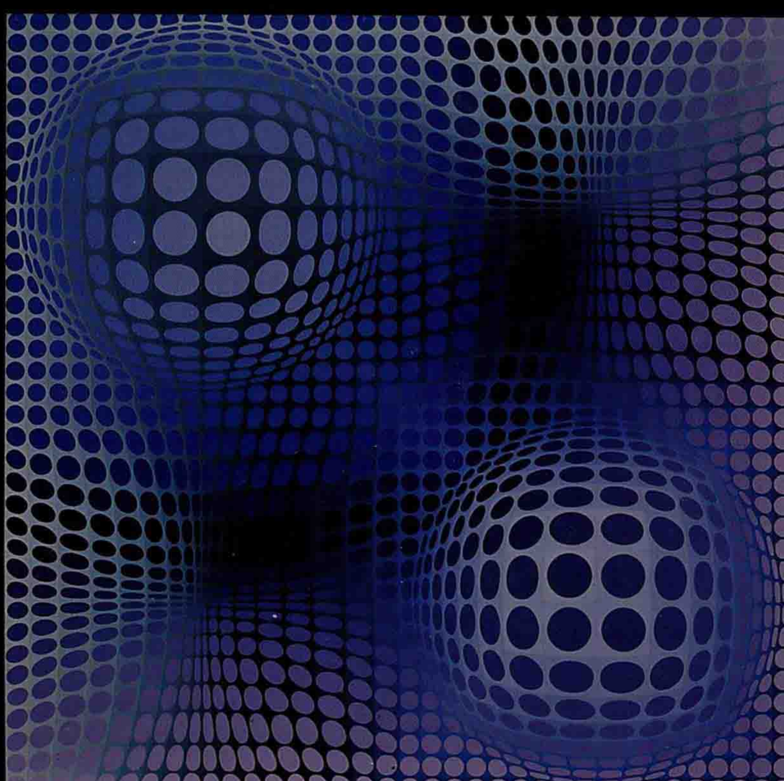
Introduction to

Optical Quantum Information Processing

Pieter Kok *and* Brendon W. Lovett



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Pieter Kok is a Lecturer in Theoretical Physics in the Department of Physics and Astronomy, the University of Sheffield. He is a member of the Institute of Physics and the American Physical Society, and his Ph.D. thesis won the Institute of Physics Quantum Electronics and Photonics thesis award in 2001.

Brendon W. Lovett is a Royal Society University Research Fellow in the Department of Materials and a Fellow of St Anne's College at the University of Oxford. He is a member of the Institute of Physics. He has been a visiting Fellow at the University of Queensland, Australia, and is an Academic Visitor at the National University of Singapore.

'The discussion of cluster state protocols and conditional optical gates is the clearest I have seen. The text includes embedded exercises for the reader, carefully constructed to illustrate important principles ... The authors make every effort to connect the abstract theory with current experimental practice ... As optics will necessarily form a part of future quantum information processing networks, this book is required reading for anyone wishing to stay abreast of the ongoing effort to make quantum computing a reality.'
Gerard J. Milburn
Centre for Quantum Computer Technology, The University of Queensland

'I have certainly enjoyed browsing through the book by Kok and Lovett ... I have no doubts that this first comprehensive and well written account of quantum information processing with optical systems will be very much welcomed and appreciated by students and experts alike ... an excellent overview of the basic insights and techniques ... done in a manner that conveys the intellectual excitement and beauty of the subject.'
Artur Ekert
Mathematical Institute, University of Oxford, and National University of Singapore

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Qubit operators

The Pauli operators:

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Single-qubit rotations:

$$U_X(\theta) = \exp(-i\theta X) = \begin{pmatrix} \cos\theta & -i\sin\theta \\ -i\sin\theta & \cos\theta \end{pmatrix}$$

$$U_Y(\phi) = \exp(-i\phi Y) = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix}$$

$$U_Z(\varphi) = \exp(-i\varphi Z) = \begin{pmatrix} e^{-i\varphi} & 0 \\ 0 & e^{i\varphi} \end{pmatrix}$$

The Hadamard H, the phase operator Φ, and the $\pi/8$ gate T:

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \qquad \Phi = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix} \qquad T = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix}$$

Important commutation relations:

$$XZ = -ZX \qquad YX = -XY \qquad ZY = -YZ$$

$$HX = ZH \qquad HZ = XH \qquad HY = -YH$$

$$\Phi X \Phi^\dagger = Y \qquad \Phi Y \Phi^\dagger = -X \qquad \Phi Z \Phi^\dagger = Z$$

Two-qubit gates:

$$\text{CX} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \; (= \text{CNOT}) \qquad \text{CZ} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

Advanced quantum mechanical relations

Von Neumann entropy:

$$S(\rho) = -\mathrm{Tr}(\rho \log_2 \rho) = -\sum_j \lambda_j \log_2 \lambda_j$$

Positive operator-valued measures of projectors P_μ:

$$\hat{E}_\nu = \sum_\mu \lambda_\mu^\nu P_\mu \quad \text{with} \quad \sum_\nu \hat{E}_\nu = \hat{\mathbb{I}}$$

In terms of Kraus operators:

$$\hat{E}_\nu = \sum_\mu \mathcal{A}_{\mu\nu} \mathcal{A}_{\mu\nu}^\dagger \quad \text{with} \quad \mathcal{A} = \sum_{\mu\nu} \alpha_\mu^\nu \, |\mu\rangle \, \langle\nu| \quad \text{and} \quad |\alpha_\mu^\nu|^2 = \lambda_\mu^\nu$$

The fidelity between two density operators:

$$F = \left[\mathrm{Tr} \left(\sqrt{\rho_2^{1/2} \, \rho_1 \, \rho_2^{1/2}} \right) \right]^2$$

The Lindblad master equation:

$$\frac{d\rho}{dt} = -\frac{i}{\hbar} [\rho, \mathcal{H}] - \sum_{n,m} \gamma_{nm} \left(\rho L + m L_n + L_m L_n \rho - 2 L_n \rho L_m \right) + \text{H.c.}$$

The Fisher information:

$$F(\theta) = \int_{-\infty}^{\infty} dx \, \frac{1}{p(x|\theta)} \left(\frac{\partial p(x|\theta)}{\partial \theta} \right)^2 = \int_{-\infty}^{\infty} dx \, p(x|\theta) \left(\frac{\partial \ln p(x|\theta)}{\partial \theta} \right)^2$$

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To Rose, Xander and Janet

Preface

The field of quantum information processing has reached a level of maturity, and spans such a wide variety of topics, that it merits further specialization. In this book, we consider quantum information processing with optical systems, including quantum communication, quantum computation, and quantum metrology. Optical systems are the obvious choice for quantum communication, since photons are excellent carriers of quantum information due to their relatively slow decoherence. Indeed, many aspects of quantum communication have been demonstrated to the extent that commercial products are now available. The importance of optical systems for quantum communication leads us to ask whether we can construct integrated systems for communication and computation in which all processing takes place in optical systems. Recent developments indicate that while full-scale quantum computing is still extremely challenging, optical systems are one of the most promising approaches to a fully functional quantum computer.

This book is aimed at beginning graduate students who are starting their research career in optical quantum information processing, and it can be used as a textbook for an advanced master's course. The reader is assumed to have a background knowledge in classical electrodynamics and quantum mechanics at the level of an undergraduate physics course. The nature of the topic requires familiarity with quantized fields, and since this is not always a core topic in undergraduate physics, we derive the quantum mechanical formulation of the free electromagnetic field from first principles. Similarly, we aim to present the topics in quantum information theory in a self-contained manner.

The book is organized as follows: in Part I, we develop the quantum theory of light, give an introduction to quantum communication and computation, and we present a number of advanced quantum mechanical techniques that are essential for the understanding of optical quantum information processing. In Part II, we consider quantum information processing using single photons and atoms. We first develop the theory of photodetection and explore what we mean by photon sources, followed by an exposition of quantum communication with single photons, quantum computation with single photons and linear optics, and quantum computing where the information carriers, the qubits, are encoded in atoms. In Part III, we explore quantum information processing in many-body systems. We revisit linear optical quantum communication and computation, but now in the context of quantum continuous variables, rather than qubits. We discuss how atomic ensembles can be used as quantum memories and repeaters, and we study in detail how to define robust qubits in solid-state systems such as quantum dots and crystal defects. The last chapter of the book deals with quantum metrology, where we explore how quantum states of light can be exploited to attain a measurement precision that outperforms classical metrology. As is inevitable in a book of this nature, a number of important topics have been omitted due to length restrictions. We have not included quantum information processing in ion traps, photonic band-gap materials, optical lattices, and Bose–Einstein condensates. We have also omitted the topic of quantum imaging.

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