwei

Numerical study on flow and dispersion in urban street canyons of asymmetrical configurations 146
WANG Jia-song, HUANG Zhen

Application of a three-point explicit compact difference scheme to the incompressible Navier-Stokes equations 151

LIN Jian-guo, XIE Zhi-hua, ZHOU Jun-tao

Session A3

Modelling of the wave-current interaction in the pearl river estuary 159
WANG Chong-hao, WAI Wing-hong, Onyx, LI Yok-sheung, CHEN Yong

The wave forces and moments on a floating rectangular box in a two-layer fluid SHI Qiang, YOU Yun-xiang, WEI Gang, MIAO Guo-ping	166
Gravity waves with effect of surface tension and fluid viscosity CHEN Xiao-bo, DUAN Wen-yang, LU Dong-qiang	171
Unsteady waves due to oscillating disturbances in an ice-covered fluid LU Dong-qiang, LE Jia-chun, DAI Shi-qiang	177
A New Model of Blowing and Suction Based on Matmatics Roughness Function Gao Lijin, Zhou Lian-di, Zhu Dexiang	181

Session B3

Numerical simulation of the oscillatory flow around two cylinders in tandem AN Hong-wei, CHENG Liang, ZHAO Ming, DONG Guo-hai	191
Numerical simulation on the water - sediment characteristics and foreshore evolution of the Lingding Estuary BAO Yun, ZHANG Wenyan	198
Numerical simulation of sediment transport near sea bed under waves ZHOU Ze-xuan, LIN Peng-zhi	203
Numerical simulation of eclipsed form arrangement of water intake-outlet project in power plant YING Bo-fen, CHENG You-liang, GUAN Ying	210
Numerical simulation of microbubble flow around an axisymmetric body WU Cheng-sheng, HE Shu-long, ZHU De-xiang, GU Min	217

Session A4

Turbulent flow prediction around appended hulls G.B. Deng, P. Queutey, M. Visonneau	225
Non-Gaussian self-similarity in the inertial range of turbulences QIAN Jian	232
Flow and sediment simulation around spur dike with free surface using 3-D turbulence model Zhangfeng CUI, Xiao-feng ZHANG	237
The wavelet analysis on the relation of the wall pressure fluctuating and turbulent coherent structures XIE Hua, LI Wan-ping	245
Numerical simulation of pulsatile blood flow through asymmetric arterial stenoses under EECP DU Jian-hang, WU Chun-liang, ZHENG Zhen-sheng, DAI Gang	251

Session B4

Large-eddy simulation for urban micro-meteorology Zhengdong XIE, Ian P. CASTRO	259
--	-----

Numerical research on the hydrodynamic stability of blasius flow with spectral method XIE Ming-liang, XIONG Hong-Bing, LIN Jian-Zhong	265
LES for near field wakes behind junction of wing and plate Li Jiang, CAI Jian-gang, LIU Chao-qun	270
The computation approach for water environmental capacity in tidal river network YAO Yi-jun, YIN Hai-Long, LI Song	275
Boundary element method for average pressure calculation of complex boundary reservoir YIN Hong-jun, LIU Zhen-lin, FU Chun-quan, WANG Jin	280

Session A5

A Precision Water-Level and Sediment Monitoring System - An update Hsien P. Pao, Sung C. Ling	287
Propulsive mechanism of forward and backward free swimming of angulla angulla HU Wen-rong, TONG Bin-gang	291
Characterization of flood inundation in Sanjo city, Japan on July 13, 2004 HUANG Guangwei	295
Critical shear stress for deposition of cohesive sediments in Mai Po CHAN, W Y, WAI, Onyx W H, LI, Y S	300
The evolution equation for the second-order internal solitary waves in stratified fluids of finite depth CHENG Youliang, Fan Zhongyao	306

Session B5

Application of CFD in ship engineering design practice and ship hydrodynamics ZHANG Zhi-rong, LIU Hui, ZHU Song-ping, ZHAO Feng	315
Design and optimization method for a two-dimensional hydrofoil Ching-Yeh Hsin, Jia-Lin Wu and Sheng-Fong Chang	323
Numerical simulation of internal waves excited by a submarine moving in the two-layer stratified fluid CHANG Yu, ZHAO Feng, ZHANG Jun, HONG Fang-Wen, LI Ping, YUN Jun	330
Hydrodynamic optimization of performance of blunt ships PAN Xiao-qiang, SHEN Qing, CHEN Xu-jun, SUN Hong-cai	337
An investigation on oil/water separation mechanism inside helical pipes ZHANG Jun, GUO Jun, GONG Dao-tong, WANG Li-yang, TANG Chi, ZHENG Zhi-chu	343

Session A6

Fluctuation of the front propagation speed of developed gravity current K. K. LEONG, K. M. MOK, HARRY YEH	351
---	-----

Cavitation resonance: the phenomenon and unknowns Li S C, Wu Y L, Dai J, Zuo Z G, Li S	356
Stability and dynamics of surfactant-covered liquid ultrathin films with slippage Guohui HU	363
The Lie-group shooting method for steady-state Burgers equation with high Reynolds number LIU Chein-shan, CHANG Jiang-Ren, CHANG Chih-Wen	367
Study on cavitation flows of underwater vehicle MA Cheng, JIA Di, QIAN Zheng-fang, FENG Ding-hua	373

Session B6

Effects of Reynolds stress on flow noise from vortex/hydrofoil interactions Z. C. ZHENG, Y. XU	381
Hydrodynamics in deepwater TLP tendon design WANG Tao, ZOU Jun	386
Studies on mechanism of sand removal from crude oil GUO Jun, GONG Dao-tong, ZHANG Jun, WANG Li-yang, ZHENG Zhi-chu, LI Ke-ming	394
Investigation on separation efficiency of liquid/solid hydrocyclone WANG Li-yang, ZHENG Zhi-chu, GUO Jun, ZHANG Jun, TANG Chi	400
Pressure drop model for gas-liquid two-phase flow and its application in underbalanced drilling PING Li-qiu, WANG Zhi-ming, WEI Jian-guang	405

Session A7

An unstructured-grid based volume-of-fluid method for extreme wave and freely-floating structure interactions YANG Chi	415
Analyses of the problems of multicolored wave diffraction by porous circular dock and disk HUANG Hua, ZHAN Jie-min	423
Closed-loop control of vortex-airfoil interaction noise ZHANG MING-ming, CHENG Li, ZHOU Yu	430
Hydrodynamics in a diamond-shaped fish school DENG Jian, SHAO Xue-ming	438
Smoothed particle hydrodynamics modeling of free surface flow Hongbing XIONG, Lihua CHEN, Jianzhong LIN	443

Session B7

Experimental study on turbulent characteristics of flow over sand rippled bed BAI Yu-chuan, XU Dong	449
---	-----

The influence of installation conditions of heat meters on interior fluid field and flux measurement accuracy DU Guang-sheng, LIU Li-ning, LI Li, LIU Zheng-gang, LIU Yong-hui, MA Yong-kun, MENG Liang	455
Experimental measurements of turbulent boundary layer flow over a square-edged rib KE Feng, LIU Ying-zheng, JIN Chun-yu, WANG Wei-zhe	461
The scaling and self-similarity of the velocity and concentration profiles of a pollutant-contained impinging jet in crossflow FAN Jing-yu, ZHANG Yan, WANG Dao-zeng	465
Measuring bubble size in aerated flow CHEN Xian-pu, SHAO Dong-chao	470

Session A8

A dynamic model for cause of formation of per current ZHANG Qing-hua, FAN Hai-mei, QU Yuan-yuan	477
A fictitious domain method for particulate flows YU Zhaosheng, SHAO Xueming, WACHS Anthony	482
Stochastic study of cavitation bubbles near boundary wall LI S, ZUO Z G, LI S C	487
Mathematical modeling of fluid flows for underwater missile Launch CHEN Yong-sheng, LIU Hua	492
Numerical Simulation for Green Water Occurrence ZHU Renchuan, LIN Zhaowei, MIAO Guoping	498

Session B8

Experimentgal studies on dragon wash phenomena H. W. PENG, H. J. YUAN, D. J. WANG, S. Z. CHEN, C. B. LEE	507
Shock action of jet flow from slit-type buckets on the botton of the trough WU Wen-ping, ZHANG Xiao-hong, ZHANG Hai-long, TIAN Jia-ning	511
Numerical Simulation of Atomized Flow Diffusion in Deep and Narrow Goeges LIU Shi-he, YIN Shu-ran, LUO Qiu-shi, ZHOU Long-cai	515
The Role of Surface Waves in the Ocean Mixed Layer Fangli QIAO, Guohong FANG, Changshui XIA, Yongzeng YANG, Jian MA, Yeli YUAN	519
Validation of RANS predictions of open water performance of a highly skewed propeller with experimental data LI Da-Qing	520
Study on Improved BP Artificial Neural Networks in Eutrophication Assessment of China Eastern Lakes JIANG Yaping, XU Zuxin, YIN Hailong	528

Keynote Lectures



LONG-PERIOD OSCILLATIONS IN A HARBOR EXCITED BY BROAD-BANDED RANDOM SEA WAVES

CHEN Mengyi, MEI Chiang-chung

Department of Civil and Environmental Engineering, Massachusetts Institute of Technology, Cambridge, MA, 02139, USA E-mail : ccmei@mit.edu

KEY WORDS: Random sea waves, nonlinear water waves, nonlinear resonance, harbor resonance, nonlinear scattering, nonlinear diffraction, slow-drift oscillations

1. Introduction

In designing the dynamic positioning systems for floating oil rigs or the mooring systems for ships and tankers, it is important to predict their oscillations at long periods in the range of one to several minutes. Forced oscillations in this period range can excite natural modes of the mooring system, and cause excessive strain and breakage of mooring lines. Semi-enclosed harbors also possess natural modes of long periods. If resonance occurs, strong current appears at the narrow entrance, creating hazards for small boats. Excessive sways of tankers or large vessels inside a harbor can slow down the loading and unloading operations, resulting in economic losses. Since many of the affected harbors are free from the threat of long-period tsunamis, but are often attacked by storm-generated short waves, harbor oscillations at long periods can only be induced by nonlinearity. Past theories on this topic are limited to narrow-banded and deterministic incident waves^[1,2,3]. A new theory for broad-banded random waves is needed.

For two-dimensional standing waves in deep water, Sclavounos^[4] has advanced a systematic stochastic theory for calculating the nonlinear contributions to the wave spectrum. He showed that the nonlinear contribution starts at the fourth order in wave steepness, and involves the frequency responses at first, second and third orders. We shall first extend his theory to more complex waves involving scattering, and then point out that long-period harbor spectra can be predicted by using just the second-order frequency response. Aided by our recent work on second-order mild-slope approximation^[5], calculation of the nonlinear spectrum is greatly simplified. Sample numerical

results are discussed for a square harbor of constant depth.

2. The Second-Order Problem

For waves of length comparable to depth, the flow field is fully three-dimensional. Let us expand the velocity potential and free-surface height in powers of the wave steepness $\varepsilon = O(kA)$

$$\Phi = \Phi_1 + \Phi_2 + \dots, \quad \zeta = \zeta_1 + \zeta_2 + \dots \quad (1)$$

where the subscripts indicate the order of magnitude in powers of ε . Let the horizontal and three-dimensional gradient operators be distinguished by ∇ and ∇_3 respectively. At each order, Φ satisfies the Laplace equation

$$\left(\nabla^2 + \frac{\partial^2}{\partial z^2} \right) \Phi_j = 0, \quad j = 1, 2, 3, \dots \quad (2)$$

in the fluid, and

$$\frac{\partial \Phi_j}{\partial z} = \nabla h \cdot \nabla \Phi_j, \quad z = -h(x, y) \quad (3)$$

$j = 1, 2, 3, \dots$ on the sloping seabed. All lateral boundaries are assumed for simplicity to be vertical throughout the depth, hence

$$\frac{\partial \Phi_j}{\partial n} = 0, \quad j = 1, 2, 3, \dots \quad (4)$$

where the unit normal \vec{n} is in the horizontal plane. On the still water surface, $z = 0$, the boundary condition for the first-order potential is homogeneous

$$g \frac{\partial \Phi_1}{\partial z} + \frac{\partial^2 \Phi_1}{\partial t^2} = 0, \quad z = 0. \quad (5)$$

while that for the second-order potential is not

$$g \frac{\partial \Phi_2}{\partial z} + \frac{\partial^2 \Phi_2}{\partial t^2} = \frac{1}{g^2} \frac{\partial \Phi_1}{\partial t} \quad (6)$$

$$\frac{\partial}{\partial z} \left[g \frac{\partial \Phi_1}{\partial z} + \frac{\partial^2 \Phi_1}{\partial t^2} \right] - \frac{\partial}{\partial t} (\nabla_3 \Phi_1)^2, z=0$$

The free-surface displacement at the first order, ζ_1 , is related to Φ_1 by

$$\zeta_1 = -\frac{1}{g} \left[\frac{\partial \Phi_1}{\partial t} \right]_{z=0} \quad (7)$$

The second-order correction, ζ_2 , is the sum of two parts,

$$\zeta_2 = \left[\frac{1}{g^2} \frac{\partial \Phi_1}{\partial t} \frac{\partial^2 \Phi_1}{\partial t \partial z} - \frac{1}{2g} (\nabla_3 \Phi_1)^2 \right]_{z=0} \quad (8)$$

$$+ \left[-\frac{1}{g} \frac{\partial \Phi_2}{\partial t} \right]_{z=0}$$

The first part can be immediately calculated from the first-order solution, while the second depends on the second-order potential Φ_2 which is to be found.

3. Correlation and Frequency Spectrum

We define the correlation function of the total free-surface height $\zeta(x, y, t)$ by the following ensemble average

$$H(x, y, t) = \zeta(x, y, t) \zeta^*(x, y, t + \tau) \quad (9)$$

where ζ^* denotes the complex conjugate of ζ . The corresponding frequency spectrum S is its Fourier transform,

$$S(x, y, \omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} H(x, y, \tau) e^{i\omega\tau} d\tau \quad (10)$$

In the sequel, the integration limits $(-\infty, \infty)$ will be omitted for brevity. Introducing the expansion (1), the total correlation function of the free surface comprises of quadratic products (ζ_1, ζ_1) at order $O(\varepsilon^2)$, (ζ_1, ζ_2) at $O(\varepsilon^3)$, and $(\zeta_2, \zeta_2), (\zeta_1, \zeta_3)$ at order $O(\varepsilon^4)$. Since $\zeta_1, \zeta_2, \zeta_3, \dots$ involve Fourier transforms with integrands proportional to A, AA, AAA, \dots respectively, the right-hand side involves ensemble averages of even and odd products of A . Assuming A to be a Gaussian random variable, all odd products give zero averages, while all even products can be reduced to averages of quadratic

products. It follows that nonlinear corrections begin at $O(\varepsilon^4)$,

$$H(\tau) = H_2(\tau) + H_4(\tau) + O(\varepsilon^6) \quad (11)$$

where

$$H_2(\tau) = \overline{\zeta_1(t) \zeta_1^*(t + \tau)}, \quad (12)$$

$$H_4 = H_{22} + H_{13} + H_{31}$$

with

$$H_{22}(\tau) = \overline{\zeta_2(t) \zeta_2^*(t + \tau)},$$

$$H_{13}(\tau) = \overline{\zeta_1(t) \zeta_3^*(t + \tau)}, \quad (13)$$

$$H_{31}(\tau) = \overline{\zeta_3(t) \zeta_1^*(t + \tau)},$$

Note that $H_2 = O(\varepsilon^2)$ and $H_4 = O(\varepsilon^4)$. The corresponding total frequency spectrum is

$$S = S_2 + S_4 + \dots = S_2 + (S_{22} + S_{13} + S_{31}) + \dots \quad (14)$$

where S_2 is the Fourier transform of H_2 as defined by (10). The nonlinear correction is represented by the three terms in the parenthesis, where S_{22}, S_{13}, S_{31} are respectively the Fourier transforms of H_{22}, H_{13}, H_{31} .

Let the free-surface displacement of the leading-order waves be

$$\zeta_1^I(x, y, t) = \int_{-\infty}^{\infty} A(\omega) \exp(i\vec{k} \cdot \vec{r} - i\omega t) d\omega \quad (15)$$

where $\omega^2 = gk \tanh kh$. By the assumption of stationarity, it can be shown that

$$A(\omega_1) A^*(\omega_2) = S_A(\omega_1) \delta(\omega_1 - \omega_2) \quad (16)$$

where $S_A(\omega)$ is the spectrum of the first-order incident wave.

To account for diffraction by lateral boundaries of the coast and the harbor, and refraction by complex bathymetry, we express the total first-order displacement by

$$\zeta_1 = \int A(\omega) \Gamma_1(x, y, \omega) e^{i\omega t} d\omega \quad (17)$$

It is easy to show that the frequency response (transfer function) Γ_1 can be found from the linear scattering theory for monochromatic incident waves. Once Γ_1 is found by any of the existing methods, the leading-order response spectrum is

$$S_2(x, y, \omega) = S_A(\omega) |\Gamma_1(x, y, \omega)|^2 \quad (18)$$

In typical sea spectra such as JONSWAP and its extension to finite water depth (see Figure 1), S_A is