

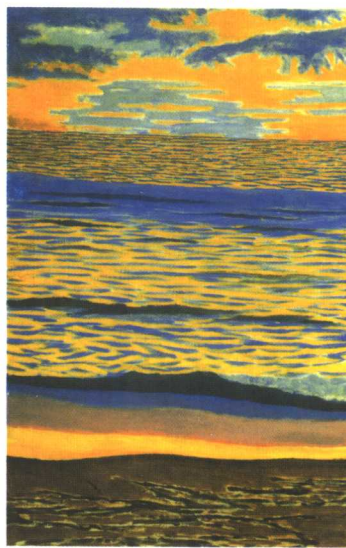
国外大学优秀教材 — 统计学系列 (影印版)

Irwin Miller, Marylees Miller

数理统计与应用

(第7版)

John E. Freund's
Mathematical Statistics
with Applications
Seventh Edition



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数理统计与应用

（第7版）

**John E. Freund's Mathematical
Statistics With Applications
(Seventh Edition)**

Irwin Miller
Marylees Miller

清华大学出版社
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出版说明

从科学研究、工农业生产、新产品开发、产品质量的提高到政治、教育、社会科学等各个领域,使用统计方法和不使用统计方法获得的结果是大不相同的。只要统计方法使用得当,就能够得到事半功倍的效果。这也是统计学能在经济发达国家兴旺发达的原因。

在我国的高等教育中,统计学的教学内容和国外先进水平相比还有一定的差距,统计学的研究和应用还没有得到足够的重视,统计学的方法还不为更多的应用工作者所了解。为了尽快地改进统计学的教学内容,促进统计学在我国的研究和应用,我们将陆续引进一批在国外知名大学受到普遍欢迎的统计学教材,希望对我国统计学的高等教育、科学研究和应用做出一点贡献。

这批教材的引进将遵循以下的基本原则:适合作为我国大学本科或研究生统计学课程的教材或主要教学参考书;原则上是近年国际上出版的最新图书或新版图书;对于某些基础课程,选择不同体系、不同风格和不同层次的教材,以满足不同层次和不同学时的需要。每本入选教材都有国内相应领域知名专家或资深教授的审阅和推荐。对于部分教学需要量较大的教材还将考虑用翻译的方式引进。统计学系列教材将分期分批出版,内容将覆盖统计学专业大多数基础课和选修课。

概率论是和统计学密切相关的学科,也是学习研究统计学的基础,所以概率论方面的教材也是这次引进的重点。我们将注重引进一批应用性强,和统计学联系紧密的优秀概率论教材,以满足各方面的需要。

本系列教材的内容将包含概率统计、随机过程、Bayes 统计、多元统计、数据分析等,同时兼顾金融经济统计、社会统计、商业统计、生物统计、质量控制、实验设计、市场调查等等。系列教材的读者是统计学专业及需要学习统计学课程和知识的其他各专业的本科生、研究生以及从事与概率统计学科相关工作的科研工作者和应用工作者。

我们希望本系列教材的引进能够对我国统计学的高等教育、科学研究和实际应用有所帮助;也希望得到广大读者的反馈意见,以便改进我们的工作。

何书元
北京大学数学学院

John E. Freund's Mathematical Statistics with Applications (Seventh Edition)

影印版序

本书是 Irwin Miller 和 Marylees Miller 所著的“John E. Freund's Mathematical Statistics with Applications”一书的最新版本(第 7 版),是专门为使用概率统计较多的理工科领域的大学生和研究生撰写的有关统计推理的理论、思维和方法的教材。

本书设计为一个学年的教学内容,包括了工科概率论入门和经典统计的基础部分,其具体内容为: 1. 引言; 2. 概率; 3. 概率分布与概率密度; 4. 数学期望; 5. 特殊概率分布; 6. 特殊概率密度; 7. 随机变量的函数; 8. 抽样分布; 9. 判决理论; 10. 估计: 理论; 11. 估计: 应用; 12. 假设检验: 理论; 13. 假设检验: 应用; 14. 回归与相关; 15. 方差分析; 16. 非参数检验。

本书理论难度适中,覆盖面比当前国内的中文教材大。学习本书只需具有初等微积分与线性代数等数学知识即可。与当前国内一般理工科流行教材中的统计部分相比,本书具有以下特点:

(1) 在理论上要深一些,包括了判决理论、博弈理论、Neyman-Pearson 理论、似然比检验、计数数据的统计、Bayes 统计和非参数统计等内容。

(2) 论述深度的把握与发展较为合理。例如,在估计方法中介绍了有效性、充分性、稳健性等理论概念。

(3) 应用面更为丰富,统计思想的阐述与算法更为具体。本书在正文与习题中引进了近代统计技术和大量在各个领域中的应用性例子,并通过统计软件 Minitab 利用计算机进行数值计算。

本书语言平实浅显,适合于理科、系统理论、控制工程、电机工程、机械工程、经济管理等方面的大学生或研究生作为教材,也可以作为广大理工科有关概率统计的教学参考书或供有关领域的决策人员、教师、工程师、技术人员作自学之用。

龚光鲁
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PREFACE

The seventh edition of John E. Freund's *Mathematical Statistics*, like the first six editions, is designed primarily for a two-semester or a three-quarter calculus-based introduction to the mathematics of statistics. It can be used, however, for a single-semester course, emphasizing probability, probability distributions and densities, sampling, and classical statistical inference. For this purpose, the authors recommend that the course be based on Chapters 1–6, 8, 11, and 13. In addition, Sections 2.8, 4.8, 5.8, 5.9, 6.7, 8.7, and 13.8 may be omitted. In teaching this abbreviated course, the instructor may facilitate fitting the material into the time allotted by choosing several other sections to de-emphasize.

The major departure in this edition is the addition of a section at the end of each chapter, called "The Theory in Practice," and dealing in greater depth with some of the applications of the theory. The applied exercises in each chapter have been grouped together at the end of this new section. Subheadings have been supplied to indicate which exercises go with which section or sections to aid the instructor in assigning homework exercises.

Many students taking this course are experiencing the ideas of statistics for the first time. It is believed that it will be helpful for them to spend some time learning how the mathematical ideas of statistics carry over into the world of applications. To emphasize this new treatment, the authors have added "with Applications" to the title of the text. In addition, further emphasis has been placed on the use of computers in performing statistical calculations. Several new exercises have been added, many of which require the use of a computer. New material has been added to Chapter 15, placing additional emphasis on experimental design and factorial experiments.

We would like to express our appreciation to the Robert E. Krieger Publishing Company for permission to base Table II on E. C. Molina's Poisson's *Exponential Binomial Limit*; to Prentice Hall, Inc., for permission to reproduce part of Table IV from R. A. Johnson and D. W. Wichern's *Applied Multivariate Statistical Analysis*; to Professor E. S. Pearson and the *Biometrika* trustees to reproduce the material in Tables V and VI; to the editors of *Biometrics* for permission to reproduce the material in Table IX from H. L. Harter's "Critical Values for Duncan's New Multiple Range Test"; to the editors of *Biometrics* for permission to reproduce the material in Tables V and VI from H. L. Harter's "Critical Values for Duncan's New Multiple Range Test"; to the American Cyanamid Company to reproduce the material in Table X from F. Wilcoxon and R. A. Wilcox's *Some Rapid Approximate Statistical Procedures*; to D. A. Auble to base Table XI on his "Extended Tables for the Mann-Whitney Statistics," *Bulletin of the Institute of Educational Research at*

Indiana University; to the editor of the *Annals of Mathematical Statistics* to reproduce the material in Table XII; and to MINITAB to reproduce the computer printouts shown in the text.

The authors would especially like to express their appreciation to the reviewers of the manuscript, Johana Hardin of Pomona College, Christopher Lake of Rowan University, Jackie Miller of Drury University, and Larry Stephens of the University of Nebraska at Omaha, whose suggestions the authors found helpful in the preparation of this new edition. The authors also would like to thank the staff at Prentice Hall for their courteous cooperation in the production of this book.

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INTRODUCTION

-
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 - 1.2 COMBINATORIAL METHODS
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 - 1.4 THE THEORY IN PRACTICE
-

1.1 INTRODUCTION

In recent years, the growth of statistics has made itself felt in almost every phase of human activity. Statistics no longer consists merely of the collection of data and their presentation in charts and tables; it is now considered to encompass the science of basing inferences on observed data and the entire problem of making decisions in the face of uncertainty. This covers considerable ground since uncertainties are met when we flip a coin, when a dietician experiments with food additives, when an actuary determines life insurance premiums, when a quality control engineer accepts or rejects manufactured products, when a teacher compares the abilities of students, when an economist forecasts trends, when a newspaper predicts an election, and so forth.

It would be presumptuous to say that statistics, in its present state of development, can handle all situations involving uncertainties, but new techniques are constantly being developed and modern statistics can, at least, provide the framework for looking at these situations in a logical and systematic fashion. In other words, statistics provides the models that are needed to study situations involving uncertainties, in the same way as calculus provides the models that are needed to describe, say, the concepts of Newtonian physics.

The beginnings of the mathematics of statistics may be found in mid-eighteenth-century studies in probability motivated by interest in games of chance. The theory thus developed for "heads or tails" or "red or black" soon found applications in situations where the outcomes were "boy or girl," "life or death," or "pass or fail," and scholars began to apply probability theory to actuarial problems and some aspects of the social sciences. Later, probability and statistics were introduced into physics by L. Boltzmann, J. Gibbs, and J. Maxwell, and by this century they have found applications in all phases of human endeavor that in some way involve an element of uncertainty or risk. The names that are connected most prominently with the growth of mathematical statistics in the first half of the 20th century are those of

R. A. Fisher, J. Neyman, E. S. Pearson, and A. Wald. More recently, the work of R. Schlaifer, L. J. Savage, and others has given impetus to statistical theories based essentially on methods that date back to the eighteenth-century English clergyman Thomas Bayes.

The approach to statistical inference presented in this book is essentially the classical approach, with methods of inference based largely on the work of J. Neyman and E. S. Pearson. However, the more general decision-theory approach is introduced in Chapter 9 and some Bayesian methods are presented in Chapter 10. This material may be omitted without loss of continuity.

This book primarily is intended as a presentation of the *mathematical theory* underlying the modern practice of statistics. Mathematical statistics is a recognized branch of mathematics, and it can be studied for its own sake by students of mathematics. Today, the theory of statistics is applied to engineering, physics and astronomy, quality assurance and reliability, drug development, public health and medicine, the design of agricultural or industrial experiments, experimental psychology, and so forth. Those wishing to participate in such applications or to develop new applications will do well to understand the mathematical theory of statistics. For only through such an understanding can applications proceed without the serious mistakes that sometimes occur. The applications are illustrated by means of examples and a separate set of applied exercises, many of them involving the use of computers. To this end, we have added at the end of most chapters a discussion of how the theory of that chapter is applied in practice.

We begin with a brief review of combinatorial methods and binomial coefficients, giving material that we shall rely on in our forthcoming discussions of probability and probability distributions.

1.2 COMBINATORIAL METHODS

In many problems of statistics we must list all the alternatives that are possible in a given situation, or at least determine how many different possibilities there are. In connection with the latter, we often use the following theorem, sometimes called the **basic principle of counting**, the **counting rule for compound events**, or the **rule for the multiplication of choices**.

THEOREM 1.1. If an operation consists of two steps, of which the first can be done in n_1 ways and for each of these the second can be done in n_2 ways, then the whole operation can be done in $n_1 \cdot n_2$ ways.

Here, "operation" stands for any kind of procedure, process, or method of selection.

To justify this theorem, let us define the ordered pair (x_i, y_j) to be the outcome that arises when the first step results in possibility x_i and the second step results in possibility y_j . Then, the set of all possible outcomes is composed of the following $n_1 \cdot n_2$ pairs:

$$\begin{aligned} &(x_1, y_1), (x_1, y_2), \dots, (x_1, y_{n_2}) \\ &(x_2, y_1), (x_2, y_2), \dots, (x_2, y_{n_2}) \\ &\quad \dots \\ &\quad \dots \\ &\quad \dots \\ &(x_{n_1}, y_1), (x_{n_1}, y_2), \dots, (x_{n_1}, y_{n_2}) \end{aligned}$$

EXAMPLE 1.1

Suppose that someone wants to go by bus, by train, or by plane on a week's vacation to one of the five East North Central States. Find the number of different ways in which this can be done.

Solution The particular state can be chosen in $n_1 = 5$ ways and the means of transportation can be chosen in $n_2 = 3$ ways. Therefore, the trip can be carried out in $5 \cdot 3 = 15$ possible ways. If an actual listing of all the possibilities is desirable, a **tree diagram** like that in Figure 1.1 provides a systematic approach. This diagram shows

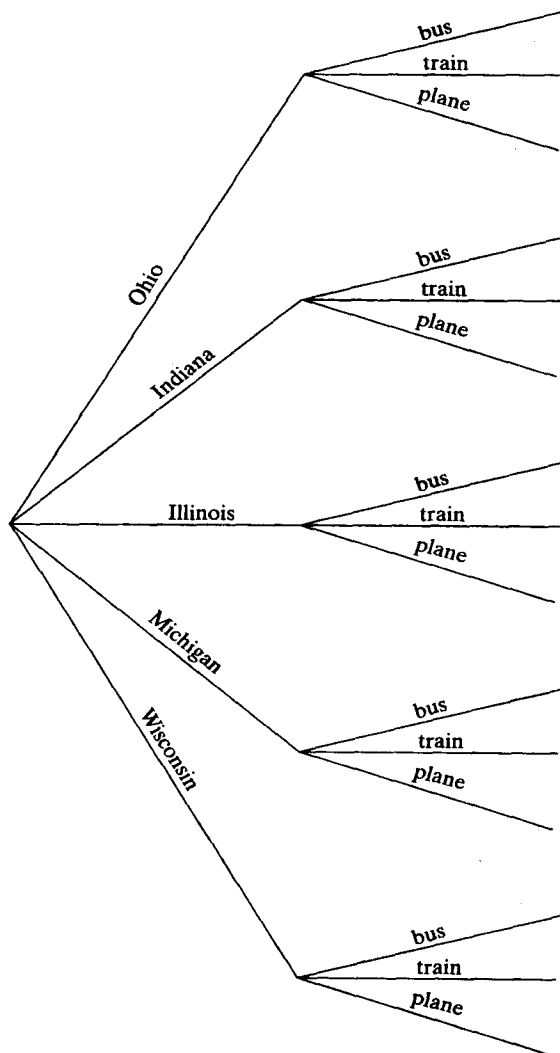


FIGURE 1.1: Tree diagram.

that there are $n_1 = 5$ branches (possibilities) for the number of states, and for each of these branches there are $n_2 = 3$ branches (possibilities) for the different means of transportation. It is apparent that the 15 possible ways of taking the vacation are represented by the 15 distinct paths along the branches of the tree. ■

EXAMPLE 1.2

How many possible outcomes are there when we roll a pair of dice, one red and one green?

Solution The red die can land in any one of six ways, and for each of these six ways the green die can also land in six ways. Therefore, the pair of dice can land in $6 \cdot 6 = 36$ ways. ■

Theorem 1.1 may be extended to cover situations where an operation consists of two or more steps. In this case,

THEOREM 1.2. If an operation consists of k steps, of which the first can be done in n_1 ways, for each of these the second step can be done in n_2 ways, for each of the first two the third step can be done in n_3 ways, and so forth, then the whole operation can be done in $n_1 \cdot n_2 \cdot \dots \cdot n_k$ ways.

EXAMPLE 1.3

A quality control inspector wishes to select a part for inspection from each of four different bins containing 4, 3, 5, and 4 parts, respectively. In how many different ways can she choose the four parts?

Solution The total number of ways is $4 \cdot 3 \cdot 5 \cdot 4 = 240$. ■

EXAMPLE 1.4

In how many different ways can one answer all the questions of a true-false test consisting of 20 questions?

Solution Altogether there are

$$2 \cdot 2 \cdot 2 \cdot 2 \cdot \dots \cdot 2 \cdot 2 = 2^{20} = 1,048,576$$

different ways in which one can answer all the questions; only one of these corresponds to the case where all the questions are correct and only one corresponds to the case where all the answers are wrong. ■

Frequently, we are interested in situations where the outcomes are the different ways in which a group of objects can be ordered or arranged. For instance, we might want to know in how many different ways the 24 members of a club can elect a president, a vice president, a treasurer, and a secretary, or we might want to know in how many different ways six persons can be seated around a table. Different arrangements like these are called **permutations**.